Chapter 2. Problems

Introduction

2.1. What are the typical ranges of resistivity for metals, plastics, and ceramics?

Solution:

Typical ranges of resistivity are as follows:

Metals: $10^{-5} \sim 10^{-6}$ Ω·cm (please note that resistivity of alloys is typically much higher than that of pure metals).

Pure Plastics and Ceramics: 10^{4} - 10^{15} Ω·cm (if donor or acceptor impurities are doped, the resistivity of plastics and ceramics could be decreased to the order of $10^{-3} \sim 10^{-4}$ Ω·cm and semiconducting or conducting behavior is observed).

2.2. What is the nature of bonding between atoms for most ceramics and plastics?

Solution:

Most ceramics the bonding is mixed covalent and ionic. Most plastics the bonding is covalent.

2.3 Compared to metals, what is the advantage in using conducting or semiconducting plastics?

Solution:

Low density, flexibility and high optical transparency of conducting polymers offer key primary advantages in several electronic applications including displays. However, the conductivity of impurity doped plastics is still smaller than that of metals, which is a fundamental problem.

2.4 Which one of these elements shows superconductivity—Ag, Au, or Al?

Solution:

At a very low temperature (~ 1.2 K) aluminum becomes superconducting. Silver and gold do not become superconductors even at very low temperatures. This is because aluminum can form superatoms that are clusters of atoms and contain a giant set of electron shells responsible for Cooper pairing. For additional information on superconductivity in elements see, " Buzea, C. and Robbie, K. in Superconducting Technology, Vol. 18, (2005), pages R1-R8" and "Avik Halder, Anthony Liang, and Vitaly V. Kresin, *Nano Letters* **2015** *15* (2), 1410-1413".

Ohm's Law

2.5 What is the difference between resistance and resistivity?

Solution:

Resistance is not inherence material property, but resistivity is material property. Resistance depends on the geometry of a component i.e. how long how wide etc. and also what a component is made from. In contrast, resistivity depends upon the composition of the material and its microstructure. Resistivity is largely independent of the size (i.e. length, cross-sectional area etc.) of the component.

2.6 Do all materials obey Ohm's law? Explain.

Solution:

Most materials do obey Ohm's law. There are a few materials that do not. For example, certain compositions of zinc oxide and other materials have a microstructure in which grain boundaries can show electrical breakdown if the internal electric field across the material is high enough. Under such conditions, these materials show non-ohmic behavior. Also, electric current passing through a junction between heterogeneous materials such as p-/n-Si and semiconductor/metal does not follow Ohm's law.

2.7. Calculate the resistance of an AWG #20 Cu wire one mile in length.

Solution:

From Table 2.2, the diameter of AWG #20 Cu wire is 0.812 mm, the length is one mile which is 1600 meters.

The resistance R is given by $R = \rho l/A$. Then, use values of resistivity of 1.72×10^{-8} ohm·met (inverse of conductivity of pure copper 5.8×10^{7} S/m), length 1609 m, and area of 5.18×10^{-7} m^2. This gives a resistance of 5.36 ohms.

Copper wires are usually not made from pure copper (because pure copper is too soft) and in this case we can substitute appropriate value of resistivity. For example, if we say the resistivity is 1.678 μΩ·cm, then the value of resistance will be 32.8 ohms for the same diameter and length.

2.8. What is the length of an AWG #16 Cu wire whose resistance is 21 Ω?

Solution:

For 16 gauge wire the diameter is 1.627734 mm (See Table 2.2). This gives an area of 2.08×10^{-6} m^2. This gives a length of 2534 m.

2.9. If the wire in Problem 2.8 carries a current of 5 A, what is the current density?

Solution:

Current density is current per unit area. Area of cross-section is of 2.08×10^{-6} m^2. Thus current density will be 2.40×10^{6} A/m^2.

2.10. Al can handle current densities of 10^5 A/cm^2 at about 150°C (Gupta 2003). What will be the maximum current allowed in an Al wire of AWG #18 operating at 150°C?

Solution:

The wire diameter is (Table 2-2) 1.023696 mm. the area of cross-section is 8.23×10^{-7} m^2. The maximum current will be the product of maximum current density and cross-sectional area and this works out to 823.06 A or 0.823 kA.

2.11. A circuit breaker connects an AWG #0000 Cu conductor wire 300 feet in length. What is the resistance of this wire? If the wire carries 150 A, what is the voltage decrease across this wire?

Solution:

From table 2-2, diameter of the wire is 11.684 mm, therefore cross-sectional area is 1.07×10^{-4} m^2. Length is 300 feet or 91.44 m (one foot is 0.3048 m). We use conductivity of copper as 5.8×10^{7} S/m. Thus, the resistance of this wire will be 0.0147 ohms. From Ohm's law the voltage drop across this will be $150 \times 0.0147 = 2.2$ volts.

2.12. You may know that a conductor carrying an electrical current generates a magnetic field. A long wire carrying a current generates a magnetic field similar to that generated by a bar magnet. This magnet is known as an electromagnet. Consider a meter of magnetic wire AWG #2. Such wires are usually made from high-conductivity soft-drawn electrolytic Cu, and the conductor is coated with a polymer to provide insulation. What will be the electrical resistance (in ohms) of this wire?

Solution:

From Table 2-2, diameter of the AWG # 2 wire is 6.543802 mm, therefore cross-sectional area is 3.36×10^{-5} m^2. Length is 1 meter. We use conductivity of copper as 5.8×10^{7} S/m. Thus, the resistance of this wire will be 0.0005 ohms.

2.13 Ground rods are used for electrical surge protection and are made from materials such as Au, Au-clad steel, or galvanized mild steel. The resistance of the actual rod itself is small; however, the soil surrounding the rod offers electrical resistance (Paschal 2001). The resistance of a ground rod is given by:

$$R = \frac{\rho}{2L\pi}\ln(\left[\frac{4L}{a}\right] - 1)$$

where R is the resistance in ohms, ρ is the resistivity of soil surrounding the ground rod (in Ω -cm), L is the length of ground rod in centimeters, and a is the diameter of the ground rod. (a) Assuming that the resistivity of a particular soil is 10^4 Ω -cm, the length of the rod is 10 feet, and the diameter is 0.75 inches, what will be the resistance (R) of the ground rod in ohms? (b) Assuming that the ground rod is made from Cu, prove that the resistance of the rod itself is actually very small. (c) What will happen to the resistance of the metallic material as it corrodes over a period of many years?

Solution:

(a) Note that we use soli resistivity which is very high compared to resistivity of copper. The value in ohm-m will be 100 ohm-m. Diameter is 0.75 inches which is 1.91 x 10^{-2} m. Length is 10 feet which is 3.048 meters. The ground resistance then works out to be 33.731 ohms. This resistance is typical of soils that are more like inorganic clays as opposed to sand or gravel mixtures (which are much higher resistivity).

(b) Since resistivity of cooper is orders of magnitude smaller compared to that of soil, we expect the electrical resistance of the rod itself will be very small.

(c) If the metallic rod corrodes, the resistance of the grounding rod will go up and it may not function as intended. This is why copper rods can be electroplated to avoid the risk of oxidation.

2.14. The electrical resistance of pure metals increases with temperature. In many ceramics, the electrical current is carried predominantly by ions (such as oxygen ions in YSZ). Based on the data shown in Figure 2.30, calculate the electrical resistance of a 50-μm-thick YSZ element at 500°C and 800°C. Assume that the cross-sectional area is 1 cm^2.

Solution: $R = \rho l/A$

In Figure 2.30, you can find the straight line that shows variation of conductivity of YSZ, it is the line that is marked as $(ZrO_2)_{0.9}(Y_2O_3)_{0.1}$.

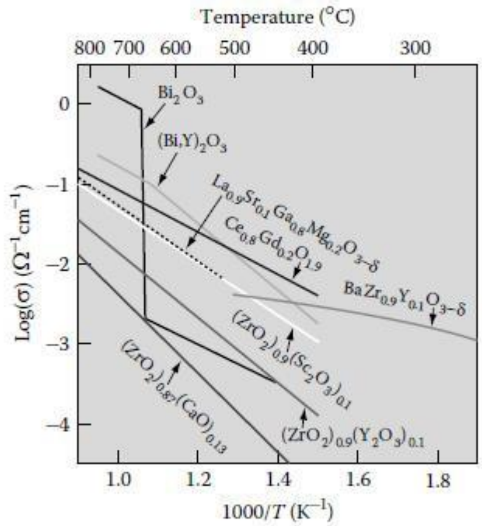

FIGURE 2.30 The conductivity of different materials used as electrolytes in the development of solid oxide fuel cells. (From Haile, S. M. 2003. *Acta Materilia* 51:5981–6000. With permission.)

At 500 C: the log σ = -3, i.e. $\sigma = 10^{-3}\ \Omega^{-1}cm^{-1}$. The area is given as 1 cm^2. Using these values we get a resistance of 5 Ω.

At 800 C: From Figure 2-30, log σ = -1.5 i.e. $\sigma = 10^{-1.5}\ \Omega^{-1}\cdot cm^{-1}$, since dimensions do not change significantly (at least that's what we assume here), the resistance will drop by 1.5 orders of magnitude. The new resistance will be 0.16 Ω.

2.15. Consider the material calcium oxide–stabilized ZrO_2 (Figure 2.30). Calculate the electrical resistance of a 50-µm-thick YSZ element at 500°C and 800°C. Assume that the cross-sectional area is 1 cm^2.

Solution: From Figure 2-30, you can find the conductivity of calcium oxide stabilized ZrO_2. Compared with YSZ, it has smaller conductivity.

At 500 C: the log σ = -4, i.e. $\sigma = 10^{-4}\ \Omega^{-1}cm^{-1}$. The area is given as 1 cm^2. Using these values we get a resistance of 50 Ω.

At 800 C: log σ = -2 i.e. $\sigma = 10^{-2}\ \Omega^{-1}\cdot cm^{-1}$, since dimensions do not change significantly (at least that's what we assume here), the resistance will drop by 2 orders of magnitude. The new resistance will be 0.5 Ω.

2.16. If high conductivity at temperatures above 700°C was the only requirement in selecting a material for a solid oxide fuel cell electrolyte, what material would you choose (Figure 2.30)? Besides cost, what additional factors must be considered in the selection of the electrolyte material?

Solution:

Bismuth oxide exhibits the highest electric conductivity above 700 ^{o}C and this will be chosen, if the only consideration were conductivity. Other most important considerations would be the environmental impact, chemical stability and long term availability (i.e. can the electrolytematerial withstand oxidizing and reducing conditions?). In addition, the thermal expansion of the electrolyte material should be comparable to that of other materials such as metal interconnects to reduce the residual thermal stress in the solid oxide fuel cell.

2.17. What is unusual about the change in resistivity as a function of temperature for bismuth oxide (B_2O_3)?

Solution:

Bismuth oxide shows a sharp change in the electric conductivity at around 680 ^{o}C. Often, such large changes in conductivity are associated with some type of phase transformation. As this transformation occurs, there is often a volume change. The volume change leads to the build-up of stress and usually ceramic materials would fracture if such phase transformations occur repeatedly during the operation.

Classical Theory of Electrical Conduction

2.18. Calculate the mobility of the electrons in Zn in $cm^2/V \cdot s$. Assume that each Zn atom contributes two conduction electrons. The atomic mass of Zn is 65. The resistivity is 5.9 $\mu\Omega \cdot cm$, and the density is 7.130 g/cm^3.

Solution:

The weight of Zn in a volume of one cubic centimeter is 7.13 grams. One mole i.e. 65 grams of Zn contains an Avogadro's number (i.e. 6.023 x 10^{23}) of atoms. Therefore, one cm^3 of Zn (i.e. 7.13 grams of Zn) contain 6.607×10^{23} atoms. Each Zn atom is assumed to contribute two valence electrons to a sea of free electrons. Thus, the conduction electron concentration for zinc is $2 \times 6.607 \times 10^{22} = 1.32 \times 10^{23}$ electrons/cm^3. The, resistivity is given to be 5.9 $\mu\Omega^{-1} \cdot cm^{-1}$, this means conductivity is 1.69×10^5 S/cm.

The conductivity is $\sigma = n \times q \times \mu$ (q: 1.6 x 10^{-19} C). Substituting the values of conductivity, conduction electron concentration, and electronic charge, we get mobility value 8.03 $cm^2/V \cdot s$.

2.19. If the mobility of electrons in Au is 31 $cm^2/V\cdot s$. Assume that the mass of electrons in Au is 9.1×10^{-31} kg. Calculate the mean free path length (λ) of electrons in Au if the average electron speed is 10^6 m/s.

Solution:

Approach 1: In Equation 2-21, the expression for conductivity is

$$J = \left(\frac{nq^2\tau}{m}\right)E$$

For gold, we use (all quantities in SI units) : n=5.91 × 10^{28} electrons/m^3 (Table 2.3, convert from electrons/cm^3), m= 9.1 x 10^{-31} kg, and q=1.6 x 10^{-19} C, and conductivity = 4.25 x 10^7 S/m (Table 2.1, convert to SI units). This gives us a mean free path (λ) of 2.556 x 10^{-8} m or 255.6 Å. This is consistent with values mentioned in Section 2.10.

The time between collisions (τ) can then be calculated as mean free path divided by average velocity.

$$\tau = 2.556 \times 10^{-8}\ \text{m}/10^6\ \text{m/s}=2.56 \text{ x } 10^{-14} \text{ seconds.}$$

Approach 2: We could also have calculated the time between collisions (τ) from the mobility value using equation 2.18. In this we use all SI units,

$$\mu_n = \frac{q\tau}{m}$$

The value of mobility will be 0.0031 $m^2/V\cdot s$. Use q=1.6 x 10^{-19} C and m: 9.1 x 10^{-31} kg. This gives us a value of τ = 1.76 x 10^{-14} s. This is similar to what we calculated before using conductivity and electron concentration values. Then the mean free path, from Equation 2.19 will be 1.76 x 10^{-8} m or 176 Å. This is similar to the value calculated in approach 1.

2.20. The thermal speed of electrons is about 10^6 m/s. However, the drift velocity is rather small because electrons are scattered off by the vibrations of atoms. Calculate the drift velocity of electrons in Cu for an electric field of 1 V/m. Assume the mobility of electrons in Cu is 32 $cm^2/V\cdot s$.

Solution:

From Equation 2.17,

$$v_{\text{drift}} = v_{\text{avg}} = \left(\frac{qE}{m}\right)\tau = \left(\frac{q\tau}{m}\right)E$$

Note that the term inside the bracket is mobility. A given mobility is 0.0032 m/s in SI units. If electric field is 1 V/m, the drift velocity is 0.0032 m/s. This is much smaller than thermal velocity, 10^6 m/s, which is due to scattering of electrons during the travel.

2.21. If the density of Ag is 10.5 g/cm^3, what is the concentration of conduction electrons in Ag?

Solution:

Atomic mass of silver is 107.868 (See Table 2.3) and 107.868 grams of silver has 6.023×10^{23} atoms. Therefore, there are 5.862×10^{22} Ag atoms in 10.5 grams or 1 cc of Ag. Since each Ag atom has one valence electron, we assume each atom donates one conduction electron. Then, the concentration of conduction electrons will also be 5.862×10^{22} electrons/cm^3.

2.22. From the information provided in Table 2.3, calculate the expected conductivity of Ag.

Solution:

When the electron concentration in 2.21 is inserted to $\sigma = n \times q \times \mu$ (here, μ is 57 cm^2/V·s), this works out to the conductivity, 53.4×10^4 S/cm and the resistivity, 1.87×10^{-6} Ω·cm. These values are close to what are reported in Table 2.1.

2.23. Au is a face-centered cubic metal with a lattice constant of 4.080 Å. If the atomic mass of Au is 196.9655, calculate the number of conduction electrons per unit volume. Express your answer as number of electrons/cm^3 . Assume that each Au atom donates one conduction electron.

Solution:

For metals such as gold (Au) with face center cubic structure we have a total of four atoms per unit cell. Each of the corner atom counts as 1/8th and each face center atom counts as half.

The volume of the unit cell is 6.7917×10^{-23} cm^3. Thus concentration of atoms in atoms/cm^3 :

$$\frac{6.7917 \text{ x } 10^{-23}}{4} = 5.88 \times 10^{22}$$

For gold, valence is one, thus concentration of conduction electrons will be the same as concentration of atoms since each atom donates only one electron. Thus, concentration of conduction electrons will be 5.88×10^{22} electrons/cm^3. This is close to the value listed in Table 2.3.

2.24 A semiconductor is made so that it carries electrical current primarily from the flow of electrons. If the mobility of electrons (μ_n) is 1350 cm^2/V·s and the conduction electron concentration is 10^{21} cm^{-3} , what is the electrical conductivity of this material?

Solution:

It is stated that the current in this particular semiconductor is predominantly carried by electrons. Then, we use the following equation.

$$\sigma = n \times q \times \mu$$

Using values given we get conductivity of 2.16×10^5 S/cm.

2.25. A heating element for a flat iron is rated at 1000 W. If the iron works at 220 V, what is the resistance of this heating element?

Solution:

Power dissipated is given by = V × I = $I^2 \times R$ (Equation 2.28). Using this, we get current value of 4.54 Amp. Then, from Ohm's law, the resistance is calculated as 48.4 Ohms.

2.26. Electronic components and devices are often tested at 125°C and −55°C to check the high- and low-temperature performances. They can then be compared with the properties observed at room temperature, 25°C. For example, using 25°C as the reference temperature and +125°C as the other temperature, α_{125} can be written as follows:

$$TCR_{125} = \alpha_{R,125} = \frac{1}{\rho_{125}} \left(\frac{\rho_{125} - \rho_{25}}{125 - 25} \right) \times 10^6 \text{ ppm/°C}$$

Write an equation to express the temperature coefficient of resistance with $T = -55$°C (note the negative sign) as the other temperature, using 25°C as the base or reference temperature (T_0).

Solution:

$$TCR_{-55} = \alpha_{R,-55} = \frac{1}{\rho_{-55}} \left(\frac{\rho_{-55} - \rho_{25}}{-55 - 25} \right) \times 10^6 \text{ ppm/°C}$$

Note that the resistivity at lower temperatures will be lower, thus the numerator in the bracketed expression will be negative and so is the denominator.

2.27. In a circuit, the TCR 125 value for a resistor is 100 ppm/°C. If the resistance (R) at 25°C is 1000 Ω, what is the resistance at 125°C?

Solution:

$$TCR_{125} = \alpha_{R,125} = \frac{1}{\rho_{125}} \left(\frac{\rho_{125} - \rho_{25}}{125 - 25} \right) \times 10^6 \text{ ppm/°C}$$

Since the resistor is the same i.e. geometry remains the same, we can use resistance instead of resistivity. Using values of TCR and R_{25} in the above equation and solving for R_{125} we get:

$$100 = \frac{1}{R_{125}} \left(\frac{R_{125} - 1000}{100}\right) \times 10^6$$

This leads to R_{125} = 1000/0.99=1010 Ω.

2.28. Assume that the bus bar discussed in Example 2.11 is heated due to the Joule losses and now operates at 70°C. Calculate the resistance, power loss, and energy consumption for 24 hours and the total energy costs per year. Ignore the change in the length of the Cu bus bar because of thermal expansion.

Solution:

The copper conductor discussed in Example 2.11 was shown to have a resistance of 2.44 x 10^{-3} Ω. From the literature, the temperature coefficient of resistance (α_R) for copper is 0.00404/°C, with a reference temperature of 20 °C. The resistance at 70 °C will be given by:

$R = R_0\ [1 + \alpha_R\ (T-T_0)] = 0.00244\ [1+0.00404(70-20)] = 2.93 \times 10^{-3}\ \Omega$.

The voltage drop will be V= 1000 Amp x 2.93 x 10^{-3} ohms = 2.93 volts. Dissipated power is $I^2 \times R$ which is 2933 Watts or 2.933 kW. Energy lost in 24 hours is 70.4 kW·h (2.993 kW × 24 hrs).

Assuming cost of electricity is 4 cents/kW·h (see Example 2.11), the cost per day will be \$2.81/day. In one year, the cost is 365 × \$2.81 = \$1027.68/year.

2.29. Nichrome wire is used for cutting materials such as polystyrene (Styrofoam) into different shapes, including large facades or insulation boards. (a) Calculate the length of an AWG #20 wire that is needed to have a resistance of R = 8 Ω. (b) What will be the resistance of this wire if it gets heated to a temperature of 200°C? See Tables 2.2 and 2.5.

Solution:

(a) From Table 2.5 the resistivity of one type of nichrome alloy is 100 μΩ·cm or 0.0001 Ω·cm. The wire gage is 20 and from Table 2.2 the diameter is 0.811814 mm which translates into an area of 0.0051761 cm^2. The resistance is 8 Ohms. Then length of the wire is 414 cm.

(b) If the wire gets heated to 200 °C, the new resistance will be given by:

$R = R_0\ [1 + \alpha_R\ (T - T_0)] = 8\ [1 + 400 \times 10^{-6}\ (200-27)] = 8.5536\ \Omega$

Note that in Table 2.5 the values of α_R should be listed as 400 × 10^{-6} and the reference temperature (T_0) of nichrome value is 300 K (i.e. 27 °C).

2.30. In the nanoscale region, why does the resistivity of thin films depend upon thickness?

Solution:

This is because of surface scattering in very thin films (See Section 2.10). If the film thickness is comparable to or smaller than the scattering length (λ: 10~100 nm) of electrons, the scattering of electrons at the surface film becomes significant in addition to the scattering by grain boundary and extended defects (e.g. dislocations). Therefore, a decrease in the film thickness decreases the carrier mobility and increases the resistivity of film.

2.31. What elements most affect the resistivity of high-purity Cu?

Solution:

According to Figure 2.13, addition of Ti into Cu changes the resistivity the most.

2.32. Why does the addition of oxygen in limited concentrations actually *increase* the conductivity of high-purity Cu?

Solution:

As mentioned in Section 2.11.2, an optimum level of oxygen can concentrate the total impurity content in form of small precipitates of copper oxide and thus the conductivity goes up after the removal of impurities in Cu matrix by the incorporation of oxygen.

2.33. Why does the resistivity of pure metals increase with temperature, whereas that of alloys is relatively stable with temperature?

Solution:

In an alloy, the second component is considered as an impurity and the impurity concentration is typically very high. Thus, in alloys, the scattering by impurities is much more significant and the effect of phonon scattering (thermal effect) is not as strong. For pure metals the carrier mobility is high and even a small change in temperature brings in considerable amount of phonon scattering. Therefore, the mobility of pure metals decreases, leading to an increase in the resistivity.

2.34 Nordheim's coefficient for Au dissolved in Cu is C = 5500 nΩ· m. If the resistivity of Cu at 300 K is 16.73 nΩ·m, calculate the resistivity of an Au–Cu alloy containing 1 weight % Au.

Solution:

Convert weight fraction (w) of gold, which is an impurity in this case, into atomic fraction using equation shown in Example 2.12. Note that Cu is the matrix.

$$x = \frac{M_{\mathrm{Cu} \times \mathrm{w}}}{(1 - w)M_{\mathrm{Au}} + w\mathrm{M}_{\mathrm{Cu}}}$$

The atomic mass of copper is 63.5, that of gold is 197 and weight fraction of gold (w) is : 0.01. This gives atomic fraction of gold as x = 0.00324534.

Using Nordheim's rule (Equation 2.32):

$$\rho_{\mathrm{alloy}} = \rho_{\mathrm{matrix}} + Cx(1 - x)$$

where C (Nordeheim's coefficient) is 5500 nΩ·m, and resistivity of the matrix (Cu, in this case) is 16.73 nΩ·m. Therefore, the resistivity of alloy is 34.52 nΩ·m [=16.73 + 5500 (0.00324534) (1-0.00324534)]

2.35 What is the electronic configuration for an Mg atom (Z = 12)?

Solution:

$1s^2\ 2s^2\ 2p^6\ 3\ 3s^2$ (refer to Example 2.13 of the textbook)

2.36 Draw a schematic of the band diagrams for a typical metal, a semiconductor, and an insulator.

Solution:

Refer to band diagrams of Figure 2.27.