Chapter 2

1.

$$T_{total} = \frac{P_{total}}{\omega} = \frac{10^6}{\left(2\pi\frac{20}{60}\right)} = 477.5 \text{ kN}$$

Torque per blade

$$T_{per\ blade} = \frac{T_{total}}{3} = \frac{477.5}{3} = 159.17 \text{ kN}$$

2.

$$P_u = \frac{P_{blade}}{C_p} = \frac{1}{0.1} = 10\,\text{MW}$$

$$A = \frac{P_u}{\frac{1}{2}\delta\ w_u^3} = \frac{10^7}{0.5*15^3} = \frac{1}{2}\left(\pi 50^2\right)*10^3 = 5926 \text{ m}^2$$

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{5926}{\pi}} = 43.43 \text{ m}$$

3.

$$P = P_0\left(\frac{h}{h_0}\right)^{3*0.1} = 729.3\ kW$$

4.

$$V = \omega\, r_c = 2\pi\frac{n_{blade}}{60} r_c = 2\pi\frac{20}{60}30 = 62.8 \text{ m/s}$$

$$\overline{W_r} = \overline{H} + \overline{W} = 62.8\angle -90^{\circ} + 15\angle 0^{\circ} = 64.56\angle -76.57^{\circ} \text{ m/s}$$

5.

$$w = \frac{w_u + w_d}{2}$$

$$w_u = 2*20 - 15 = 25 \text{ m/s}$$

6.

$$P_{blade} = \frac{1}{2}\delta\, A_{blade}\frac{w + w_d}{2}\left(w^2 - w_d^2\right) = 0.5*\left(\pi*100\right)\frac{20+10}{2}\left(400-100\right) = 706.5 \text{ kW}$$

7.

$$V = \omega r_c = 2\pi \frac{30}{60} 20 = 62.82 \text{ m/s}$$

$$\overline{w_r} = \overline{H} + \overline{w} = 62.82\angle -90^\circ + 15\angle 0^\circ = 64.59\angle -76.57^\circ \text{ m/s}$$

$$90^\circ - 76.57^\circ = \alpha + \beta$$

$$\alpha = 8.43^\circ$$

8.

$$V = \omega * r_c = \frac{2 * \pi * 20}{60} * 20 = 41.89 \frac{m}{s}$$

$$w_r = H + W = 41.89 \angle -90 \quad + \quad 15 \angle 20$$

$$w_r = 39.37 \angle -69.022 \ m/s$$

9.

$$w_r = 15 \angle -20 \ m/s$$
$$\alpha + \beta = 90 - 20$$
$$\alpha = 40^\circ$$

10.

$$Wb \approx \frac{Wu + Wd}{2}$$

$$Wu \approx 2 * Wb - Wd \approx 25 \, m/s$$

11.

$$P_w = \frac{1}{2} \delta A_b w_u^3 = 1.256 \, MW$$
$$\gamma = \frac{10}{20} = 0.5$$
$$Cp = \frac{1}{2} (1 + \gamma)(1 - \gamma^2) = 56.25\%$$
$$P_b = Cp \, P_w = 706.5 \, kW$$

12.

$$C_p = 0.2 = \frac{1}{2}(1 + \gamma)(1 - \gamma^2)$$

$$\gamma = 0.887754$$

$$W_d = \gamma W_u = 8.87754 \text{ m/s}$$

$$W = \frac{Wu + Wd}{2} = 9.43875 \text{ m/s}$$

$$fmass = Aw\sigma = \pi 50^2 * 9.43875 * 1 = 74.13*10^3 \text{ kg/s}$$

$$P_w = 0.5 * A * \sigma * Wu^3 = 3.927 \text{ MW}$$

$$P_{blade} = Cp * P_w = 785.4 \text{ kW}$$

13.

a. $f_{mass} = \frac{m}{t} = Aw\rho$

$$A_u = \frac{f_{mass}}{w\delta} = \frac{10^5}{10} = 10^4 \text{ m}^2$$

$$A_d = \frac{f_{mass}}{w_d \delta} = \frac{10^5}{8} = 1.25*10^4 \text{ m}^2$$

$$A_{blade} = \frac{f_{mass}}{w\delta} = \frac{10^5}{\left(\frac{8+10}{2}\right)} = 1.11*10^4 \text{ m}^2$$

$$d_u = 2\sqrt{\frac{10^4}{\pi}} = 112.8 \text{ m}$$

$$d_d = 2\sqrt{\frac{1.25*10^4}{\pi}} = 126.2 \text{ m}$$

$$P_u = \frac{1}{2}\delta A_u w^3 = \frac{1}{2}10^4 * 10^3 = 5 \text{ MW}$$

$$P_d = \frac{1}{2}\delta A_d w_{downwind}^3 = \frac{1}{2}1.25*10^4 * 8^3 = 3.2 \text{ MW}$$

$$P_{blade} = P_u - P_d = 5 - 3.2 = 1.8 \text{ MW}$$

b. $C_p = \frac{P_{blade}}{P_u}\frac{A_u}{A_{blade}} = \frac{1.8}{5}\frac{10^4}{1.11*10^4} = 0.324$

c. $\gamma = \frac{w_d}{w} = \frac{8}{10} = 0.8$

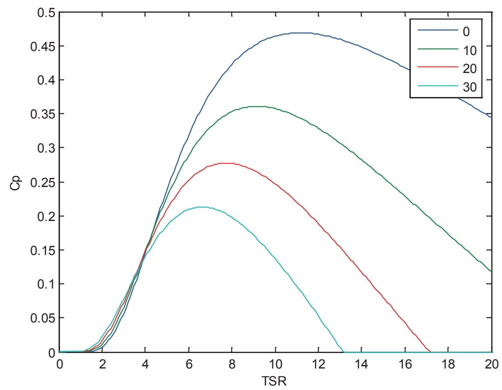
$$C_p = \frac{1}{2}(1+0.8)(1-0.8^2) = 0.324$$

The difference is because of the assumption that the area in the upstream is the same as the sweep area of the blade

14.

$$\Lambda = \frac{1}{\lambda + k_6\beta} - \frac{k_7}{1+\beta^3}$$

$$C_p = k_1(\Lambda - k_2\beta - k_3\beta^3 - k_4)e^{-\Lambda k_5}$$



15.

$$D = 2\,r\,S = 100 * 8 = 800 \text{ m}$$

The number of wind turbines in each row or column is

$$N_{row} = \text{int}\left(\frac{10{,}000}{800}\right) + 1 = 12 + 1 = 13 \text{ turbines}$$

The total number of turbines in the farm is

$$N_{total} = N_{row}^2 = 169 \text{ turbines}$$

16.

$$P_w = \rho\, A_{blade} = 400 * \left(\pi * 50^2\right) = 3.14 \text{ MW}$$

The output power of the turbine is

$$P_{out} = P_w * C_p * \eta = 3.14 * 0.3 * 0.85 = 800 \text{ kW}$$

The power production per land area

$$\frac{P_{out} * n_{total}}{A_{land}} = \frac{800 * 169}{10 * 10} = 1.35 \text{ MW/km}^2$$

Chapter 3

11.

$$w_{ave} = \frac{1}{N}\sum_{i=1}^{\infty} n_i\ w_i$$

$$= \frac{1}{8765}(2*600 + 4*700 + \ldots\ \ldots + 26*15) = 10.54\ \text{m/s}$$

$$Var = \frac{1}{N}\sum_{i=1}^{\infty} n_i\left(w_i - w_{ave}\right)^2$$

$$= \frac{1}{8765}\left(600(2-10.54)^2 + 700(4-10.54)^2 + \ldots\ldots + 15(26-10.54)^2\right) = 25.538\ \ \text{m}^2/\text{s}^2$$

$$\sigma = \sqrt{25.538} = 5.053\ \text{m/s}$$

$$F_{4-16} = \frac{1}{N}\sum_{i=3}^{5} n_i = \frac{1}{8765}(700 + 800 + 900 + 1000 + 1200 + 1100 + 900) = 75.3\%$$

12.

$$W_{avg} = c * \Gamma\left(1 + \frac{1}{k}\right) = 3 *\ \Gamma\left(1 + \frac{1}{0.5}\right) =\ 6\ m/s$$

13.

$$P(w \geq 9) =\ e^{-\left(\frac{9}{10}\right)^{2.17}} = 0.4513$$

$$Total\ number\ of\ hours = 0.4513 * 8760 \approx 3953\ \frac{hrs}{yr}$$