4.1 Since the charging current due to the capacitance of a short line may be neglected,

$$I_S = I_R = I$$

Using I as the reference vector,

$$V_S = [(V_R\cos\Phi_R + IR)^2 + (V_R\sin\Phi_R \pm IX)^2]^{1/2}$$

therefore, $\Phi_S = \tan^{-1}\dfrac{V_R\ \sin\Phi_R \pm IX}{V_R\ \cos\Phi_R + IR}$

The phasor diagram is shown below, in Figure P 2.12.

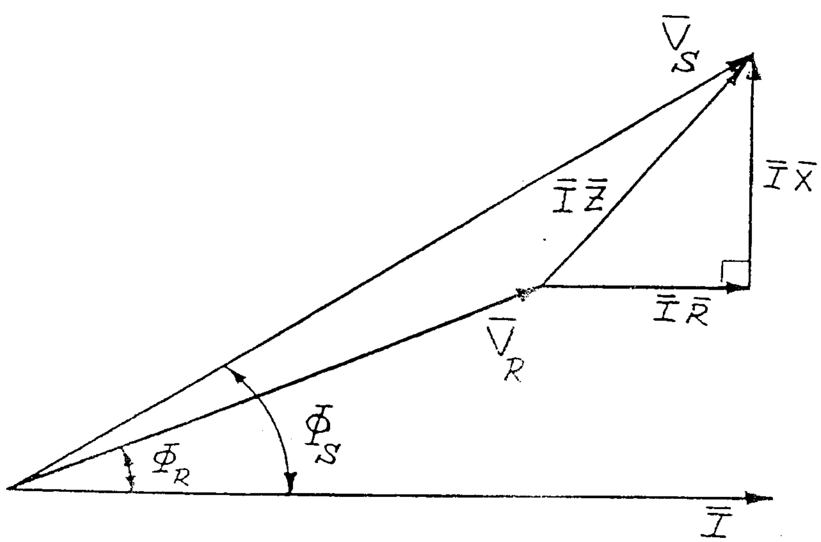

Figure P 2.12

4.2 From Table A.3, X_a = 0.458 Ω/mi/Φ, or X_a = 0.458 (15 mi) = 6.87 Ω/Φ.

From Table A. 8 , for 3 ft separation, X_d = 0.1333 Ω/mi/Φ, or X_d = 1.9995 Ω/Φ. Therefore,

$X_L = X_a + X_d = 6.87 + 1.9995 \cong 8.87\ \Omega/\Phi$

Also, $R_a = 0.311\ \Omega/\text{mi}/\Phi$, or $R_a = 4.665\ \Omega/\Phi$

Thus, $Z_L = 4.665 + j8.87\ \Omega/\Phi$

$$I_L = \frac{P_{3\Phi}}{\sqrt{3}\ V_{L-L}\cos\Phi} = \frac{10\text{x}10^6}{\sqrt{3}\ (34.5\text{x}10^3)0.9} = 185.942\ \text{A}$$

or $\bar{I}_L = 185.942\underline{/-25.84^\circ}$ A

Therefore,

$$\bar{V}_{S(L-N)} = \bar{V}_{R(L-N)} + \bar{I}_L\bar{Z}_L$$

$$= 19{,}917\underline{/0^\circ} + (185.942\underline{/-25.84^\circ})(4.665 + j8.87)$$

$$= 21{,}414 + j1103.9 = 21.442\underline{/2.95}\ \text{kV}$$

or $\bar{V}_{S(L-L)} = \sqrt{3}(21.442\underline{/2.95^\circ + 30^\circ}) = 37.149\underline{/32.95^\circ}$ kV

(b) $\cos\Phi_S = \cos[\Theta_{\bar{V}_S} - \Theta_{\bar{I}_S}] = \cos[2.95 - (-25.84)$

$$= \cos 28.74^\circ = 0.876 \text{ lagging.}$$

(c) The transmission efficiency is

$$\eta = \frac{\text{Output}}{\text{Input}}\ 100 = \frac{\sqrt{3}\ V_{R(L-L)}I_R\cos\Theta_R}{\sqrt{3}\ V_{S(L-L)}I_S\cos\Theta_S}\ 100$$

$$\eta = \frac{V_{R(L-L)}\cos\Theta_R}{V_{S(L-L)}\cos\Theta_S}\ 100 = \frac{(19.919)0.9}{(21.442)0.8768}\ 100 = 95.4\%$$

(d)

$$\%\text{VReg.} = \frac{|V_S| - |V_R|}{|V_R|}\ 100 = \frac{21.442 - 19.919}{19.919}\ 100 = 7.66\%$$

4.3 (a) From Table A.3, $X_a = 0.458\ \Omega/\text{mi}/\Phi$, or

$X_a = 0.458\ (15\ \text{mi}) = 6.87\ \Omega/\Phi$

From Table A.8 , for 3 ft separation,

$X_d = 0.1333\ \Omega/\text{mi}/\Phi$, or $X_d = 1.9995\ \Omega/\Phi$

Therefore,

$X_L = X_a + X_b = 6.87 + 1.9995 = 8.8695\ \Omega/\Phi$

Also, $R_a = 0.311\ \Omega/\text{mi}/\Phi$, or $R_a = 4.665\ \Omega/\Phi$

Thus, $Z_L = 4.665 + j8.8695\ \Omega/\Phi$

$$I_L = \frac{10\text{x}10^6}{\sqrt{3}(34.5\text{x}10^3)0.8} = 209.1849$$

or $\bar{I}_L = 209.1849\underline{/-36.87^\circ}$ A

$$\bar{V}_S = \bar{V}_R + \bar{I}_L\bar{Z}_L = 21{,}812.443 + j898.8652$$

$$= 21{,}830.9557\underline{/2.36^\circ}\ \text{V}$$

or $\bar{V}_{S(L-L)} = \sqrt{3}(21{,}830.9557\underline{/2.36^\circ + 30^\circ})$

$$= 37{,}812.3856\underline{/32.36^\circ}\ \text{V}$$

(b) $\cos\Phi_S = \cos[\Theta_{\bar{V}_S} - \Theta_{\bar{I}_S}] = \cos[2.36^\circ - (-36.87^\circ)]$

$$= 0.7746 \text{ lagging}$$

(c) $\eta = \dfrac{19{,}918.5843\text{x}0.8}{21{,}830.9557\text{x}0.7746}\ 100 = 94.23\%$

(d) $\%\text{VReg} = \dfrac{|V_{S(N-L)}| - |V_{R(F-L)}|}{|V_{R(F-L)}|}\ 100$

$$= \frac{21{,}830.9557 - 19{,}918.5843}{19{,}918.5843}\ 100 = 9.6\%$$

4.4 (a) $\bar{I}_L = 209.1849\underline{/36.87^\circ}$ A

$$\bar{V}_S = \bar{V}_R + \bar{I}_L\bar{Z}_L = 19{,}695.1012\underline{/6.03^\circ} \text{ V}$$

$$\text{or } \bar{V}_{S(L-L)} = \sqrt{3}\ \bar{V}_{S(L-N)} = 34{,}112.9159\underline{/6.03^\circ + 30^\circ}$$

$$= 34{,}112.9159\underline{/36.03^\circ} \text{ V}$$

(b) $\cos\Phi_S = \cos[\Phi_{\bar{V}_S} - \Phi_{\bar{I}_S}]$

$$= \cos[6.03 - 36.87] = \cos[-30.84] = 0.86 \text{ leading}$$

(c) $$= \frac{(34.5\text{x}10^3)(0.8)}{34{,}112.9159 \text{ x } 0.86} = 94.23\%$$

(d) $$\%\text{VReg} = \frac{19{,}695 - 19{,}918}{19{,}918}\ 100 = -1.12\%$$

Note that the voltage rise is due to the Ferranti Effect. The circuit provides for the absorption of reactive power by the series impedance I^2X, but does not provide for its supply by the shunt admittance V^2B which becomes more important as the voltage is increased, and therefore does not take into account the increase of the receiving-end voltage on light load. For longer lines at higher voltages, this voltage rise will be considerably greater. On the other hand, quite a small load is sufficient to eliminate the Ferranti Effect. Therefore, when a long line is terminated in a transformer, the magnitizing current is often enough to eliminate V_R becoming greater than V_S.

4.5 The load is connected to the secondary-side of the transformer tnat is supplied by the feeder. Thus, the transformer is connected to the receiving-end of the feeder. Since the load voltage is $V_{L(LV)}$ = 2250 V or referred to the high-voltage (i.e., primary) side of the transformer,

$$V_{L(HV)} = 2250\left(\frac{19,900}{2,400}\right) = 18,656.25 \text{ V}$$

with $\cos\Theta_L = 0.85$ leading

thus, $$S_L = \frac{200 \times 10^3}{0.85} = 235,294.12 \text{ VA}$$

Therefore, the load current at the primary-side of the transformer is

$$I_{L(HV)} = \frac{235,294.12}{18,656.25} = 12.6121 \text{ A}$$

The transformer impedance referred to the primary side of the transformer is

$$\bar{Z}_{T(HV)} = (0.24+j0.99)\left(\frac{19,900}{2,400}\right)^2 = 16.5 + j68.0642\ \Omega$$

Thus, $$\bar{Z}_{eq} = \bar{Z}_{feeder} + \bar{Z}_{T(HV)}$$

$$= (95 + j340) + (16.5 + j68.0642)$$

$$= 111.5 + j408.0642 = 423.0232\underline{/74.72^\circ}\ \Omega$$

(a) Taking $I_{L(HV)}$ as reference,

$$\bar{V}_{S(L-N)} = \bar{V}_{R(L-N)} + \bar{I}_{L(HV)}\bar{Z}_{eq}$$

where $\bar{V}_{R(L-N)} = \bar{V}_{L(HV)} = 18,656.25\underline{/-31.79^\circ}$ V thus, $\bar{V}_{S(L-N)} = 18{,}656.25(0.85 - j0.5268)$

$$+ (12.6121/\underline{0^{\circ}})(423.0232/\underline{74.72^{\circ}}$$

$$= 17{,}263.8345 - j4{,}681.5041$$

$$= 17{,}887.3269/\underline{-15.17^{\circ}}\ \text{V}$$

(b) $\bar{V}_{T(HV)} = \bar{V}_{L(HV)} + \bar{I}_{L(HV)}\bar{Z}_{T(HV)}$

$$= 18{,}656.25/\underline{-31.79^{\circ}} + (12.6121/\underline{0^{\circ}})(70.0356/\underline{76.37^{\circ}})$$

$$= 16{,}065.9126 - j8{,}969.6801$$

$$= 18{,}400.2367/\underline{-29.17^{\circ}}\ \text{V}$$

(c) $P_S = P_L + (P_{loss}$ in feeder and transformer)

$$= 200\text{x}10^3 + 17{,}735.75 = 217{,}735.75\ \text{W}$$

where $P_{loss} = I^2_{L(HV)}R_{eq}$

$$Q_S = Q_L + I^2_{L(HV)}X_{eq}$$

$$= -123{,}948.87 + 64{,}908.76 = -59{,}040.11\ \text{vars}$$

Alternatively,

$$\bar{S}_S = \bar{V}_{S(L-N)}\bar{I}^*_{L(HV)}$$

$$= (17{,}887.3269/\underline{-15.17})(12.6121/\underline{0^{\circ}})$$

$$= 217{,}735.53 - j59{,}035.03\ \text{VA}$$

Checks!

4.6 (a) $\bar{I}^* = \dfrac{\bar{S}_R}{\bar{V}_R} = \dfrac{(15 + j12)x10^6}{(115 + j0)x10^3} = 167.04\underline{/38.66^\circ}$ A

thus, $\bar{V}_S = \bar{V}_R + \bar{I}_R\bar{Z}_L = (115{+}j0)x10^3{+}(167.04\underline{/-38.66^\circ})15\underline{/90^\circ}$

$= 116{,}565.24 + j1{,}956.54 = 116{,}581.3\underline{/0.96^\circ}$ V

(b) $\bar{I}_S = \bar{I}_R = 167.04\underline{/-38.66^\circ}$ A

4.7 (a) $\bar{V}_{S(L-N)} = \bar{V}_{R(L-N)} + \bar{I}_R\bar{Z}_L$

$= 39{,}837.2\underline{/0^\circ} + (300\underline{/-30^\circ})(44.94\underline{/57.7^\circ})$

$= 52{,}150.17\underline{/6.9^\circ}$ V

$$\%VReg = \frac{|V_S| - |V_R|}{|V_R|}\,100 = \frac{52{,}150.17 - 39{,}837.2}{39{,}837.2}\,100 = 30.9\%$$

(b) $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 44.94\underline{/57.72^\circ} \\ 0 & 1 \end{bmatrix}$

(c) The phasor diagram is shown below, in Figure P2.18.

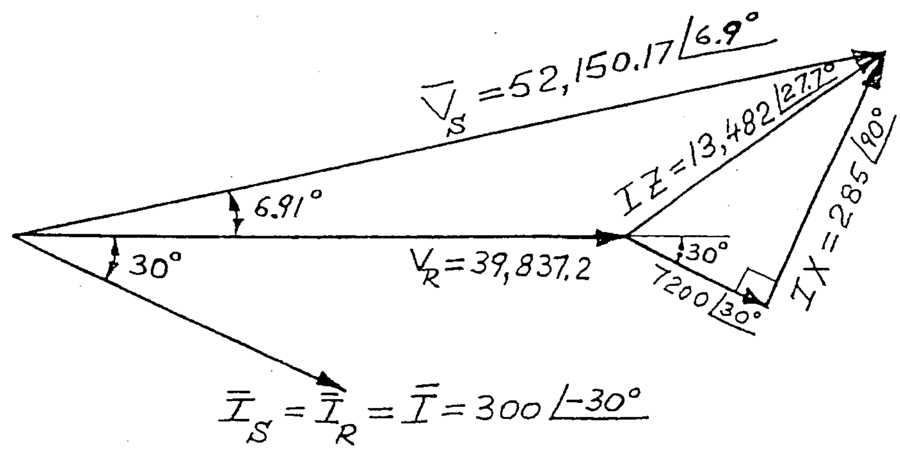

Figure P2.18

4.8 (a) $\bar{I}_R = 300\angle{-45^\circ} = 212.1 - j212.1$ A

$$\bar{V}_S = \bar{V}_R + \bar{I}_R\bar{Z} = 39,837.17 + (300\angle{-45^\circ})(44.94\angle{57.7^\circ})$$

$$= 52,989.58 + j2,968.85 = 53,072.6\angle{3.2^\circ} \text{ V}/\Phi$$

$$\%VR_{eg} = \frac{|V_S| - |V_R|}{|V_R|} 100 = \frac{53,072.6 - 39,837.17}{39,837.17} 100 = 33.22\%$$

(b) $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 44.94\angle{57.7^\circ} \\ 0 & 1 \end{bmatrix}$

(c) The phasor diagram is shown below, in Figure P2.19

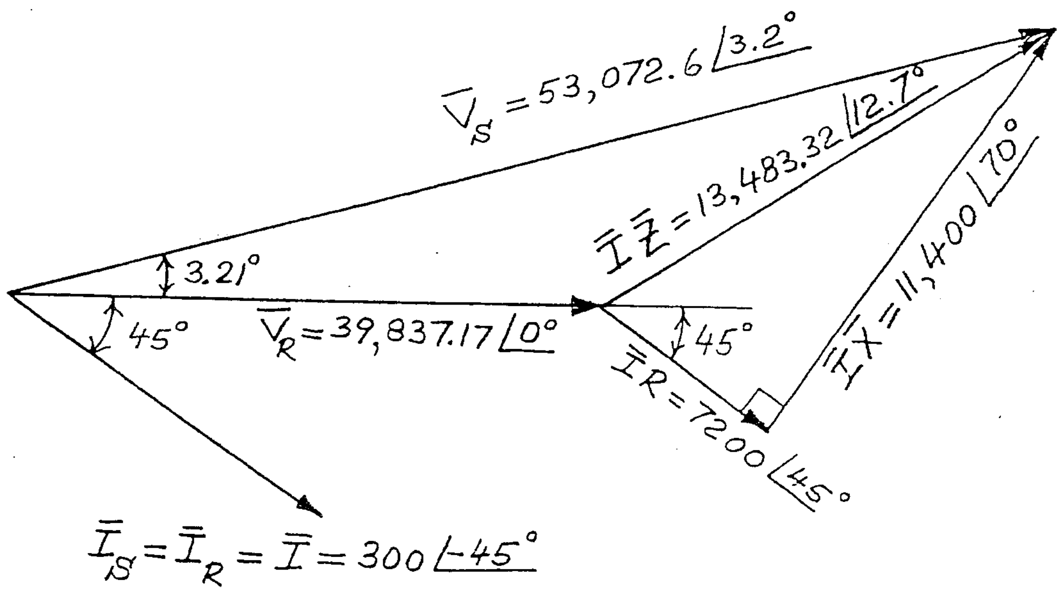

Figure P2.19

4.9 (a)

$$I_L = \frac{P_{3\Phi}}{\sqrt{3}\ V_L \cos\Phi} = \frac{12\text{x}10^6}{\sqrt{3}(138\text{x}10^3)0.85} = 59.06 \text{ A}$$

$$\bar{I} = 59.06(0.85 - j0.527) = 50.2044 - j31.1246 = 59.06/\underline{-31.8^\circ}$$

$$R = r.\ell = 4 \text{ x } 40 = 160\Omega/\Phi$$

$$X_L = x_a \cdot \ell = 2\pi 60(14\text{x}10^{-3})40 = 211.112\ \Omega/\Phi$$

$$Z_L = R + jX_L = 160 + j211.112\ \Omega/\Phi$$

$$\bar{V}_{R(L\text{-}N)} = \frac{138\text{x}10^3/\underline{0^\circ}}{\sqrt{3}} = 79,674.3/\underline{0^\circ} \text{ V}$$

$$\bar{V}_{S(L-N)} = \bar{V}_{R(L-N)} + \bar{I}_R \bar{Z}_L = 94{,}445.54\angle 3.4^\circ \text{ V}/\Phi$$

$$|V_{S(L-L)}| = \sqrt{3}\, V_{S(L-N)} = 163{,}584.47 \text{ V}$$

(b) $P_{LOSS} = 3I^2R = 3(59.06)^2 160 = 1.67$ MW

thus, $P_S = P_R + P_{LOSS} = 12 + 1.67 = 13.67$ MW

or $P_{3\Phi(LOSS)} = P_{3\Phi(in)} - P_{3\Phi(out)}$

$$= \sqrt{3}\, V_{S(L-L)} I_L \cos(\Theta_{\bar{V}_S} - \Theta_{\bar{I}}) - 12 \times 10^6$$

$$= \sqrt{3}\,(163{,}584.47)(59.06)\cos(3.4^\circ + 31.8^\circ)$$

$$\cong 1.67 \text{ MW}$$

(c) $$I_L = \frac{P_{3\Phi}}{\sqrt{3}\, V_{R(L-L)} \cos\Theta_R} = \frac{12\times 10^6}{\sqrt{3}\,(138\times 10^3) \times 1.0} = 50.2044 \text{A}$$

$P_{L(new)} = P_{3\Phi(LOSS)} = 3I^2R = 3(50.2044)^2 160 = 1.208$ MW

Reduction in $P_{LOSS} = P_{L(old)} - P_{L(new)}$

$$= 1.67 - 1.208 = 0.462 \text{ MW}$$

4.10 $\bar{Z} = 18 + j57 = 59.7746\angle 72.47^\circ\ \Omega$

From Eq. (2.180), $$P_{R,max} = \frac{V_R^2}{Z^2}\left(\frac{V_S \cdot Z}{V_R} - R\right)$$

where

$$V_{R(L-N)} = \frac{34.5 \times 10^3}{\sqrt{3}} = 19,918.6 \text{ V}$$

$$V_{S(L-N)} = \frac{39 \times 10^3}{\sqrt{3}} = 22,516.6 \text{ V}$$

Thus,

$$P_{R,max} = \frac{19,918.6^2}{59.7746^2} \left(\frac{22,516.6 \times 59.7746}{19,918.6} - 18\right)$$

$$= 5.504 \text{ MW}$$

4.11 (a) From Tables A.1 and A.8 ,

$$Z_1 = [r_a + j(x_a + x_d)]\ell$$

$$= [0.303 + j(0.497 + 0.1682)]45 = 32.8931\underline{/65.5^\circ} \ \Omega$$

$$I_R = \frac{20\text{x}10^6}{\sqrt{3}\ (161\text{x}10^3)} = 71.72 \text{ A or } \bar{I}_R = 71.72\underline{/-31.8^\circ} \text{ A}$$

$$\bar{V}_{S(L-N)} = \bar{V}_{R(L-N)} + \bar{I}\bar{Z}$$

$$= 92.9534\text{x}10^3\underline{/0^\circ} + (71.72\underline{/-31.8^\circ})(32.8931\underline{/65.5^\circ})$$

$$= 94,915.83 + j1,309.27 = 94,924.86\underline{/0.8^\circ} \text{ V}$$

$$\%\text{VReg} = \frac{|V_S| - |V_R|}{|V_R|} 100 = \frac{94,925 - 92,953}{92,953} 100 = 2.12\%$$

(b) $\cos\Phi_S = \cos(\Theta_{\bar{V}_S} - \Theta_{\bar{I}_S}) = \cos(0.8^\circ - (-31.8^\circ)) = 0.8426$

(c) If the line is 1Φ,

$$\eta = \frac{V_R I \cos\Phi_R}{V_R I \cos\Phi_R + 2I^2 R} \times 100$$

$$= \frac{92{,}953 \times 71.72 \times 0.85}{92{,}953 \times 71.72 \times 0.85 + 2 \times 71.72^2 \times 13.635} 100$$

$= 97.572\%$

(d) If the line is 3Φ,

$$\eta = \frac{\sqrt{3}\ V_R I\cos\Phi_R}{\sqrt{3}\ V_R I\cos\Phi_R + 3I^2R} 100$$

$$= \frac{\sqrt{3} \times 92{,}953.4 \times 71.72}{\sqrt{3} \times 92{,}953.4 \times 71.72 \times 0.85 + 3 \times 71.72^2 \times 13.635} 100 = 97.89\%$$

4.12 $z_1 = 0.7 + j1.2\ \Omega/\Phi/\text{mi}$

$Z_1 = (0.7 + j1.2)10 = 7 + j12 = 13.8924\underline{/59.7^\circ}\ \Omega/\Phi$

$$V_S = \frac{71 \times 10^3}{\sqrt{3}} = 40{,}991.9\ \text{V}$$

$$V_R = \frac{69 \times 10^3}{\sqrt{3}} = 39{,}837.2\ \text{V}$$

$VD_{line} = 40{,}991.9 - 39{,}837.2 = 1154.73 = IZ_{eq}$

$$\text{where } I = \frac{15 \times 10^6}{\sqrt{3}\ (69 \times 10^3)0.9} = 139.456\ \text{A}$$

$$Z_{eq} = \frac{IZ_{eq}}{I} = \frac{1154.73}{139.456} = 8.28\ \Omega$$

$$P_S = \frac{15 \times 10^6}{0.98} = 15{,}306{,}122.5\ \text{W}$$

$P_{LOSS} = 3I^2R = 306{,}122.5\ \text{W}$

Therefore, the equivalent resistance of the two lines in parallel is

$$3I^2R_{eq} = 306,122.5$$

$$R_{eq} = \frac{306,122.5}{3 \times 139.456^2} = 5.25\ \Omega$$

Thus, $X_{eq} = (Z_{eq}^2 - R_{eq}^2)^{1/2} = (8.28^2 - 5.25^2)^{1/2} = 1.74\ \Omega$

$$Z_{eq} = 5.25 + j1.74 = 5.5308/\underline{18.3^\circ}\ \Omega$$

Since $Z_1 = 7 + j12\ \Omega$ and $\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$

$$Z_2 = \frac{Z_1 \cdot Z_{eq}}{Z_1 - Z_{eq}}$$

where $Z_1 - Z_{eq} = (7 + j12) - (5.25 + j1.74) = 1.75 + j10.26$

Therefore,

$$Z_2 = \frac{(7 + j12)(5.25 + j1.74)}{(7 + j12) - (5.25 + j1.74)} = 7.3766\text{-}j0.2885 = 7.3923/\underline{-2.4^\circ}$$

That is, $R_2 = 7.366\ \Omega$ and $X_2 = 0.2885/\underline{-90^\circ}\ \Omega$

4.13 $$\frac{\cosh\gamma\ell - 1}{\sinh\gamma\ell} = \frac{\cosh\gamma\ell - (\cosh^2\gamma\ell - \sinh^2\gamma\ell)}{\sinh\gamma\ell}$$

$$= \frac{\frac{1}{2}(e^{\gamma\ell} - e^{-\gamma\ell}) - [\frac{1}{2}(e^{\gamma\ell} - {}^{\gamma\ell})]^2 + [\frac{1}{2}(e^{\gamma\ell} - e^{-\gamma\ell})]^2}{\frac{1}{2}(e^{\gamma\ell} - e^{-\gamma\ell})}$$

$$= \frac{e^{\gamma\ell} + e^{-\gamma\ell} - 2e^{-\gamma\ell}e^{\gamma\ell}}{e^{\gamma\ell} - e^{-\gamma\ell}}$$

$$= \frac{(e^{\gamma\ell/2} - e^{-\gamma\ell/2})^2}{(e^{\gamma\ell/2} - e^{-\gamma\ell/2})(e^{\gamma\ell/2} + e^{-\gamma\ell/2})}$$

$$= \frac{e^{\gamma\ell/2} - e^{-\gamma\ell/2}}{e^{\gamma\ell/2} + e^{-\gamma\ell/2}} = \tanh \tfrac{1}{2}\gamma\ell$$

4.14 Left to the reader.

4.15 Considering the network as a series connection of single two-port elements,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & j4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0.04 & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j0.02 & 1 \end{bmatrix} \begin{bmatrix} 1 & j5 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9433/\underline{17.3^\circ} & 10.5226/\underline{64.1^\circ} \\ 0.0471/\underline{31.8^\circ} & 1.1338/\underline{10.2^\circ} \end{bmatrix}$$

$$= \begin{bmatrix} 0.9008 + j0.28 & 4.6 + j9.464 \\ 0.04 + j0.0248 & 1.116 + j0.2 \end{bmatrix}$$

Alternatively, the network can be considered as the series connection of a π-network (made up of Y_1, Z_2, and Y_2 elements) with the elements Z_1 and Z_3. Therefore,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & j4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1+ZY_2 & Z \\ Y_1+Y_2+ZY_1Y_2 & 1+ZY_1 \end{bmatrix} \begin{bmatrix} 1 & j5 \\ 0 & 1 \end{bmatrix}$$

The answer turns out to be the same.

4.16 $$Z_1 = \frac{A-1}{C} = \frac{(0.9008 + j0.28) - 1}{0.04 + j0.0248} = \frac{0.2971/\underline{109.5^\circ}}{0.0471/\underline{31.8^\circ}}$$

$$= 6.3079/\underline{77.7^\circ}\ \Omega$$

$$Y_2 = C = 0.0471/\underline{31.8^\circ}\ S$$

$$Z_3 = \frac{D-1}{C} = \frac{(1.116 + j0.2) - 1}{0.0471/\underline{31.8^\circ}} = \frac{0.2312/\underline{59.9^\circ}}{0.0471/\underline{31.8^\circ}}$$

$$= 4.9087/\underline{28.1^\circ}$$

4.17 (a) $$\gamma = \sqrt{YZ} = [(95/\underline{78^\circ})(0.001/\underline{90^\circ})]^{1/2} = 0.3082/\underline{84^\circ}$$

$$\gamma = \alpha + j\beta = 0.03221 + j0.30651$$

$$A = \cosh\gamma\ell = \cosh(\alpha + j\beta)\ell = \frac{1}{2}\left(e^{\alpha\ell}/\underline{\beta\ell} + e^{-\alpha\ell}/\underline{-\beta\ell}\right)$$

Thus,

$$A = \frac{1}{2}\left(e^{0.03221} \cdot e^{j0.30651} + e^{-0.03221} \cdot e^{-j0.30651}\right)$$

$$= \frac{1}{2}(1.033/\underline{17.56^\circ} + 0.968/\underline{-17.56^\circ})$$

$$= 0.9539 + j0.0197 = 0.9541/\underline{1.18^\circ}$$

$$B = Z_c \sinh\gamma\ell = Z_c \sinh(\alpha + j\beta)\ell$$

$$= 93.5258/\underline{78.14^\circ}\ \Omega$$

$$C = Y_c \sinh\gamma\ell = \frac{\sinh\gamma\ell}{Z_c}$$

$$= 9.8448\times10^{-4}/\underline{90.14^\circ}\ S$$

$$D = A = 0.9541/\underline{1.18^\circ}$$

Thus,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.9541\underline{/1.18^\circ} & 93.5258\underline{/78.14^\circ} \\ 9.8448\text{x}10^{-4}\underline{/90.14^\circ} & 0.9541\underline{/1.18^\circ} \end{bmatrix}$$

(b) Using the T representation,

$$\frac{Z_T}{2} = \frac{A-1}{C} = 50.9203\underline{/66.72^\circ}\ \Omega$$

$$Y_T = C = 9.8448\text{x}10^{-4}\underline{/90.14^\circ}\ S$$

$$\begin{bmatrix} \bar{V}_S \\ \bar{I}_S \end{bmatrix} = \begin{bmatrix} 1 + \dfrac{Z_T Y_T}{2} & Z_T\left(1 + \dfrac{Z_T Y_T}{4}\right) \\ Y_T & 1 + \dfrac{Z_T Y_T}{2} \end{bmatrix} \begin{bmatrix} \bar{V}_R \\ \bar{I}_R \end{bmatrix}$$

$$= \begin{bmatrix} 0.9541\underline{/1.18^\circ} & 93.5258\underline{/78.14^\circ} \\ 9.8448\text{x}10^{-4}\underline{/90.14^\circ} & 0.9541\underline{/1.18^\circ} \end{bmatrix} \begin{bmatrix} 79{,}674.34\underline{/0^\circ} \\ 241.18\underline{/-31.8^\circ} \end{bmatrix}$$

Thus,

$$\bar{V}_{S(L-N)} = 93{,}303.697\underline{/11.05^\circ}\ V$$

$$\bar{V}_{S(L-L)} = 161{,}606.7438\underline{/41.05^\circ}\ V$$

(c) $\bar{I}_S = 201.5953\underline{/-11.09^\circ}$ A

(d) $\Phi_S = 11.05^\circ + 11.09^\circ = 22.14^\circ$

(e) $\cos\Phi_S = 0.9263$ lagging

(f) $\eta = 90.7\%$

4.18 Using the π representation,

(a) $$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.9541\underline{/1.18^\circ} & 93.5258\underline{/78.17^\circ} \\ 9.8448\text{X}10^{-4}\underline{/90.14^\circ} & 0.9541\underline{/1.18^\circ} \end{bmatrix}$$

(b) $\bar{V}_{S(L-L)} = 161{,}606.7438\angle 41.05^\circ$ V

(c) $I_S = 201.5953\angle -11.09^\circ$ A

(d) $\cos\Phi_S = 0.9263$ lagging

(e) $\eta = 90.7\%$

4.19 Assuming a short line,

(a) $$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 95\angle 78^\circ \\ 0 & 1 \end{bmatrix}$$

(b) $\bar{V}_S = \bar{V}_R + \bar{I}_R\bar{Z}_L$

$= 79{,}674.34\angle 0^\circ + (241.18\angle -31.8^\circ)(95\angle 78^\circ)$

$\bar{V}_{S(L-N)} = 97{,}001.0\angle 9.84^\circ$ V

$\bar{V}_{S(L-L)} = 168{,}010.6\angle 30^\circ + 9.84^\circ = 168{,}010.6\angle 39.84^\circ$ V

(c) $\bar{I} = \bar{I}_S = \bar{I}_R = 241.18\angle -31.8^\circ$ A

(d) $\cos\Phi_S = \cos(9.84^\circ + 31.8^\circ) = 0.7473$ lagging

(e) $$\eta = \frac{(138\times10^3)(0.85)}{(168{,}010.6)(0.7473)}\,100 = 93.4\%$$

4.20 (a) $$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 95\angle 78^\circ \\ 0 & 1 \end{bmatrix}$$

(b) $\bar{V}_{S(L-L)} = 168{,}010.6\angle 39.84^\circ$ V

(c) $\bar{I}_S = 241.18\angle -31.8^\circ$ A

(d) $\cos\Phi_S = 0.7473$ lagging

(e) $\eta = 93.4\%$

4.21 $z = 0.1858 + j120\pi(2.6 \times 10^{-3}) = 0.9977\angle 79.27^{\circ}\ \Omega/\text{mi}$

$y = j120\pi(0.012 \times 10^{-6}) = 4.5239 \times 10^{-6}\angle 90^{\circ}\ \text{S/mi}$

Thus,

$Z = z\ell = (0.9977\angle 79.27^{\circ})75 = 74.828\angle 79.27^{\circ}\ \Omega$

$Y = y\ell = (4.5239 \times 10^{-6}\angle 90^{\circ})75 = 339.29 \times 10^{-6}\angle 90^{\circ}\ \text{S}$

(a) Since the line has medium length,

$$A = 1 + \frac{YZ}{2} = 987.53 \times 10^{-3}\angle 0.14^{\circ}$$

$$B = Z + \frac{YZ^2}{4} = 74.361\angle 79.34^{\circ}$$

$$C = Y = 339.29 \times 10^{-6}\angle 90^{\circ}$$

$$D = A = 987.53 \times 10^{-3}\angle 0.14^{\circ}$$

(b) $$\begin{bmatrix} \bar{V}_S \\ \bar{I}_S \end{bmatrix} = \begin{bmatrix} 987.53\text{x}10^{-3}\angle 0.14^{\circ} & 74.361\angle 79.34^{\circ} \\ 339.29\text{x}10^{-5}\angle 90^{\circ} & 987.53\text{x}10^{-3}\angle 0.14^{\circ} \end{bmatrix} \begin{bmatrix} 79{,}674.34\angle 0^{\circ} \\ 209.18\angle -31.8^{\circ} \end{bmatrix}$$

$$\bar{V}_{S(L-N)} = (987.53\text{x}10^{-3}\angle 0.14^{\circ})(79{,}674.34\angle 0^{\circ} + (74.361\angle 79.34^{\circ})$$
$$(209.18\angle -31.8^{\circ}) = 87{,}624.628\angle 7.65^{\circ}$$

$$\bar{V}_{S(L-L)} = 151{,}770.3077\angle 30^{\circ} + 7.65^{\circ} = 151{,}770.3077\angle 37.65^{\circ}$$

(c) $$\bar{I}_S = (339.29\text{x}10^{-6}\angle 90^{\circ})(79{,}674.34\angle 0^{\circ}) + (987.53\text{x}10^{-3}\angle 0.14^{\circ})$$
$$(209.18\angle -31.8^{\circ})$$
$$= 193.7538\angle -24.82^{\circ}\ \text{A}$$

(d) $\cos\Phi_S = \cos(7.65^{\circ} + 24.82^{\circ}) = \cos 32.47^{\circ} = 0.8437$ lagging

(e) $P_S = \sqrt{3}\ V_{S(L-L)} I_S \cos\Phi_S$

$= \sqrt{3}\ (151{,}770.3077)(193.7538)(0.8437) = 42{,}971.4$ kW

(f) $P_R = \sqrt{3}\ V_{R(L-L)} I_R \cos\Phi_R$

$= \sqrt{3}\ (138 \times 10^3)(209.18)(0.85) = 42{,}499$ kW

Thus, $P_{LOSS} = P_S - P_R = 472.4$ kW

(g) $\eta = \dfrac{P_R}{P_S} \times 100 = 98.9\%$

(h) $\%VReg = \dfrac{87{,}624.628 - 79{,}674.34}{79{,}674.34} \times 100 = 9.98\%$

(i) $\bar{I}_c = \dfrac{\bar{Y}}{2} \bar{V}_{S(L-N)} = \left(\dfrac{j339.29 \times 10^{-6}}{2}\right)(87{,}624.628/\underline{7.65^\circ})$

$= 14.8651/\underline{97.64^\circ}$ A

(j) $\bar{V}_{R(L-N)} = \bar{V}_{S(L-N)} - \bar{Z}\bar{I}_c$

$= 87{,}624.628/\underline{7.65^\circ} - (74.828/\underline{79.27^\circ})(14.8651/\underline{97.65^\circ})$

$= 88{,}717.7469/\underline{7.52^\circ}$ V

$\bar{V}_{R(L-L)} = 153{,}663.6452/\underline{30^\circ + 7.52^\circ} = \underline{153{,}663.6452/\underline{37.52^\circ}}$

4.22 $$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ Y & 1 + YZ \end{bmatrix}$$

4.23 $$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1 + YZ & Z \\ Y & 1 \end{bmatrix}$$

4.24 Consider the series combination of two networks, as shown in Figure 2.43, the various voltages and currents being as shown.

$$V = A_2V_R + B_2I_R$$

$$I = C_2V_R + D_2I_R$$

$$V_R = A_1(A_2V_R + B_2I_R) + B_1(C_2V_R + D_2I_R)$$

$$= (A_1A_2 + B_1C_2)V_R + (A_1B_2 + B_1D_2)I_R$$

$$I_R = C_1V + D_1I$$

$$= C_1(A_2V_R + B_2I_R) + D_1(C_2V_R + D_2I_R)$$

$$= (C_1A_2 + D_1C_2)V_R + (C_1B_2 + D_1D_2)I_R$$

Therefore, the equivalent A, B, C, D constants for two networks connected in series are

$$A = A_1A_2 + B_1C_2 \qquad B = A_1B_2 + B_1D_2$$

$$C = C_1A_2 + D_1C_2 \qquad D = C_1B_2 + D_1D_2$$

4.25 If the two networks are connected in parallel, as shown in Figure 2.45,

$$V_S = A_1V_R + B_1I_{R1} = A_2V_2 + B_2I_{R2}$$

$$I_{S1} = C_1V_R + D_1I_{R1}$$

$$I_{S2} = C_2V_R + D_2I_{R2}$$

$$I_{R1} = \frac{V_S - A_1V_R}{B_1}$$

$$I_{R2} = \frac{V_S - A_2V_R}{B_2}$$

$$I_R = I_{R1} + I_{R2}$$

$$V_S = A_1V_R + B_1(I_R - I_{R2})$$

$$= A_1V_R + B_1[I_R - \frac{V_S - A_2V_R}{B_2}]$$

$$= \frac{A_1B_2 + A_2B_1}{B_1 + B_2} V_R + \frac{B_1B_2}{B_1 + B_2} I_R$$

$$I_S = I_{S1} + I_{S2}$$

$$= (C_1 + C_2)V_R + D_1I_{R1} + D_2(I_R - I_{R1})$$

$$= (C_1 + C_2)V_R + \frac{(D_1 - D_2)(V_S - A_1V_R)}{B_1} + D_2I_R$$

Substituting for V_S and collecting terms,

$$I_S = [C_1 + C_2 + \frac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2}]V_R + [\frac{D_1B_2 + D_2B_1}{B_1 + B_2}]I_R$$

therefore,

$$A = \frac{A_1B_2 + A_2B_1}{B_1 + B_2}$$

$$B = \frac{B_1B_2}{B_1 + B_2}$$

$$C = C_1 + C_2 + \frac{(A_1 - A_2)(D_2 - D_1)}{B_1 + B_2}$$

$$D = \frac{D_1B_2 + D_2B_1}{B_1 + B_2}$$

4.26 From Example 2.8,

$$z = \frac{Z}{52} = \frac{95\underline{/78^\circ}}{52} = 1.8269\underline{/78^\circ}\ \Omega/\text{mi}$$

$$y = \frac{Y}{52} = \frac{0.001\underline{/90^\circ}}{52} = 1.9231 \times 10^{-5}\underline{/90^\circ}\ \text{S/mi}$$

(a) $\gamma = \sqrt{yz} = 5.927319 \times 10^{-3}\underline{/84^\circ}$

$$= 6.19574 \times 10^{-4} + j5.894851 \times 10^{-3}$$

Thus, $\alpha = 6.19574 \times 10^{-4}$ nepers/mi

$\beta = 5.894851 \times 10^{-3}$ radians/mi

(b) $$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5.894851 \times 10^{-3}} = 1{,}065.8769 \text{ mi}$$

$$v = \lambda f = 1{,}065.8769 \times 60 = 63{,}952.615 \text{ mi/s}$$

(c) $$\bar{I}_R = \frac{49 \times 10^6}{\sqrt{3}\,(138 \times 10^3)(0.85)} \underline{/-31.79^\circ} = 241.1779\underline{/-31.79^\circ} \text{ A}$$

$$\bar{Z}_c = \left(\frac{z}{y}\right)^{1/2} = 308.2169\underline{/-6^\circ}\ \Omega$$

$$\bar{V}_{R(\text{incident})} = \frac{\bar{V}_R + \bar{I}_R\bar{Z}_c}{2} = 72{,}860.71\underline{/-18.21^\circ} \text{ V}$$

$$\bar{V}_{R(\text{reflected})} = \frac{\bar{V}_R - \bar{I}_R\bar{Z}_c}{2} = 25{,}063.13\underline{/65.32^\circ} \text{ V}$$

(d) $$\bar{V}_{R(L-N)} = \bar{V}_{R(\text{incident})} + \bar{V}_{R(\text{reflected})} = 79{,}674.33\underline{/0^\circ}$$

$$V_{R(L-L)} = \sqrt{3}\, V_{R(L-N)} = 138{,}000 \text{ V}$$

(e) $$\bar{V}_{S(\text{incident})} = \frac{\bar{V}_R + \bar{I}_R\bar{Z}_c}{2} e^{\gamma \ell}$$

where $\gamma\ell = 1.18546\underline{/84^\circ}$

$$= 123.915 \times 10^{-3} + j1.17897$$

$$\bar{V}_{S(\text{incident})} = (72{,}860.71\underline{/-18.21})(e^{123.915\times 10^{-3} + j1.17897})$$

$$= 82{,}472.5\underline{/49.34^\circ} \text{ V}$$

$$\bar{V}_{S(\text{reflected})} = (25{,}063.13\underline{/65.32^\circ})(e^{-123.915\times 10^{-3} - j1.17897})$$
$$= 22{,}142.2\underline{/-2.23^\circ} \text{ V}$$

(f) $\bar{V}_{S(L-N)} = \bar{V}_{S(incident)} + \bar{V}_{S(reflected)}$

$$= 97{,}787.4\underline{/39.12^{\circ}}\ V$$

$$V_{S(L-L)} = \sqrt{3}V_{S(L-N)} = 169{,}373\ V$$

4.27 From Prob. 2.37 , $\bar{Z}_c = 308.2169\underline{/-6^{\circ}}\ \Omega$

$$SIL \cong \frac{138^2}{308.2169} = 61.7877\ MW$$

4.28 (a) Since the distributed reactance of the line has not been altered, Z_c and SIL of the line are the same as in Example 2.17.

(b) $X_{L(new)} = 0.40X_{L(old)} = 0.40 \times 117.6 = 47.04\ \Omega/\Phi$

$$\text{Thus, } P_{max} = \frac{V_S R_R}{X_L} = \frac{(345 \times 103)^2}{47.04}$$

$$= 2530.3\ MW/3\Phi$$

4.29 (a) $2 \times SIL = 2 \times 416.5 = 833\ MVA/3\Phi/mi$

$$X_L = 0.60X_{L(old)} = 0.60 \times 117.6 = 70.56\ \Omega/\Phi$$

$$\text{Therefore, } I = \frac{833 \times 10^6}{\sqrt{3}\ (345 \times 10^3)} = 1394\ A$$

$$V_c = IX_c = 1394 \times 70.56 = 98{,}360.64\ V$$

Since series connection builds up the voltage rating and parallel connection builds up the current rating,

$$\text{No. of series capacitors} = \frac{V_c}{12{,}000} = \frac{98{,}360.64}{12{,}000}$$

$$= 8.2 \text{ or } 9/\Phi$$

$$\text{No. of parallel capacitors} = \frac{1394}{12.5} = 111.5 \text{ or } 112/\Phi$$

36

Since each capacitor can stand for 12 kV and (150 kvar/12 kV) = 12.5 A. Therefore, the numbers of series- and parallel-connected capacitors are 9 and 112, respectively. Thus, the total number of capacitors required is 9x112 = 1008 per phase or 3024 per three phase.

(b) Therefore,

$I_{actual} = 112(12.5A) = 1400 \text{ A} > 1304 \text{ A}$

$V_{actual} = 9(12 \text{ kV}) = 108 \text{ kV} > 98{,}360.64 \text{ V}$

Thus, $Q_c = VI = (108 \text{ kV})(1400) = 151{,}200 \text{ kvar}/\Phi$

Cost of capacitor bank = (151,200 kvar)($1.50/kvar)

= $226,800/Φ

= $680,400/3Φ

4.30 (a) From Table 2.6 , $y = 1.08 \times 10^{-4}/\underline{89.7^\circ}$ S/mi

thus, $b = y\sin 89.7^\circ = (1.08\text{x}10^{-4})(1.0) = 1.08\text{x}10^{-4}$ S/mi

Alternatively, the charging current is

$i_c = bV_{an}$

from which, $b = \dfrac{i_c}{V_{an}} = \dfrac{21.6\text{x}\sqrt{3}}{345{,}000} = 1.08 \times 10^{-4}$ S/mi

(b) Since $I_{\ell o} = I_T$,

$$\ell_o = \frac{I_{\ell o}}{V_s b} = \frac{585 \times \sqrt{3}}{(345{,}000)(1.08 \times 10^{-4})} = 27.194 \text{ mi}$$

for cable ampacity of 585 A. Note that it is given as 27.1 mi in Table 2.6 .

4.31

$$\bar{I}_{\ell o} = \frac{\bar{V}_S}{\bar{Z}_c} \tanh \bar{\gamma}\ell_o$$

$$\tan\bar{\gamma}\ell_o = \tanh[0.179\underline{/83.9^\circ}]$$

$$= \frac{0.179\cos 83.9^\circ + j0.179\sin 83.9^\circ + 0.000956\cos 251.7^\circ + j0.000956\sin 251.7^\circ}{1 + 0.016\cos 167.8^\circ + j0.016\sin 167.8^\circ + j0.000956\sin 251.7^\circ}$$

$$= 0.181\underline{/83.75^\circ}$$

Therefore,

$$\bar{I}_{\ell o} = \left[\frac{345 \times 10^3\underline{/0^\circ}}{\sqrt{3}\ (61.2\underline{/-5.9})}\right][0.181\underline{/83.75^\circ}]$$

$$= 590\underline{/89.65^\circ}\ \text{A}$$

Note that it is given in Table 2.6 as

$$I_{\ell o} = I_T = 585\ \text{A}$$

4.32

$$P_R = \frac{V_S \cdot V_R}{Z} \cos(\Theta - \delta) - \frac{V_R^2}{Z} \cos\Theta$$

If $\Theta = 90$, $\sin\delta \triangleq \cos(90^\circ - \delta) \triangleq \sin(180 - \delta)$

and $\cos\Theta = 0$.

Therefore,

$$P_{R,max} = \frac{V_S V_R}{Z} \cos(90^\circ - \delta) = \frac{V_S V_R}{Z} \sin\delta$$

If $R = 0$: $Z = X$

$$P_{R,max} = \frac{|V_S||V_R|}{X} \sin\delta$$

4.33 Since, $\bar{S} = \bar{V}\bar{I}^*$ and

$$\bar{I} = \frac{\bar{V}_S - \bar{V}_R}{\bar{Z}} = \frac{V_S\underline{/\Theta} - 115 \times 10^3\underline{/0^\circ}}{15\underline{/90^\circ}}$$

$$\frac{19.22 \times 10^6\underline{/38.66^\circ}}{V_S\underline{/\Theta}} = \frac{(V_S\underline{/\Theta})\left(\dfrac{V_S\underline{/-\Theta} - 115 \times 10^3\underline{/0^\circ}}{15\underline{/-90^\circ}}\right)}{V_S\underline{/\Theta}}$$

and by multiplying both sides by $15\underline{/-90^\circ}$

$$\frac{288 \times 10^6\underline{/-51.34^\circ - \Theta}}{V_S} + 115 \times 10^3\underline{/0^\circ} = V_S\underline{/-\Theta}$$

$$V_S\underline{/\Theta} = 115 \times 10^3\underline{/0^\circ} + \frac{288 \times 10^6\underline{/51.34^\circ + \Theta}}{V_S}$$

By using the trial and error method,

$$\bar{V}_S \cong 116{,}517\underline{/0.96^\circ} \text{ V}$$

Alternatively, since R = 0, i.e., Z = 0 + j15 Ω there are no losses.

$$\bar{S} = P + jQ = 15 + j12 = 19.2094\underline{/38.66^\circ} \text{ MVA}$$

therefore, $|\bar{S}| = 19.2094$ MVA

$$I_R = \frac{19.2094 \times 10^6}{115 \times 10^3} = 167.038 \text{ A}$$

$$\bar{I}_R = 167.038\underline{/-38.66^\circ} \text{ A}$$

Using $\bar{V}_R$ as the reference,

$\bar{V}_S = \bar{V}_R + \bar{I}_R\bar{Z}_L$

$= 115 \times 10^3 + (167.038\underline{/-38.66^\circ})(15\underline{/90^\circ})$

$= 116,581.64\underline{/0.96^\circ}$ A

(b) $\bar{I}_S = \dfrac{\bar{V}_S - \bar{V}_R}{\bar{Z}}$

$$= \frac{116,517\underline{/0.96^\circ} - 115\underline{/0^\circ}}{15\underline{/90^\circ}} = 164.78\underline{/-37.39^\circ} \text{ A}$$

Alternatively,

$\bar{S}_S = \bar{V}_S\bar{I}_S^*$

$$\bar{I}_S^* = \frac{(15 + j12) \times 10^6}{116,581.64\underline{/0.96^\circ}} = 164.7719\underline{/37.7^\circ}$$

Therefore, $\bar{I}_S = 164.7719\underline{/-37.7^\circ}$ A

4.34 (a) $D = (D_{ab} \times D_{bc} \times D_{ca})^{1/3} = (10\text{x}8\text{x}18)^{1/3} = 11,292$ ft

From Table A.1, $X_a = 0.422\ \Omega/\text{mi}$

From Table A. 8 , $X_d = 0.2941\ \Omega/\text{mi}$

Therefore, $X_L = X_a + X_d = 0.422 + 0.2941 = 0.7161\ \Omega/\text{mi}$

(b) From Table A.1, $X'_a = 0.0954$ MΩ - mi

From Table A.10, $X'_d = 0.0718$ MΩ-mi

Thus, $X_c = X'_a + X'_d = 0.0954 + 0.0718 = 0.1672$ MΩ-mi

(c) From Table A.1, r = 0.0871 Ω/mi. Therefore,

$R = r \times \ell = 0.0871 \times 100 = 8.71\ \Omega$

(d) $X_L = 0.7161 \times 100 = 71.61\ \Omega$

(e) $X_c = \dfrac{0.1672}{100} = 1.672\ k\Omega$

4.35 (a) $\gamma = \sqrt{yz} = [(0.7688\underline{/77.4^\circ})(4.5239 \times 10^{-6}\underline{/90^\circ})]^{1/2}$

$$= [0.7688 \times 4.5239 \times 10^{-6}]^{1/2}\underline{/\dfrac{77.4^\circ + 90^\circ}{2}}$$

$$= 0.0019\underline{/83.7^\circ} = 0.0002 + j0.0019$$

(b) Thus, $\alpha = 0.0002$ nepers/mi and $\beta = 0.0019$ rad/mi

(c) $Z_c = (\dfrac{z}{y})^{1/2} = (\dfrac{0.7688}{4.5239 \times 10^{-6}})^{1/2}\underline{/\dfrac{77.4^\circ - 90^\circ}{2}}$

$$= 412.24\underline{/-6.3^\circ}\ \Omega$$

(d) $SIL = \dfrac{(138 \times 10^3)^2}{412.24} = 46.1964$ MW

(e) $I_R = \dfrac{50 \times 10^6}{\sqrt{3} \times 138 \times 10^3} = 209.4$ A or $\bar{I}_R = 209.4\underline{/-31.8^\circ}$ A

(f) $\bar{V}_{S(incident)} = \dfrac{\bar{V}_R + \bar{I}_R \bar{Z}_c}{2} e^{\alpha \ell} \cdot e^{j\beta \ell}$

$$= (78{,}505.1\underline{/-19.8^\circ}) e^{0.0307} \cdot e^{j0.278}$$

$$= (78{,}505.1\underline{/-19.8^\circ})(1.0312\underline{/15.9^\circ})$$

$$= 80{,}954.5\underline{/-3.9^\circ}\ V$$

where $(0.278\ rad)(57.3^\circ/rad) = 15.9^\circ$

(g) $\bar{V}_{S(reflected)} = \dfrac{\bar{V}_R - \bar{I}_R \bar{Z}_c}{2} e^{-\alpha \ell} \cdot e^{-j\beta \ell}$

$$= (27{,}282\underline{/77.5^\circ})(0.9698\underline{/-15.9^\circ})$$

$$= 26{,}458.1\underline{/6.6^\circ}\ V$$

4.36 (a) $\bar{V}_{R(\text{incident})} = \dfrac{\bar{V}_R + \bar{I}_R\bar{Z}_c}{2}$

$$= \frac{79,674\angle 0^\circ + (209.18\angle -31.8^\circ)(469.12\angle -5.37^\circ)}{2}$$

$$= 84,367.77\angle -20.59^\circ \text{ V}$$

$$\bar{V}_{R(\text{reflected})} = \frac{\bar{V}_R - \bar{I}_R\bar{Z}_c}{2} = 29,684.15\angle 88.65^\circ \text{ V}$$

(b) $e^{j\beta\ell} = e^{j0.3202} = (0.3202 \text{ rad})(57.3^\circ/\text{rad}) = 18.35^\circ$

$$\bar{V}_{S(\text{incident})} = \frac{\bar{V}_R + \bar{I}_R\bar{Z}_c}{2} e^{\alpha\ell} \cdot e^{j\beta\ell}$$

$$= (84,366.77\angle -20.59^\circ)e^{0.0301}\angle 18.35^\circ$$

$$= 86,946\angle -2.24^\circ \text{ V}$$

$$\bar{V}_{S(\text{reflected})} = \frac{\bar{V}_R - \bar{I}_R\bar{Z}_c}{2} e^{-\alpha\ell} \cdot {}^{-j\beta\ell}$$

$$= (29,684.15\angle 88.65^\circ)e^{-0.0301}\angle -18.35$$

$$= 28,802.5\angle 70.3^\circ \text{ V}$$

(c) $\bar{V}_{S(L-N)} = \bar{V}_{S(\text{incident})} + \bar{V}_{S(\text{reflected})} = 99,458.1\angle 13.8^\circ$ V

Thus, $|V_{S(L-L)}| = \sqrt{3}\, V_{S(L-N)} = 172,266.5$ V

4.37 (a) From Table A.3, $D_s = 0.0493$ ft, therefore

$$D_s^b = (D_s \times d)^{1/2} = (0.0493\text{x}0.3048\text{x}12\text{x}0.0254)^{1/2} = 0.0677 \text{ m}$$

$$D_{eq} = (D_{12}\cdot D_{23}\cdot D_{31})^{1/3} = (25\text{x}25\text{x}50\text{x}0.3048^3)^{1/3} = 9.6006 \text{ m}$$

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s^b} = 2 \times 10^{-7} \ln \frac{9.606}{0.0677} = 0.9909\ \mu H/m$$

(b) $X_L = 2\pi f L_a = 2\pi 60 \times 0.9909 \times 10^{-6} \times 10^3 = 0.3736\ \Omega/km$

$X_L = 0.3736 \times 1.609 = 0.6011\ \Omega/mi$

(c) $Z_B = \dfrac{345^2}{100} = 1190.25\ \Omega$

$$X_L = \frac{0.3736 \times 200}{1190.25} = 0.0628 \text{ pu}$$

(d) From Table A.3, the outside diameter of the subconductor is 1.465 in, therefore its radius is

$$r = \frac{1.465 \times 0.3048}{2 \times 12} = 0.0186 \text{ m}$$

$$D_{sc}^b = (rd)^{1/2} = (0.0186 \times 12 \times 0.0254)^{1/2} = 0.0753 \text{ m}$$

Thus, the line-to-neutral capacitance of the line is

$$C_n = \frac{55.63 \times 10^{-12}}{\ln \dfrac{D_{eq}}{D_{sc}^b}} = \frac{55.63 \times 10^{-12}}{\ln \dfrac{9.606}{0.0753}} = 11.473 \times 10^{-12} \text{ F/m}$$

(e) $X_c = \dfrac{1}{2\pi f C_n} = \dfrac{10^{12} \times 10^{-3}}{2\pi 60 \times 11.473} = 0.231 \times 10^6\ \Omega\text{-km}/\Phi$

$$X_c = \frac{0.231 \times 10^6}{1.609} = 0.14369 \times 10^6\ \Omega\text{-mi}/\Phi$$