CHAPTER 2

P. P. 2.1

$$f(t) = e^{-4t^2}, \qquad g(t) = t$$

Let y(t) = f(t)*g(t),

$$y(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau = \int_{-\infty}^{\infty} e^{-4\tau^2}(t-\tau)d\tau$$

$$= t\int_{-\infty}^{\infty} e^{-4\tau^2}d\tau - \int_{-\infty}^{\infty} \tau e^{-4\tau^2}d\tau \text{(the second term is zero)}$$

$$= t\frac{\sqrt{\pi}}{2(2)} = 0.4431t$$

P.P. 2.2

$$x(t) = tu(t), \qquad h(t) = u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

$$y(t) = x(t) * h(t) = tu(t) * u(t) = \int_{-\infty}^{\infty} \tau u(\tau)u(t-\tau)d\tau$$

But

$$u(\tau)u(t-\tau) = \begin{cases} 1, & 0 < t < \tau \\ 0, & \text{otherwise} \end{cases}$$

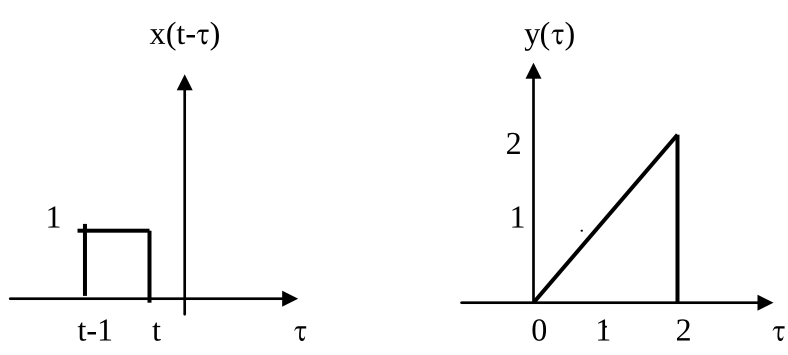
Hence,

$$y(t) = \int_0^t \tau d\tau = \frac{\tau^2}{2}\Bigg|_0^t = \frac{t^2}{2}, t > 0$$

or

$$y(t) = 0.5t^2u(t)$$

P.P. 2.3

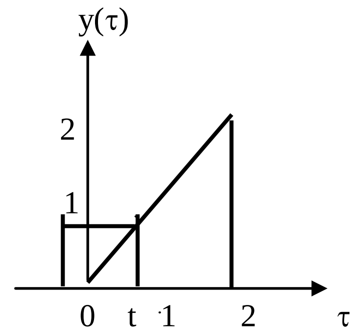


y(τ) = τ, 0 < τ <2

Let $z(t) = x(t) * y(t) = \int y(\tau)x(t-\tau)d\tau$

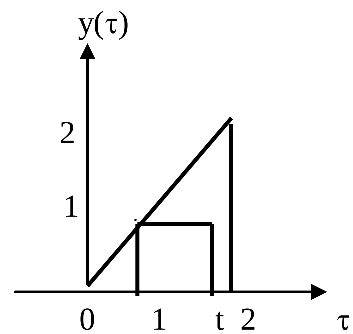
For 0 < t <1

$$z(t) = \int_0^t \tau(1)d\tau = \frac{\tau^2}{2}\Big|_0^t = \frac{1}{2}t^2, 0 < t < 1$$



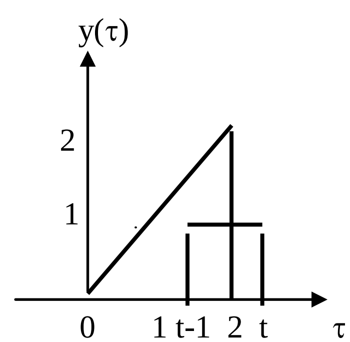
For 1< t < 2,

$$z(t) = \frac{\tau^2}{2}\Big|_{t-1}^{t} = \frac{1}{2}(2t-1), 1 < t < 2$$



For 2<t<3

$$z(t) = \frac{\tau^2}{2}\Big|_{t-1}^{2} = \frac{1}{2}\left[4-(t-1)^2\right] = \frac{1}{2}(3+2t-t^2), 2 < t < 3$$



For t>2, there is no overlapping/ z(t) = 0.
Thus, in summary,

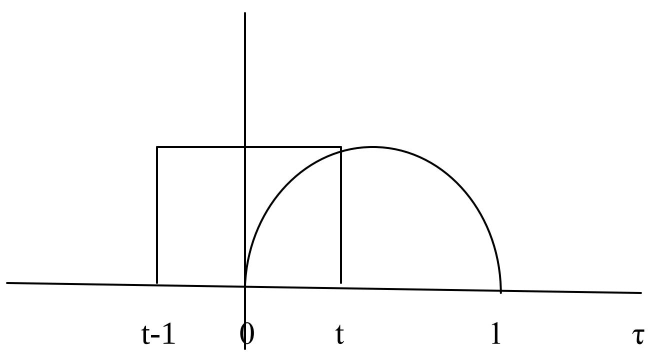
$$z(t) = \begin{cases} \frac{1}{2}t^2, & 0 < t < 1 \\ \frac{1}{2}(2t-1). & 1 < t < 2 \\ \frac{1}{2}(3+2t-t^2), & 2 < t < 3 \\ 0, & \text{otherwise} \end{cases}$$

P. P. 2.4

Let z(t) = x(t)*y(t)

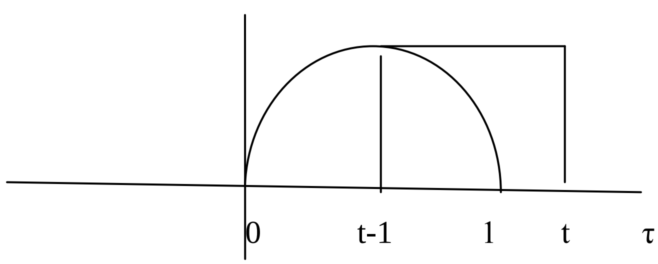
$$z(t) = \int x(t-\tau)y(\tau)d\tau$$

For t<0, z(t) = 0
For 0 < t < 1,



$$z(t) = \int_0^t (1)\sin\pi\tau d\tau = -\frac{1}{\pi}\cos\pi\tau \Big|_0^t = \frac{1}{\pi}(1-\cos\pi t)$$

For 1 < t < 2,



$$z(t)=\int_{t-1}^{1}(1)\sin\pi\tau d\tau=-\frac{1}{\pi}\cos\pi\tau\Big|_{t-1}^{1}=-\frac{1}{\pi}(\cos\pi-\cos\pi(t-1))$$
$$=-\frac{1}{\pi}(-1-\cos\pi t\cos\pi-\sin\pi t\sin\pi)=\frac{1}{\pi}(\cos\pi t-1)$$

For t > 2. There is no overlap and z(t) = 0.
Thus,

$$z(t)=\begin{cases}\frac{1}{\pi}(1-\cos\pi t), & 0<t<1\\ \frac{1}{\pi}(\cos\pi t-1), & 1<t<2\\ 0, & \text{otherwise}\end{cases}$$

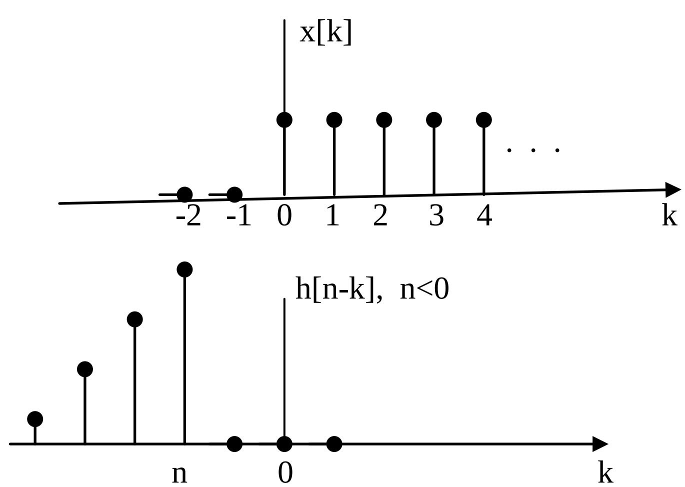
P.P. 2.5

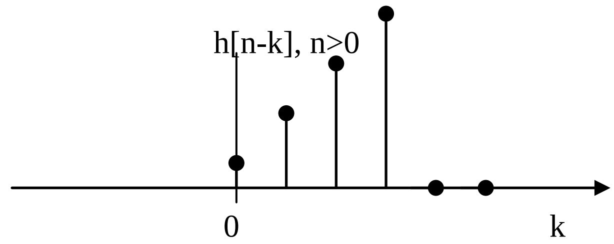
$$h(t)=\int_{-\infty}^{\infty}h_1(\tau)h_2(t-\tau)d\tau=\int_{-\infty}^{\infty}2e^{-\tau}u(\tau)4e^{-2(t-\tau)}u(t-\tau)d\tau=8e^{-2t}\int_{-\infty}^{\infty}e^{\tau}u(\tau)u(t-\tau)d\tau$$
$$=8e^{-2t}\int_0^t e^{\tau}d\tau, t>0=8e^{-2t}(e^t-1), t>0$$
$$=8(e^{-t}-e^{-2t})u(t)$$

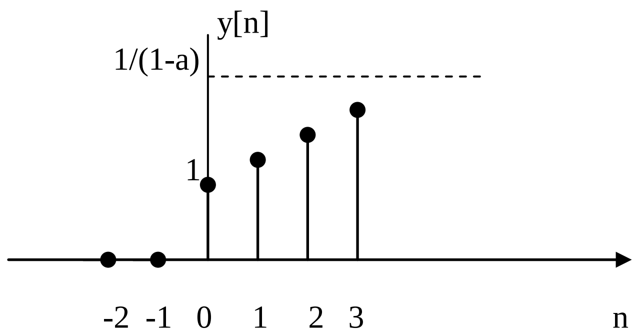
P.P. 2.6

$$y[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]$$

Sequences x[k] and h[k] are shown below for n<0 and n>0.







For n<0, x[k[and h[n-k] do not overlap. For n $\geq$ 0, they overlap from k=0 to k=n.

$$y[n] = \sum_{k=0}^{n} a^{n-k}$$

Changing the variable of summation k to m=n-k gives

$$y[n] = \sum_{m=n}^{0} a^m = \sum_{m=0}^{n} a^m$$

But

$$\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a}, a \neq 1$$

$$y[n] = \frac{1-a^{n+1}}{1-a}, \qquad n \geq 0$$

Thus,

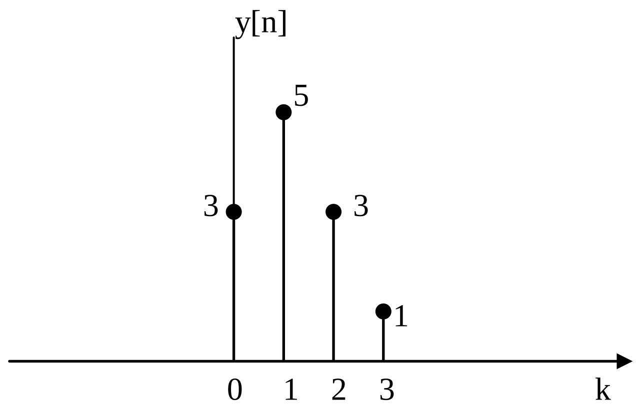
$$y[n] = \frac{1-a^{n+1}}{1-a} u[n]$$

P.P. 2.7

Analytically, y[n] = x[n]*h[n]

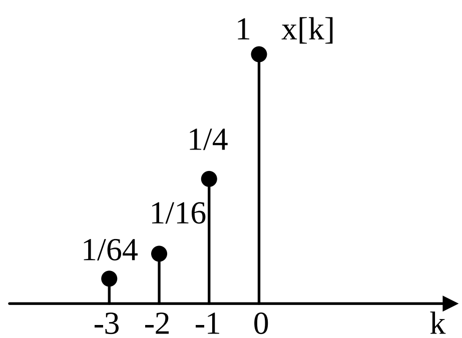
$$x[n] = \delta[n] + \delta[n-1]$$
$$h[n] = 3\delta[n] + 2\delta[n] + \delta[n-2]$$
$$y[n] = x[n] * \left(\delta[n] + 2\delta[n] + \delta[n-2]\right) = 3x[n] + 2x[n-1] + x[n-2]$$
$$= 3\delta[n] + 3\delta[n-1] + 2\delta[n-1] + 2\delta[n-2] + \delta[n-2] + \delta[n-3]$$
$$= 3\delta[n] + 5\delta[n-1] + 3\delta[n-1] + \delta[n-3]$$

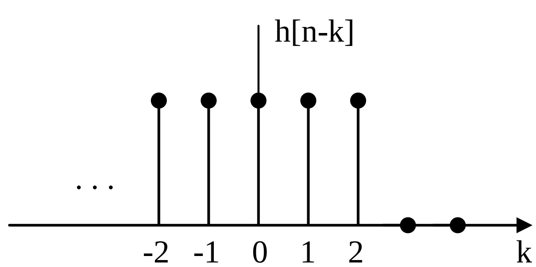
which is sketched below.

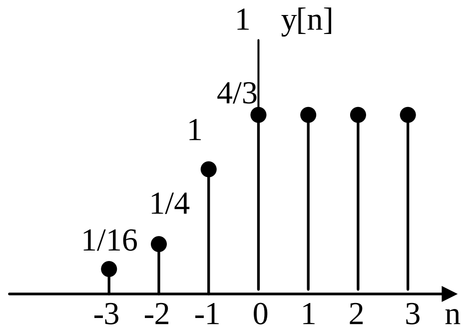


P.P. 2.8

The sequences x[k] and h[n-k] are shown below.







For n ≥0,

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{0} 4^k$$

Changing the variable of summation from k to r=-k,

$$\sum_{k=-\infty}^{0} 4^k = \sum_{r=0}^{\infty} \left(\frac{1}{4}\right)^k$$

But,

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}, \qquad 0 < |a| < 1$$

Hence,

$$\sum_{r=0}^{\infty} \left(\frac{1}{4}\right)^k = \frac{1}{1-1/4} = \frac{4}{3}$$

$$y[n] = \frac{4}{3}$$

When n<0, x[k]h[n-k] has nonzero values for k≤ n, Thus, for n < 0,

$$y[n] = \sum_{k=-\infty}^{0} x[k]h[n-k] = \sum_{k=-\infty}^{n} 4^k$$

By performing a change of variable l=-k, then m = l+n

$$y[n] = \sum_{l=-n}^{\infty} \left(\frac{1}{4}\right)^{m-n} = \sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^{m-n} = \left(\frac{1}{4}\right)^{-n} \sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^{m} = 4^{n+1}$$

Thus,

$$y[n] = \begin{cases} 4^{n+1}, & n < 0 \\ \dfrac{4}{3}, & n \geq 0 \end{cases}$$

P.P. 2.9

x[n] = { 1, 8, 4, 5}, y [n]={4, 38, 66, 61, 46, 14, 5}

$$x(z) = z^3 + 8z^2 + 4z + 5$$

$$y(z) = 4z^6 + 38z^5 + 66z^4 + 61z^3 + 46z^2 + 14z + 5$$

$$
\begin{array}{r|l}
 & 4z^3 + 6z^2 + 2z + 1 \\
\hline
z^3 + 8z^2 + 4z + 5 & 4z^6 + 38z^5 + 66z^4 + 61z^3 + 46z^2 + 14z + 5 \\
 & 4z^6 + 32z^5 + 16z^4 + 20z^3 \\
\hline
 & 6z^5 + 50z^4 + 41z^3 + 46z^2 + 14z + 5 \\
 & 6z^5 + 48z^4 + 24z^3 + 30z^2 \\
\hline
 & 2z^4 + 17z^3 + 16z^2 + 14z + 5 \\
 & 2z^4 + 16z^3 + 8z^2 + 10z \\
\hline
 & z^3 + 8z^2 + 4z + 5 \\
 & z^3 + 8z^2 + 4z + 5 \\
\hline
 & 0
\end{array}
$$

Hence, h(z) = $4z^3 + 6z^2 + 2z + 1$

or

h[n] = {4 6 2 1}

P.P. 2.10

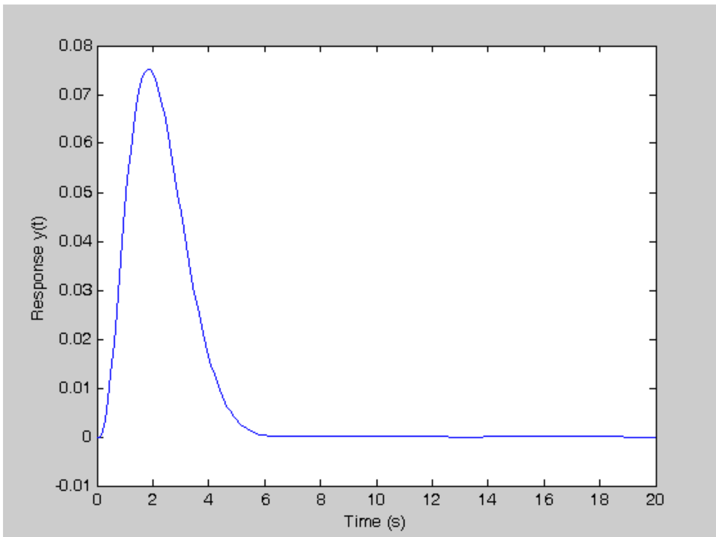
```
x1 = [ 2 5 4 3 5 1 0];
x2 = [ 1 6 3 9  4  7 2];
y = conv(x1,x2)
y = [2   17   40   60   88  110  103   98   58   45   17   2   0]
```

P.P. 2.11

The MATLAB code and the plot are given below.

```
T = 0.1;  % sampling period
t = 0:T:10;
x = exp(-t).*sin(t); % calculates x(t)
```

```
h = 0.5*( exp(-t) -  exp(-4*t) );  % calculates h(t)
y = T*conv(x,h);  % calculate y[n] = T x(n)*h(n)
t0 = (0:200)*T
plot(t0,y)  % or use this plot(t,y(1:101))
xlabel('Time (s)')
ylabel('Response y(t)')
```



P.P. 2.12

The MATLAB code with the result is shown below.

```
>> x = [1 8 4 5];

>> y = [4 38 66 61 46 14 5];

>> h= deconv(y,x)

h =

    4     6     2     1
```

P.P. 2.13

If h(t) = u(t), then

$$\int_{-\infty}^{\infty} |\, h(\tau)\,|\, d\tau = \int_{0}^{\infty} (1) d\tau = \tau \Big|_{0}^{\infty} = \infty$$

h(t) is not integrable and hence the system is unstable.

P.P. 2.14

$$\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{\infty} (-0.6)^n = \frac{1}{1-0.6} = 2.5 < \infty$$

which is finite. Hence the system is stable.

P.P. 2.15

$$v_o(t) = v_s(t) * h(t) = \int u(\tau) h(t-\tau) d\tau = \int 5e^{-2\tau} e^{-(t-\tau)} d\tau$$

$$= 5e^{-t} \int_0^t e^{-\tau} d\tau = 5e^{-t}(1-e^{-t})$$

$$= 5(e^{-t} - e^{-2t}) u(t)$$

Prob.2.1

Change variable by letting $\lambda = t - \tau$.

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau) d\tau = \int_{-\infty}^{\infty} x_1(t-\lambda) x_2(\lambda) d\lambda$$

$$= \int_{-\infty}^{\infty} x_2(\lambda) x_1(t-\pi) d\lambda = x_2(t) * x_1(t)$$

Prob. 2.2

(a) $$f(t) * \delta(t) = \int_{-\infty}^{\infty} f(\tau) \delta(t-\tau) d\tau = f(\tau)\Big|_{\tau = t} = f(t)$$

(b) $$f(t) * \delta(t-t_o) = \delta(t-t_o) * f(t) = \int_{-\infty}^{\infty} \delta(t-t_o) f(t-\tau) d\tau$$

$$= f(t-\tau)\Big|_{\tau = t_o} = f(t-t_o)$$

(c) $$f(t)*u(t)=\int_{-\infty}^{\infty} f(\tau)u(t-\tau)d\tau$$

But $$u(t-\tau)=\begin{cases}1, & \tau<t\\ 0, & t>t\end{cases}$$

Hence,

$$f(t)*u(t)=\int_{-\infty}^{t} f(\tau)d\tau$$

Prob. 2.3

$$x(t)*\frac{d}{dt}\delta(t)=\int_{-\infty}^{\infty} x(\tau)\frac{d}{dt}\delta(t-\tau)d\tau=\frac{d}{dt}x(t)$$

Prob. 2.4

(a) $x(t)*y(t)=2\delta(t)*4u(t)=8u(t)$

(b) $x(t)*z(t)=2\delta(t)*e^{-2t}u(t)=2e^{-2t}u(t)$

(c) $y(t)*z(t)=4u(t)*e^{-2t}u(t)=4\int_0^t e^{-2\lambda}d\lambda=\left.\frac{4e^{-2\lambda}}{-2}\right|_0^t=2(1-e^{-2t})$

(d) $y(t)*[y(t)+z(t)]=4u(t)*[4u(t)+e^{-2t}u(t)]=4\int[4u(\lambda)+e^{-2\lambda}u(\lambda)]d\lambda$

$$=4\int_0^t[4+e^{-2\lambda}]d\lambda=4[4t+\left.\frac{e^{-2\lambda}}{-2}]\right|_0^t=16t-2e^{-2t}+2$$

Prob. 2.5

(a) t*e$^{\alpha t}$u(t) =

$$\int_0^t e^{a\lambda}(t-\lambda)d\lambda=t\left.\frac{e^{a\lambda}}{a}\right|_0^t-\left.\frac{e^{a\lambda}}{a^2}(a\lambda-1)\right|_0^t=\frac{t}{a}(e^{at}-1)-\frac{1}{a^2}-\frac{e^{at}}{a^2}(at-1)$$

(b) $\cos t*\cos t u(t)=\int_0^t \cos\lambda\cos(t-\lambda)d\lambda=\int_0^t\{\cos t\cos\lambda\cos\lambda+\sin t\sin\lambda\cos\lambda\}d\lambda$

$$=\left[\cos t\int_0^t\frac{1}{2}[1+\cos 2\lambda]d\lambda+\sin t\int_0^t\cos\lambda d(-\cos\lambda)\right]=\left[\frac{1}{2}\cos t[\lambda+\left.\frac{\sin 2\lambda}{2}]\right|_0^t-\sin t\left.\frac{\cos\lambda}{2}\right|_0^t\right]$$

= 0.5cos(t)(t+0.5sin(2t)) – 0.5sin(t)(cos(t) – 1).

Prob. 2.6

(a) We use the property $f(t) * \delta(t - t_o) = f(t - t_o)$

$$y(t) = x(t-2) = (t-2)^2$$

(b) $y(t) = x(t-2) = (t-2)(t-2+0.5)^2 = (t-2)(t-1.5)^2$

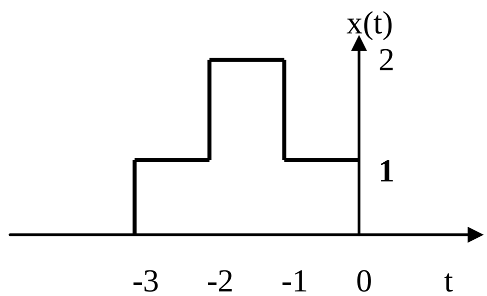
(c) $y(t) = x(t-2) = 4e^{-2(t-2)} \cos\left[2\pi(t-2) - \pi/2\right]$

$$= 4e^4 e^{-2t} \cos\left[2\pi t - 4\pi - \pi/2\right] = 4e^4 e^{-2t} \cos\left(2\pi t - \pi/2\right)$$

Prob. 2.7

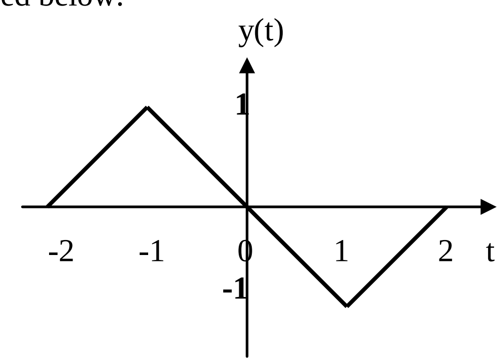
(a) $x(t) = \Pi(t/2) * \left[\delta(t+1) + \delta(t+2)\right] = \Pi((t+1)/2) + \Pi((t+2)/2)$

which is a combination of two unit rectangular pulses, shown below.

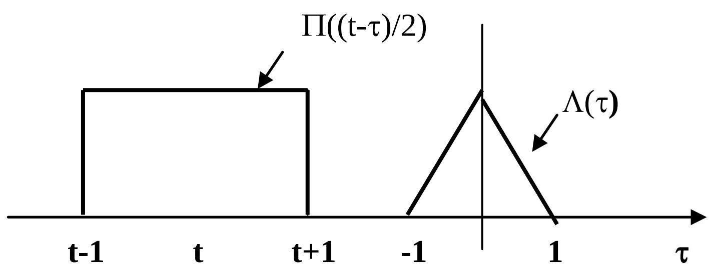


(b) $y(t) = \left[\Lambda(t+1) - \Lambda(t-1)\right] * \delta(t) = \Lambda(t+1) - \Lambda(t-1)$

which is sketched below.

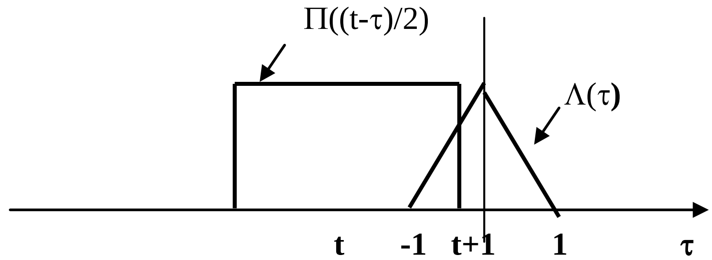


(c) Both $\Pi((t-\tau)/2$ and $\Lambda(\tau)$ are sketched below.

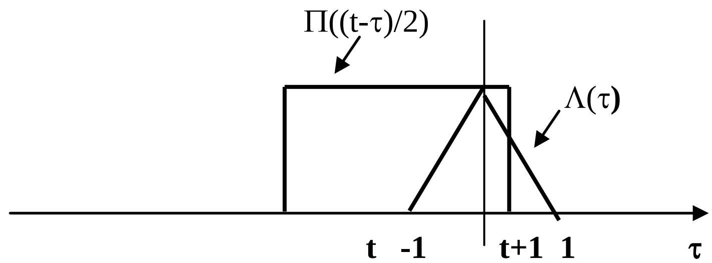


When t < 2, there is no overlap. For -2 < t < -1,

$$z(t) = \int_{-1}^{t+1} (1)(1+\tau)d\tau = \tau + \frac{\tau^2}{2}\Big|_{-1}^{t+1} = 0.5t^2 + 2t + 2, \qquad -2 < t < -1$$



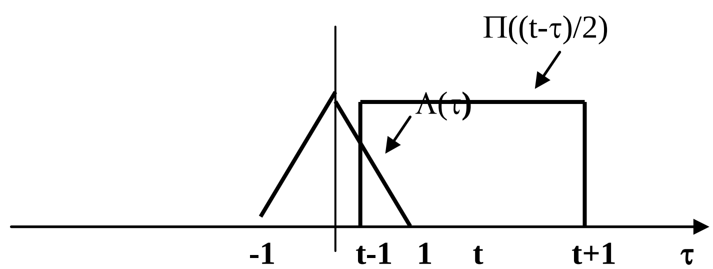
For -1 < t <1,



$$z(t) = \int_{-1}^{0} (1)(1+\tau)d\tau + \int_{0}^{t+1} (1)(1-\tau)d\tau = \left(\tau + \frac{\tau^2}{2}\right)\Big|_{-1}^{0} + \left(\tau - \frac{\tau^2}{2}\right)\Big|_{0}^{t+1}$$

$$= 1 - \frac{1}{2}t^2, \qquad -1 < t < 1$$

For 1 < t <2,

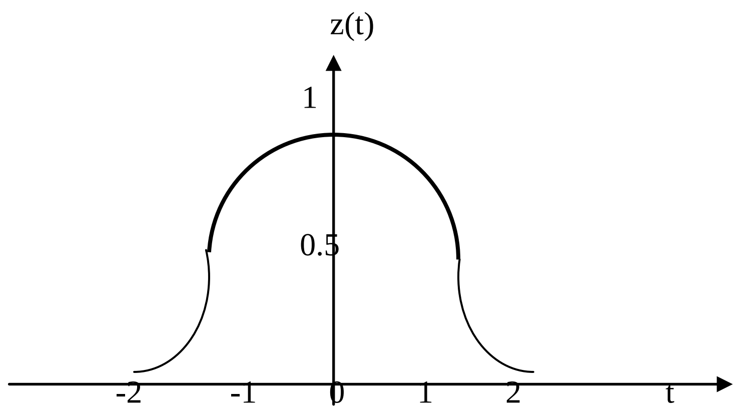


$$z(t) = \int_{t-1}^{1} (1)(1-\tau)d\tau = \tau - \frac{\tau^2}{2}\Big|_{t-1}^{1} = 0.5t^2 - 2t + 2, \qquad 1 < t < 2$$

Thus.

$$z(t) = \begin{cases} 0.5t^2 + 2t + 2, & -2 < t < -1 \\ 1 - 0.5t^2, & -1 < t < 1 \\ 0.5t^2 - 2t + 2, & 1 < t < 2 \\ 0, & \text{otherwise} \end{cases}$$

$z(t)$ is sketched below.



Prob. 2.8

(a) $$u(t)*u(t) = \int_0^t u(\tau)u(t-\tau)d\tau = \int_0^t (1)(1)d\tau = tu(t) = r(t)$$

(b) We use one of the properties of convolution, i.e.

$$y(t) = x(t)*h(t) \quad \longrightarrow \quad x(t+t_1)*h(t+t_2) = y(t+t_1+t_2)$$

Hence,

$$u(t+t_1)*u(t+t_2) = r(t+t_1+t_2)$$

If $t_1 = -1$ and $t_2 = -3$, then

$$u(t-1)*u(t-3) = r(t-4)$$

(c) $\Pi(\mathrm{t/2}) = u(t+0.5\tau) - u(t-0.5\tau)$

$$\Pi(t/2) * \Pi(t/2) = u(t+0.5\tau) * u(t+0.5\tau) - 2u(t+0.5\tau) * u(t-0.5\tau)$$
$$+ u(t-0.5\tau) * u(t-0.5\tau)$$
$$= r(t+\tau) - 2r(t) + r(t-\tau)$$

But $\Lambda(t/\tau) = \frac{1}{\tau}\left[r(t+\tau) - 2r(t) + r(t-\tau)\right]$

Hence,

$$\Pi(t/2) * \Pi(t/2) = \tau\Lambda(t/\tau)$$

Prob. 2.9

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} e^{-2\tau} 10 e^{-(t-\tau)} d\tau$$
$$= 10e^{-t} \int_0^t e^{-\tau} d\tau = 10e^{-t}(-e^{-t} + 1)$$
$$= 10(e^{-t} - e^{-2t})u(t)$$

Prob. 2.10

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$x(\tau) = \cos 2\tau, \qquad h(t-\tau) = \begin{cases} e^{(t-\tau)}, & t-\tau < 0 \\ e^{(\tau-t)}, & t-\tau \geq 0 \end{cases} = \begin{cases} e^{(t-\tau)}, & \tau < t \\ e^{(\tau-t)}, & \tau \geq t \end{cases}$$

$$y(t) = \int_{-\infty}^{t} \cos 2\tau e^{(\tau-t)} d\tau + \int_t^{\infty} \cos 2\tau e^{(t-\tau)} d\tau = e^{-t} \int_{-\infty}^{t} \cos 2\tau e^{\tau} d\tau + e^{t} \int_t^{\infty} \cos 2\tau e^{-\tau} d\tau$$
$$= e^{-t} \left[\frac{e^{\tau}(\cos 2\tau + 2\sin 2\tau)}{5}\right]\Bigg|_{-\infty}^{t} + e^{t} \left[\frac{e^{-\tau}(-\cos 2\tau + 2\sin 2\tau)}{5}\right]\Bigg|_{t}^{\infty}$$
$$= 0.2\cos 2t$$

Prob. 2.11

$$y(t) = h(t) * x(t) = \left(e^{-2t}u(t) - \delta(t)\right) * e^{-t}u(t) = \frac{1}{2-1}\left(e^{-t} - e^{-2t}\right)u(t) - e^{-t}u(t)$$
$$= e^{-t}u(t) - e^{-2t}u(t) - e^{-t}u(t) = -e^{-2t}u(t)$$

Prob. 2.12

For t<0, the product of x(τ) and h(t-τ) is zero, i.e. y(t) = 0. For t >0,

$$x(\tau)h(t-\tau) = \begin{cases} e^{-2\tau}, & 0 < \tau < t \\ 0, & \text{otherwise} \end{cases}$$

$$y(t) = \int_0^t e^{-2\tau} d\tau = -\frac{1}{2} e^{-2\tau} \Big|_0^t = \frac{1}{2}(1 - e^{-2t})$$

i.e. $y(t) = \frac{1}{2}(1 - e^{-2t})u(t)$

Prob. 2.13

(a) Using Table 2.2,

$$e^{-2t}u(t) * u(t) = \frac{e^{-2t} - 1}{-2} u(t) = \frac{1}{2}(1 - e^{-2t})u(t)$$

(b) Using Table 2.2,

$$e^{-2t}u(t) * e^{-t}u(t) = \frac{1}{1-2}(e^{-t} - e^{-2t})u(t) = (e^{-2t} - e^{-t})u(t)$$

(c) $\cos 2t u(t) * e^{-2t}u(t) = \int_0^t \cos 2\tau e^{-2(t-\tau)} d\tau = e^{-2t} \int_0^t \cos 2\tau e^{2\tau} d\tau$

$$= e^{-2t} \left[\frac{e^{2\tau}}{4+4} \left(2\cos 2\tau + 2\sin 2\tau \right) \right] \Bigg|_0^t$$

$$= \frac{e^{-2t}}{8} \left[e^{2t}(2\cos 2t + 2\sin 2t) - 2 \right]$$

$$= \frac{1}{4} \left(\cos 2t + \sin 2t - e^{-2t} \right) u(t)$$

Prob. 2.14

$x(t) = u(t)$

$$y(t) = x(t) * h(t) = \int_0^t e^{-\tau} u(\tau) d\tau = (1 - e^{-t})u(t)$$

Prob. 2.15

(a) $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau) = x(t)$

i.e. a unity system.

(b) $y(t) = \int_{-\infty}^{\infty} x(\tau) \frac{d}{dt} \delta(t-\tau) d\tau = \frac{d}{dt} x(t)$

i.e. the system is a differentiator.

(c) $$y(t) = \int_{-\infty}^{\infty} x(\tau)u(t-\tau)d\tau = \int_{-\infty}^{t} x(\tau)d\tau$$

i.e. the system is an integrator

Prob. 2.16

$$y(t) = h(t) * x(t) = \left[4e^{-2t}u(t)\right] * \left[\delta(t) - 2e^{-2t}u(t)\right]$$

$$= 4e^{-2t}u(t) * \delta(t) - 4e^{-2t}u(t) * 2e^{-2t}u(t) = 4e^{-2t}u(t) - 8e^{-2t}\int_0^t e^o d\lambda$$

$$= 4e^{-2t}u(t) - 8te^{-2t}u(t)$$

Prob. 2.17

In Example 2.3,

$$A_{x1} = 2\times 1 = 2, \qquad A_{x2} = 1\times 2 = 2, \qquad A_y = \frac{1}{2}(1+3)(2) = 4$$

Hence, $A_y = A_{x1}A_{x2}$

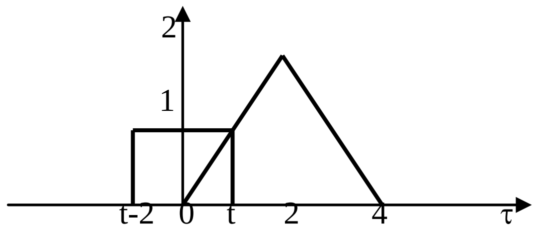
Prob. 2.18

$$x_1(t) = \begin{cases} t, & 0 < t < 2 \\ 4-t, & 2 < t < 4 \end{cases}$$

Let $y(t) = x_1(t) * x_2(t) = \int_0^t x_2(t-\tau)x_1(\tau)d\tau$

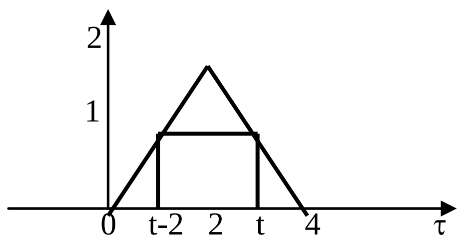
For t < 0. there is no overlapping, y(t) = 0.

For 0 < t < 2, there is overlapping as shown below.



$$y(t) = \int_0^t (1)\tau d\tau = \frac{\tau^2}{2}\Big|_0^t = \frac{1}{2}t^2, \qquad 0 < t < 2$$

For 2 < t < 4. there is overlapping as shown below.



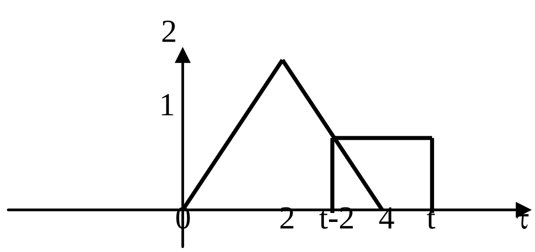
$$y(t) = \int_{t-2}^{2} (1)\tau d\tau + \int_2^t (1)(4-\tau) d\tau = \frac{\tau^2}{2}\Big|_{t-2}^{2} + \left(4\tau - \frac{\tau^2}{2}\right)\Big|_2^t$$

$$= 2 - \frac{1}{2}(t-2)^2 + \left(4t - \frac{1}{2}t^2 - 8 + 2\right)$$

$$= 2 - \frac{1}{2}t^2 + 2t - 2 + 4t - \frac{1}{2}t^2 - 6$$

$$= -t^2 + 6t - 6, \qquad 2 < t < 4$$

For 4 < t < 6. the overlapping is as shown below.



$$y(t)=\int_{t-2}^{4}(1)(4-\tau)d\tau=\left(4\tau-\frac{\tau^2}{2}\right)\Bigg|_{t-2}^{4}$$

$$=16-8-4(t-2)+\frac{1}{2}(t-2)^2$$

$$=8-4t+8+\frac{1}{2}t^2-2t+2$$

$$=\frac{1}{2}t^2-6t+18,\quad 4<t<6$$

Thus,

$$y(t)=\begin{cases}0.5t^2, & 0<t<2\\ -t^2+6t-6, & 2<t<4\\ 0.5t^2-6t+18, & 4<t<6\\ 0, & \text{otherwise}\end{cases}$$

Prob. 2.19

Let $y(t)=x(t)*h(t)=\int x(t-\tau)h(\tau)d\tau$

For t < 0. there is no overlap, y(t) = 0.
For t > 0,

$$y(t)=\int_0^t(1)\left(2e^{-\tau}\right)d\tau=\frac{2e^{-\tau}}{-1}\Bigg|_0^t=-2(e^{-t}-1)$$

$$=2(1-e^{-t}),\qquad t>0$$

Prob. 2.20

(a) $x_1(t) = u(t-1) - u(t-3), \qquad x_2(t) = u(t) - u(t-1)$

$y(t) = x_1(t) * x_2(t) = u(t-1)*u(t) - u(t-3)*u(t) - u(t-1)*u(t-1) + u(t-3)*u(t-1)$

But

$x_1(t+t_1) * x_2(t) = y(t+t_1)$

$x_1(t+t_1) * x_2(t+t_2) = y(t+t_1+t_2)$

Hence,

$$y(t) \; = r(t\text{-}1) - r(t\text{-}2) - r(t-3) + r(t-4) = \begin{cases} t-1, & 1<t<2 \\ 1, & 2<t<3 \\ 4-t, & 3<t<4 \\ 0, & \text{otherwise} \end{cases}$$

(b) $x_1(t) = u(t+2) - 2u(t) + u(t-1), \qquad x_2(t) = 4e^{-2t}u(t)$

$$y(t) = x_1(t) * x_2(t) = u(t+1) * x_2(t) - 2u(t) * x_2(t) + u(t-1) * x_2(t)$$
$$= y_o(t+1) - 2y_o(t) + y_o(t-1)$$

where

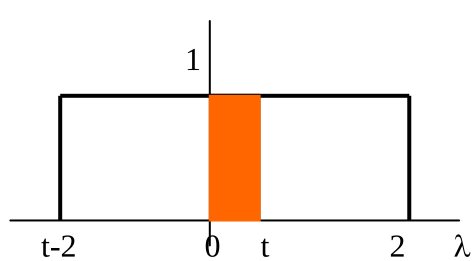
$$y_o(t) = u(t) * x_2(t) = \int_{-\infty}^{t} x_2(\tau)d\tau = \begin{cases} 0, & t \le 0 \\ 4\int_0^t e^{-2\varsigma}d\tau \end{cases} = 2(1-e^{-2t})u(t)$$

$$y(t) = 2(1-e^{-2t-2})u(t+1) - 4(1-e^{-2t})u(t) + 2(1-e^{-2t+2})u(t-1)$$

$$= \begin{cases} 0, & t < -1 \\ 2(1-e^{-2t-2}), & -1 \le t \le 0 \\ 2(2e^{-2t} - e^{-2t-4} - 1), & 0 \le t \le 1 \\ 2(2e^{-2t} - e^{-2t-4} - e^{-2t+2}), & t \ge 1 \end{cases}$$

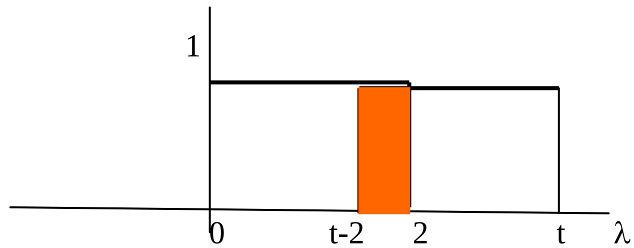
Prob. 2.21

For 0<t<2, the signals overlap as shown below.



$$y(t) = x(t) * x(t) = \int_0^t (1)(1)d\lambda = t$$

For 2 < t< 4, they overlap as shown below.



$$y(t) = \int_{t-2}^{2} (1)(1)d\lambda = t \Big|_{t-2}^{2} = 4 - t$$

Thus,

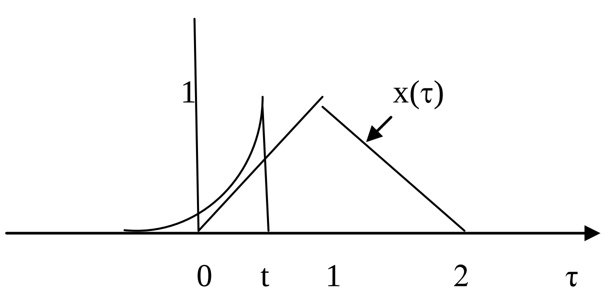
$$y(t) = \begin{cases} t, & 0 < t < 2 \\ 4 - t, & 2 < t < 4 \\ 0, & \text{otherwise} \end{cases}$$

Prob. 2.22

$$x(t) = \begin{cases} t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{cases}$$

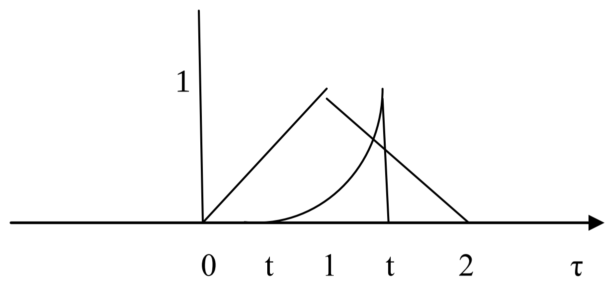
Let $y(t) = x(t) * h(t) = \int_0^t x(\tau)h(t-\tau)d\tau$

For t<0, y(t)= 0. For 0 < t < 1, x and h overlap as shown below.



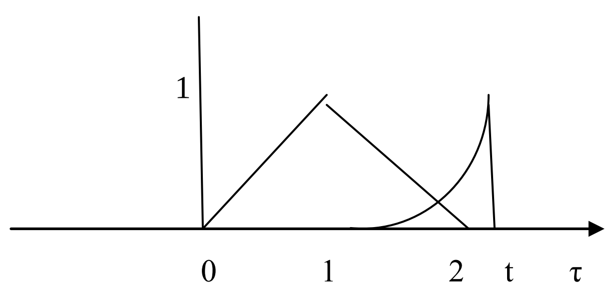
$$y(t) = \int_0^t \tau e^{-(t-\tau)}d\tau = e^{-t}\int_0^t \tau e^{\tau}d\tau = e^{-t}\left[e^t(t-1)+1\right] = e^{-t} + t - 1$$

For 1 < t < 2, they overlap as shown below.



$$y(t)=\int_0^1 \tau e^{-(t-\tau)}d\tau+\int_1^t (2-\tau)e^{-(t-\tau)}d\tau = e^{-t}\left[e^{\tau}(\tau-1)\right]\Big|_0^1+e^{-t}\left[2e^{\tau}-e^{\tau}(\tau-1)\right]\Big|_0^t$$
$$=e^{-t}-2e^{-t+1}-t+3,\qquad 1<t<2$$

For t>2, they overlap as shown below.



$$y(t)=\int_0^1 \tau e^{-(t-\tau)}d\tau+\int_1^2 (2-\tau)e^{-(t-\tau)}d\tau = e^{-t}+e^{-t}\left[2e^{\tau}-e^{\tau}(\tau-1)\right]\Big|_1^2$$
$$=e^{-t}-2e^{-t+1}+e^{-t+2},\qquad t>2$$

Thus,

$$y(t)=\begin{cases} 0, & t<0 \\ e^{-t}+t-1, & 0<t<1 \\ e^{-t}-2e^{-t+1}-t+3, & 1<t<2 \\ e^{-t}-2e^{-t+1}+e^{-t+2}, & t>2 \end{cases}$$

Prob. 2.23

$h(t)=\left(4e^{-t}-e^{-2t}\right)u(t)$

Prob. 2.24

$$h(t) = h_5(t) * \left[h_4(t) + h_2(t) * h_3(t) - h_1(t)\right]$$

Prob. 2.25

(a) When $a = 1$,

$$\sum_{k=0}^{N-1} 1^k = 1 + 1 + 1 + \cdots + 1 \text{ (in N places) } = N$$

For $a \neq 1$, let

$$F = \sum_{k=0}^{N-1} a^k = 1 + a + a^2 + a^3 + \cdots + a^{N-1} \qquad (1)$$

$$aF = a\sum_{k=0}^{N-1} a^k = a + a^2 + a^3 + a^4 + \cdots + a^N \qquad (2)$$

Subtracting (2) from (1) gives

$$(1-a)F = 1 - a^N$$

$$F = \sum_{k=0}^{N-1} a^k = \frac{1-a^N}{1-a}$$

Thus,

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N, & a = 1 \\ \dfrac{1-a^N}{1-a}, & a \neq 1 \end{cases}$$

(b) From part (a)

$$\sum_{k=0}^{N-1} a^k = \frac{1-a^N}{1-a}$$

We take the limit of this as N approaches infinity

$$\sum_{k=0}^{\infty} a^k = \lim_{N\to\infty} \frac{1-a^N}{1-a} = \frac{1}{1-a}, \qquad |a| < 1$$

(c) From part (b),

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$$

Taking the derivative of both sides,

$$\sum_{k=0}^{\infty} k a^{k-1} = \frac{1}{(1-a)^2}$$

Multiplying both sides by a,

$$\sum_{k=0}^{\infty} k a^k = \frac{a}{(1-a)^2}$$

Prob. 2.26

$$y[n] = h[n] * u[n] = (0.4)^n u[n] * \left[\delta[n-1] + 3\delta[n]\right]$$
$$= (0.4)^{n-1} u[n-1] + 3(0.4)^n u[n]$$
$$y[2] = (0.4)^1 u[-1] + 3(0.4)^2 u[2] = 0 + 3(0.4)^2 (1) = 0.48$$
$$y[5] = (0.4)^4 u[4] + 3(0.4)^5 u[5] = 0.4^4 + 3(0.4)^5 = 0.05632$$

Prob. 3.27

(a) For n < 0, $y[n] = 0$. For n ≥ 0,

$$y[n] = \sum_{k=0}^{n} 4^k (1)(1) = \frac{4^{n+1} - 1}{4 - 1} = \frac{1}{3}\left(4^{n+1} - 1\right) u[n]$$

(b) $$y[n] = 2^n u[n] * 2^n u[n] = \sum_{k=0}^{n} 2^k u[k] 2^{n-k} u[n-k] = 2^n \sum_{k=0}^{n} (1) = 2^n (n+1) u[n]$$

(c) $$y[n] = \sum_{k=0}^{n} x[k] h[n-k] = 2^n \sum_{k=0}^{n} \left(\frac{0.3}{2}\right)^k = 2^n \left(\frac{1.5^n - 1}{1.5 - 1}\right) u[n]$$
$$= 2^{n+1} (1.5^n - 1) u[n]$$

Prob. 3.28

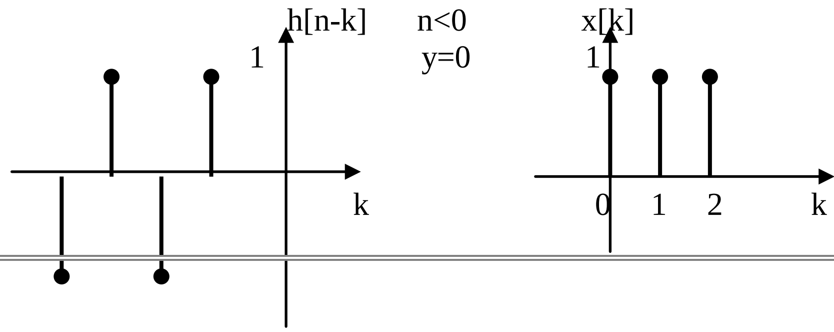
Using entry 4 in Table 2.4,

$$y[n] = 6(3)^n u[n] * (2)^n u[n] = 6\frac{\left[3^{n+1} - 2^{n+1}\right]}{3 - 2} = 6\left[3^{n+1} - 2^{n+1}\right]$$

Prob. 3.29

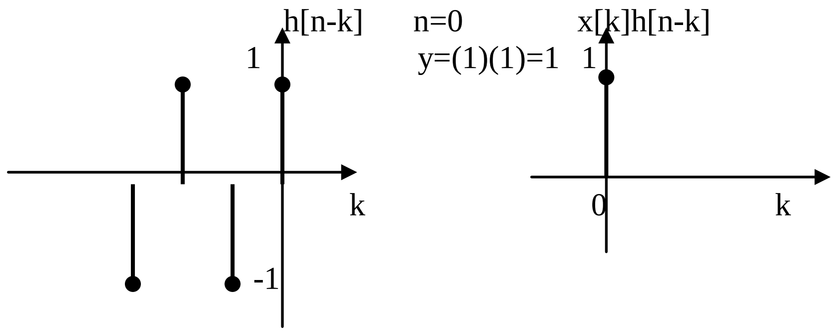
$$y[n] = x[n] * h[n] = \sum_{k=0}^{n} x[k] h[n-k]$$

y[n] is shown below for n<0.

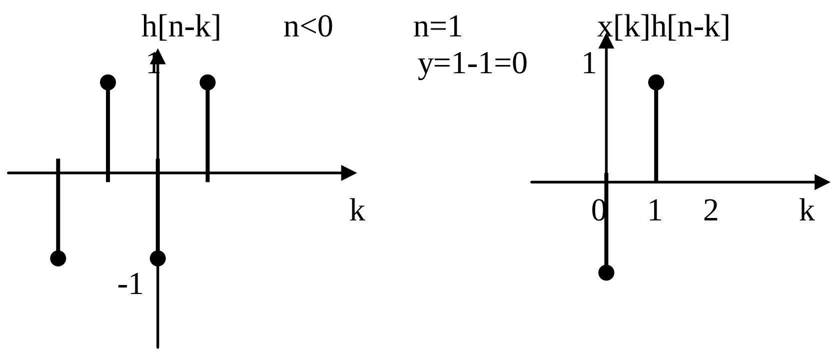


-1

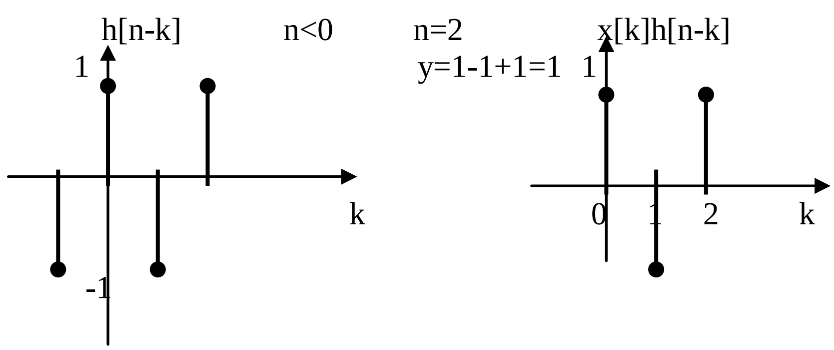
For n=0, y[0] is calculated as shown below.



For n=1, y[1] is calculated as shown below



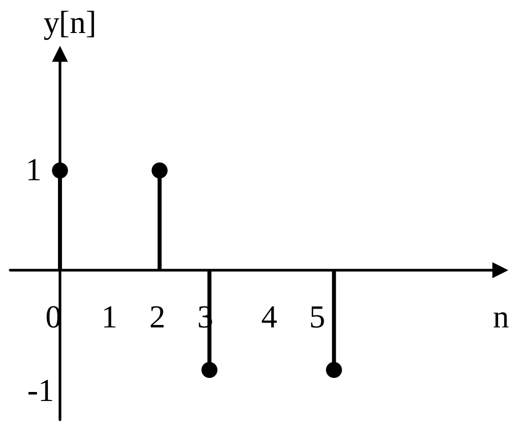
For n=2, y[2] is calculated as shown below.



Continuing this way, we obtain

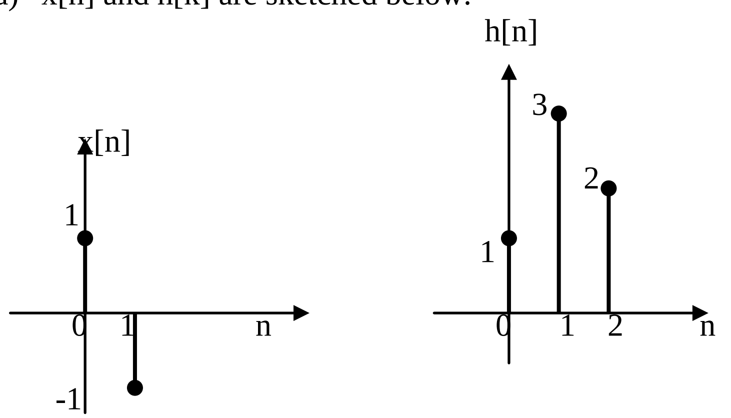
$$y[n]=\begin{cases}1, & n=0\\ 0, & n=1\\ 1, & n=2\\ -1, & n=3\\ 0, & n=4\\ -1, & n=5\\ 0, & \text{otherwise}\end{cases}$$

which is sketched below.



Prob. 2.30

(a) x[n] and h[k] are sketched below.



(b)This can be done in three ways.

<u>Method 1:</u> Using MATLAB,

x = [1 -1];

h = [1 3 2];
y = conv(x,h) = [1 2 -1 -2]

Method 2: Analytically, we can apply x[n]*δ[n-k] = x[n-k]

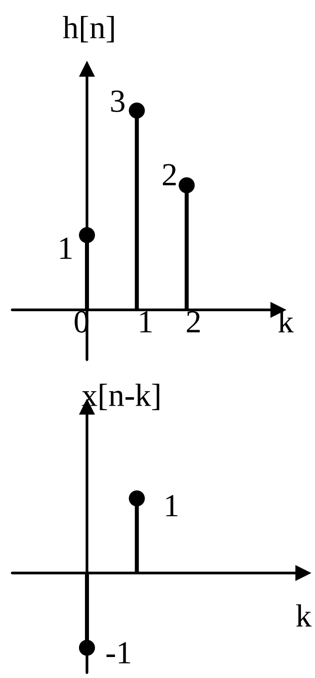
$x[n] = \delta[n] - \delta[n-1]$

$h[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-2]$

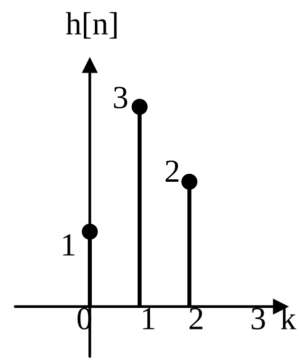
$$y[n] = x[n]*h[n] = \delta[n]*\delta[n] + 3\delta[n]*\delta[n-1] + 2\delta[n]*\delta[n-2]$$
$$-\delta[n]*\delta[n-1] - 3\delta[n-1]*\delta[n-1] - 2\delta[n-1]*\delta[n-2]$$
$$= \delta[n] + 2\delta[n-1] - \delta[n-2] - 2\delta[n-3]$$

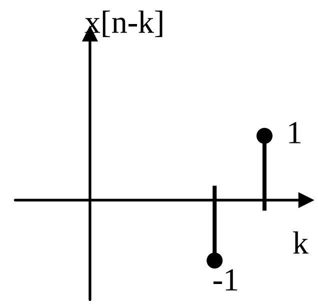
Method 3: Graphically, $y[n] = x[n]*h[n] = \sum_{k=0}^{n} h[k]x[n-k]$

For n = 0, y]0] = (1)(1)=1. For n=1, we have the figure below



y[1] = (1)(-1)+(1)(3)=2
For n=2, y[2] = (3)(-1) + (1)(2)= -1
For n=3, we have the figure shown below.



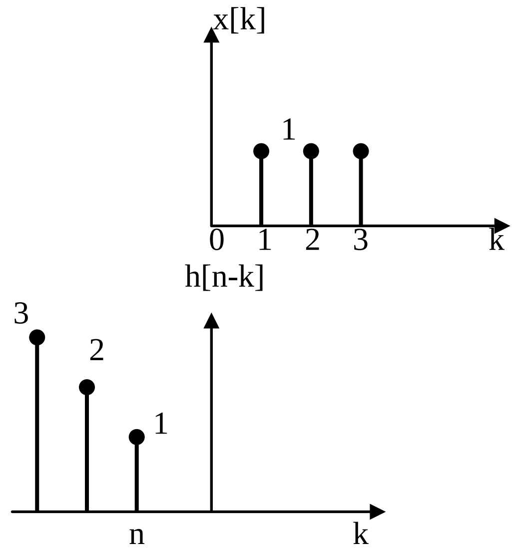


y[3]=(2)(-1)= -2. Thus,

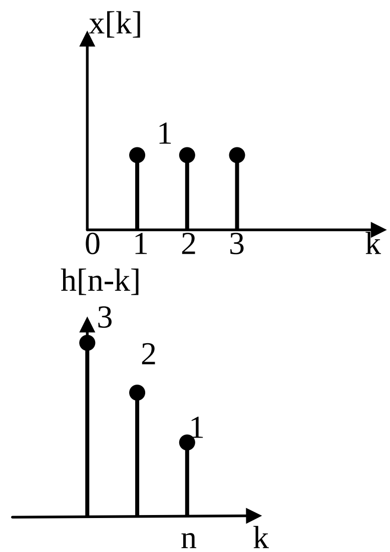
$$y[n]=\begin{cases} 1, & n=0 \\ 2, & n=1 \\ -1, & n=2 \\ -2, & n=3 \\ 0, & \text{otherwise} \end{cases}$$

Prob. 2.31

<u>Method 1</u>: Graphically, y[n] = x[n]*h[n]



For n =0, there is no overlap, y[0] = 0.
For n = 1, y[1] = 1(1) = 1
For n=2, they overlap as shown below.



y[2] = 2(1) + 1(1) = 3
For n=3, y[3]= 3(1) + 2(1) + 1(1) = 6 and so on.

y[n] = [0 1 3 6 5 3]

<u>Method 2:</u>

$$y[n] = x[n]*h[n] = \sum_{k=0}^{\infty} x[k]h[n-k]$$
$$= x[1]h[n-1] + x[2]h[n-2] + x[3]h[n-3]$$
$$= h[n-1] + h[n-2] + h[n-3]$$
$$y[1] = h[0] + h[-1] + h[-2] = 1 + 0 + 0 = 1$$
$$y[2] = h[1] + h[0] + h[-1] = 1 + 2 + 0 = 3$$
$$y[3] = h[2] + h[1] + h[0] = 3 + 2 + 1 = 6$$
$$y[4] = h[3] + h[2] + h[1] = 0 + 3 + 2 = 5$$
$$y[5] = h[4] + h[3] + h[2] = 0 + 0 + 3 = 3$$

Prob. 2.32

$$y[n] = \sum_{k=0}^{n} u[k]\left[(0.4)^{n-k} + (0.5)^{n-k+1}\right]u[n-k]$$
$$= \sum_{k=0}^{n}\left[(0.4)^{n-k} + (0.5)^{n-k}\right] = (0.4)^n \sum_{k=0}^{n}(0.4)^{-k} + (0.5)^{n+1}\sum_{k=0}^{n}(0.5)^{-k}$$
$$= (0.4)^n \sum_{k=0}^{n}(1/0.4)^k + (0.5)^{n+1}\sum_{k=0}^{n}(1/0.5)^k$$
$$= (0.4)^n \frac{[1-(2.5)^{n+1}]}{1-2.5} + (0.5)^{n+1}\frac{[1-2^{n+1}]}{1-2}$$
$$= \frac{2}{3}(0.4)^n(2.5^{n+1}-1) - (0.5)^{n+1}(2^{n+1}-1)$$

Prob. 2.33

(a) When they are connected in parallel,

$$h = h_1 + h_2 = (0.4)^n u[n] + \delta[n] + 0.5\delta[n-1]$$
$$y[n] = h[n]*x[n] = (0.4)^n u[n]*\left[(0.4)^n u[n] + \delta[n] + 0.5\delta[n-1]\right]$$
$$= (n+1)(0.4)^n u[n] + (0.4)^n u[n] + 0.5(0.4)^{n-1}u[n-1]$$
$$= (n+2)(0.4)^n u[n] + 1.25(0.4)^n u[n-1]$$

(b) When they are cascaded

$$h = h_1 * h_2 = (0.4)^n u[n] * \left[\delta[n] + 0.5\delta[n-1]\right]$$

$$= (0.4)^n u[n] + 0.5(0.4)^{n-1} u[n-1]$$

$$y[n] = h[n] * x[n] = (0.4)^n u[n] * \left[(0.4)^n u[n] + 0.5(0.4)^{n-1} u[n-1]\right]$$

But if $z[n] = a^n u[n] * a^n u[n] = (n+1)a^n u[n] = r[n]a^n$

$$a^n u[n] * a^{n-1} u[n-1] = z[n-1] = (n-1+1)\, a^{n-1} u[n-1] = na^{n-1} u[n-1]$$

Hence,

$$y[n] = (n+1)(0.4)^n u[n] + n(0.4)^{n-1} u[n-1]$$

Prob. 2.34

$$h[n] = h_1[n] * \left(h_2[n] * h_3[n] - h_4[n]\right)$$

Prob. 2.35

```
h = deconv(y,x)
  = [ 2  3]
```

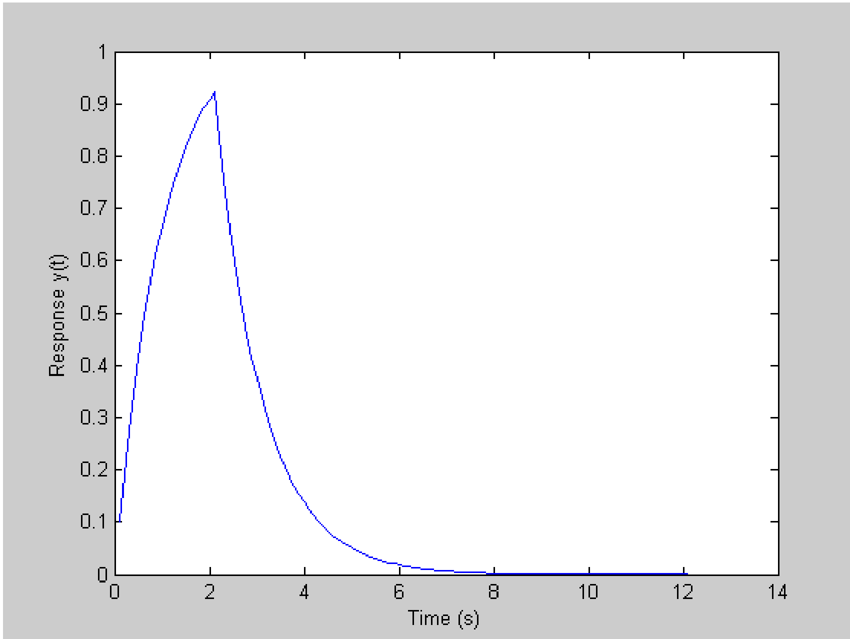
Prob. 2.36

```
h = deconv(y,x)
  = [4  6  11]
```

Prob. 2.37

(a) The MATLAB code and the plot of y(t) are presented below.

```
clear
T = 0.1;  % sampling period
t = 0:T:10;
t1=0:T:2
len1 = max(size(t1));
x =ones(len1,1); %calculates x(t)
h =  exp(-t) ;  % calculates h(t)
y = T*conv(x,h);
len2 = max(size(y));
t0 = (1:len2)*T;
plot(t0,y)     % or use this to  plot(t,y(1:101))
xlabel('Time (s)')
ylabel('Response y(t)')
```



(b) $$y(t) = x(t) * h(t) = e^{-t}u(t) * [u(t) - u(t-2)] = \frac{e^{-t} - 1}{-1}u(t) - \frac{e^{-(t-2)} - 1}{-1}u(t-2)$$

$$= (1 - e^{-t}) - (1 - e^{-(t-2)})u(t-2)$$

$$= \begin{cases} 1 - e^{-t}, & 0 < t < 2 \\ e^{-t}(e^2 - 1), & t > 2 \end{cases}$$

which agrees with the plot.

Prob. 2.38

The MATLAB code and the result are shown below.

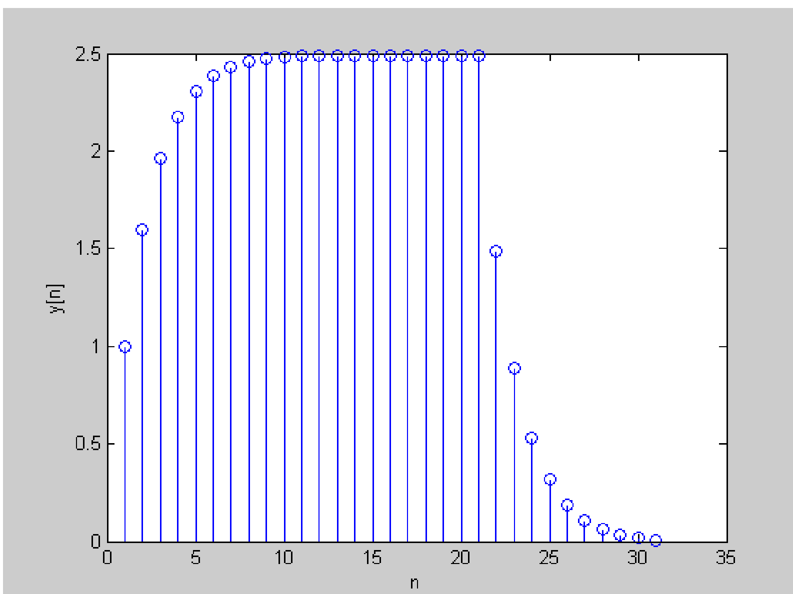
```
clear
n = 0:10;
x = cos(0.5*pi*n);
h =  [ 1 1 1 1 1 -1 -1 -1 -1 -1];
y =  conv(x,h);

y =

  Columns 1 through 11

    1.0000    1.0000         0   -0.0000    1.0000   -1.0000   -2.0000   -0.0000    1.0000   -
1.0000   -1.0000

  Columns 12 through 20
```

1.0000 0.0000 -2.0000 -1.0000 1.0000 0 -0.0000 1.0000 1.0000

Prob. 2.39

The MATLAB code and the plot of y are provided below.

```
m=0:1:20;
len1 = max(size(m));
x =ones(len1,1); %calculates x[n]
n = 0:10;
h = (0.6).^n;  % calculates h[n]
y =  conv(x,h);
u= max(size(y));
nn=1:1:u
stem(nn,y)
xlabel('n');
ylabel('y[n]');
```

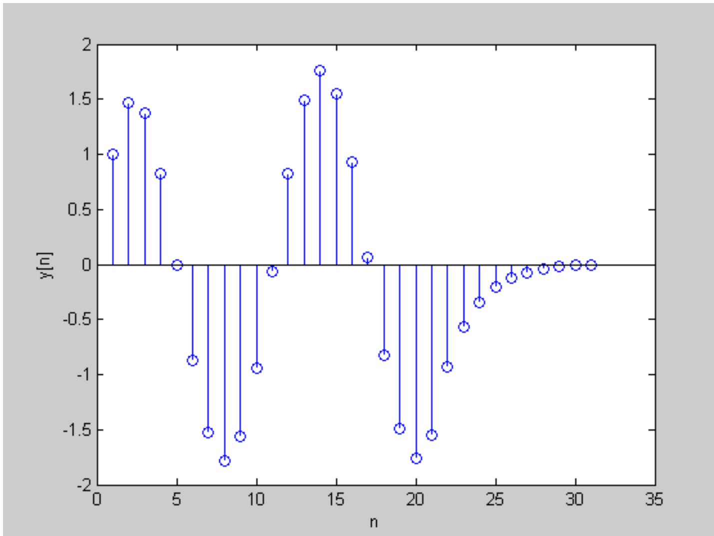


Prob. 2.40

The MATLAB code and the results are shown below.

```
m=0:1:20;
x = cos(m*pi/6); %calculates x[n]
n = 0:10;
h = (0.6).^n;  % calculates h[n]
y =  conv(x,h);
u= max(size(y));
```

```
nn=1:1:u
stem(nn,y)
xlabel('n');
ylabel('y[n]');
```



Prob. 2.41

```
clear
x = [ 1  -1  2  4];
y = [ 2 6 4  0 8 5 12 ];
h = deconv(y,x);
h =

    2     8     8   -16
```

Prob. 2.42

(a) $$\int_{-\infty}^{\infty} |h(\tau)|\,d\tau = \int_{0}^{\infty} e^{2\tau} d\tau = \left.\frac{e^{2\tau}}{\tau}\right|_{0}^{\infty} = \infty$$

Hence, the system is unstable.

(b) $$\int_{-\infty}^{\infty} |h(\tau)|\,d\tau = \int_{0}^{\infty} \sin 2\tau d\tau = \left.\frac{-\cos 2\tau}{2}\right|_{0}^{\infty} < \infty$$

The value of cos2τ varies between 1 and -1. Hence the system is stable.

(c) $$\int_{-\infty}^{\infty} |h(\tau)|\,d\tau = \int_{0}^{\infty} e^{-\tau} \cos 2\tau d\tau \Big| = \frac{e^{-\tau}}{1+4}\left(-\cos 2\tau + 2\sin 2\tau\right)\Big|_{0}^{\infty} = \frac{1}{4} < \infty$$

Hence the system is stable.

Prob. 2.43

$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau = \int_{-1}^{1} (1) d\tau = \tau \Big|_{-1}^{1} = 2 < \infty$$

Hence the system is stable.

Prob. 3.44

(a) $$\sum_{k=0}^{\infty} |h[k]| = \sum \delta[k] = 1$$

i.e. the system is stable.

(b) $$\sum_{k=0}^{\infty} |h[k]| = \sum_{k=0}^{\infty} (-0.5)^k = \frac{1}{1-0.5} = 2$$

i.e. the system is stable.

(c) $$\sum_{k=0}^{\infty} |h[k]| = \sum_{k=0}^{\infty} \left[u[k] - u[k-10]\right] = 10$$

i.e. the system is stable.

Prob. 3.45

$$\sum_{k=0}^{\infty} |h[k]| = \sum_{k=0}^{\infty} u[k] = \infty$$

i.e. the sum of |h[k]| is inifinte. Hence, the system is not BIBO stable.

Prob. 3.46

$$\begin{aligned} v_o(t) &= h(t) * v_s(t) = e^{-t}u(t) * \left[u(t-1) + \delta(t-2)\right] \\ &= \frac{e^{-t}-1}{-1} u(t) - e^{-(t-2)} u(t-2) = (1-e^{-t})u(t) - e^{-t+2} u(t-2) \\ &= \begin{cases} 1-e^{-t}, & 0 < t < 2 \\ 1-e^{-t}(1+e^2), & t > 2 \end{cases} \end{aligned}$$

Prob. 3.47

$$\begin{aligned} v_o(t) &= h(t) * v_s(t) = 2e^{-t}u(t) * u(t) \\ &= \frac{2(e^{-t}-1)}{-1} u(t) = 2\left(1-e^{-t}\right)u(t) \end{aligned}$$

Prob. 3.48

$x(t) = u(t)$

Let $\quad y(t) = x(t) * h(t) = u(t) * \left[\delta(t) - 2e^{-2t}u(t)\right] = u(t) * \delta(t) - u(t) * 2e^{-2t}u(t)$

$$= u(t) - 2\int_{-\infty}^{t} e^{-2\tau} d\tau = u(t) + 2\frac{e^{-2t}}{2}\Bigg|_{-\infty}^{t} = (1 + e^{-2t})u(t)$$