

Chapter 2: Electrical Conduction in Metals and Alloys

Key Topics to be Understood

- The fundamentals of what causes a material to be a conductor of electricity.
- Different types of conducting materials based on metals and alloys and their “real-world” technological applications.
- The classical theory of conductivity
- The band theory of solids and its use for examining the differences among conductors, semiconductors, and insulators
- The effects of chemical composition, microstructure, and temperature on the conductivity of metals and alloys
- Real-world applications of conductive materials

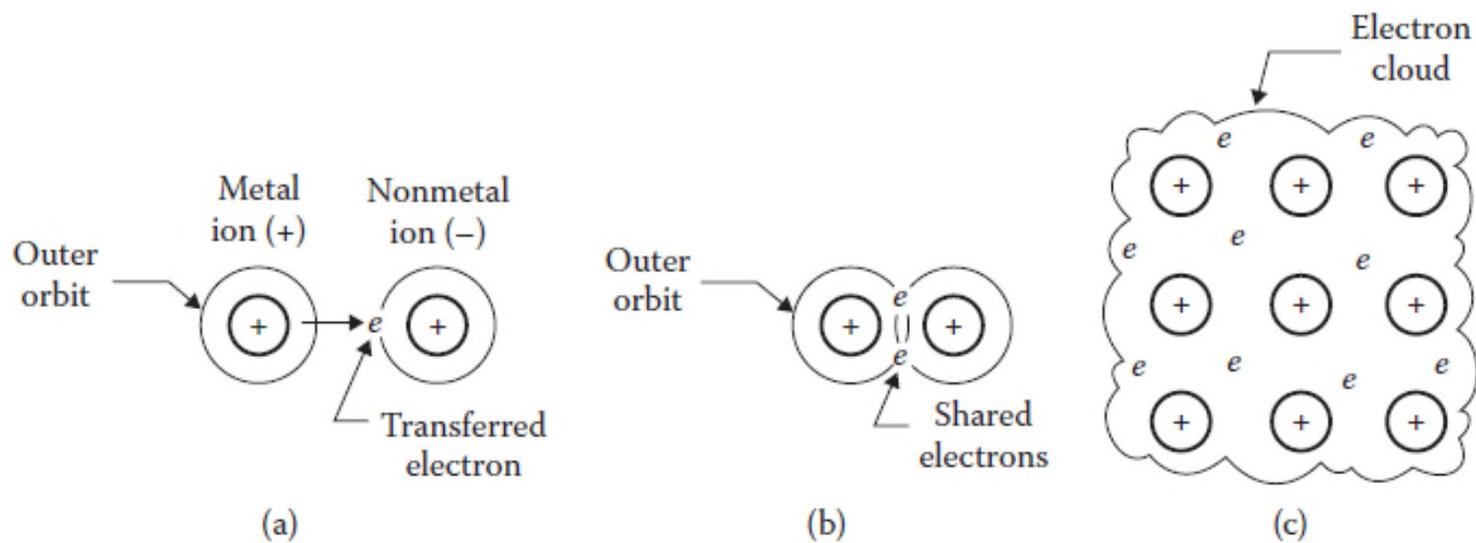


FIGURE 2.1 Illustration of (a) ionic bonds, such as those in sodium chloride and many other ceramics; (b) covalent bonds, such as those in silicon, many ceramics, and polymers; and (c) metallic bonds. (From Groover, M.P., *Fundamentals of Modern Manufacturing: Materials, Processes, and Systems*, Wiley, New York, 2007. With permission.)

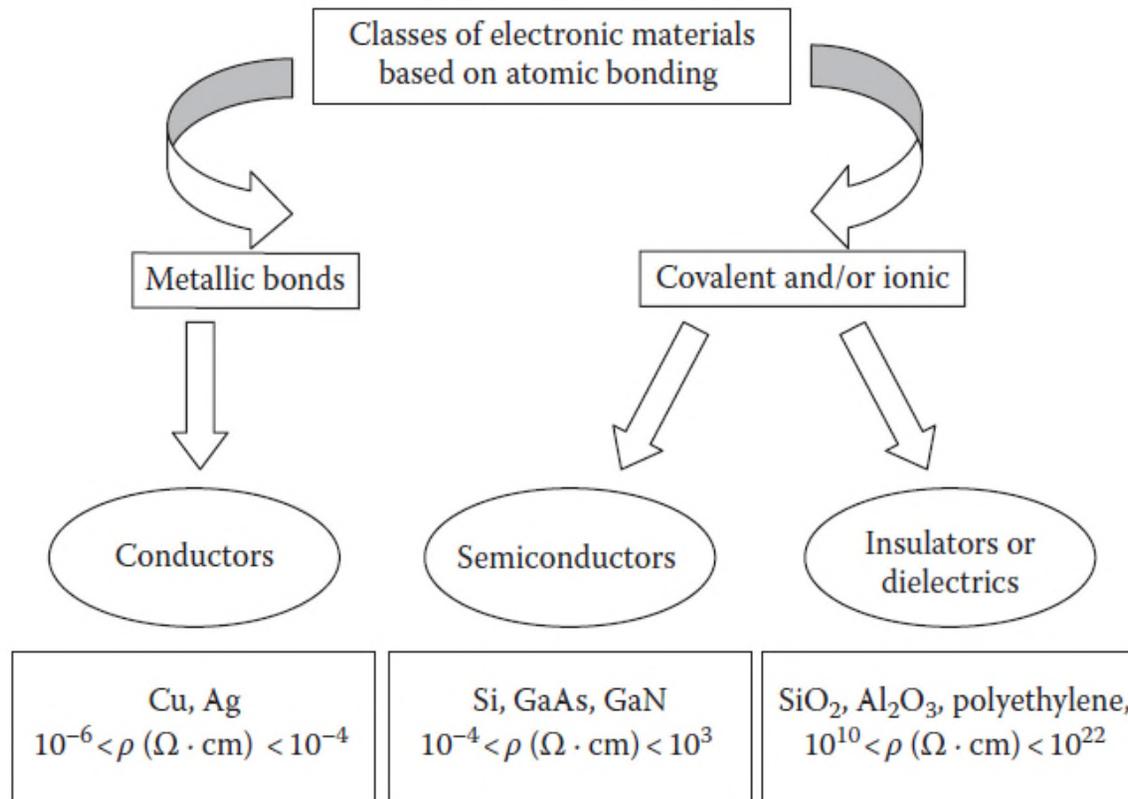


FIGURE 2.2 Classification of materials based on the nature of bonding. Typical ranges of resistivity are also shown.

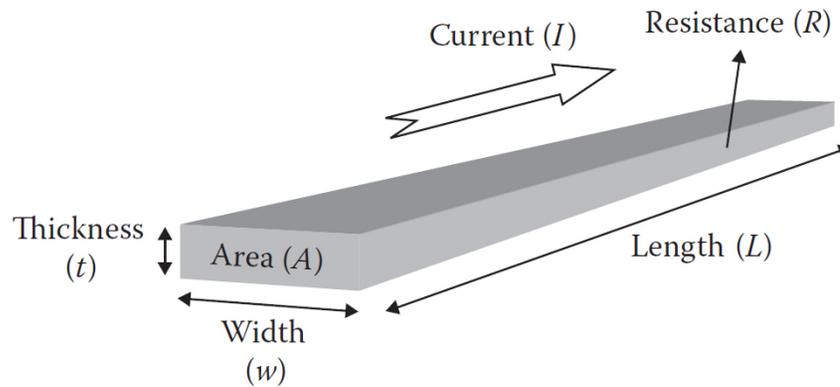


FIGURE 2.3 Geometry of a resistor used in describing Ohm's law.

Unit of electrical resistance (R) is ohm (Ω). The inverse of R is known as *conductance*, which has units of Siemens or Ω^{-1} .

ρ (rho) is defined as the *electrical resistivity*, *bulk resistivity*, or *volume resistivity*. A commonly used unit for resistivity is $\Omega\cdot\text{cm}$. However, resistivity values may be listed as $\Omega\cdot\text{cm}$, $\Omega\cdot\text{m}$, $\mu\Omega\cdot\text{cm}$ (micro-ohm-centimeter), $\text{n}\Omega\cdot\text{m}$ (nano-ohm-meter), and so on.

The inverse of ρ is known as *conductivity* (σ) and its common unit is $\Omega^{-1}\cdot\text{cm}^{-1}$.

$$\sigma = \frac{1}{\rho}$$

TABLE 2.1
Conductivity/Resistivity Values for Selected Metals

Material	Conductivity (σ) S/cm	Resistivity (ρ) $\mu\Omega \cdot \text{cm}$	Temperature Coefficient of Resistivity (α_R) ($\Omega/\Omega \cdot ^\circ\text{C}$)
Aluminum	37.7×10^4	2.65	4.3×10^{-3}
Beryllium	25×10^4	4.0	25×10^{-3}
Cadmium	14.6×10^4	6.83	4.2×10^{-3}
Chromium	7.75×10^4 (at 0°C)	12.9 (at 0°C)	3.0×10^{-3}
Cobalt (magnetic)	16.0×10^4	6.24	5.30×10^{-3}
Copper	59.7×10^4	1.673	4.3×10^{-3}
Gold	42.5×10^4	2.35	3.5×10^{-3}
Iridium	18.8×10^4	5.3	3.93×10^{-3}
Iron (magnetic)	10.3×10^4	9.7	6.51×10^{-3}
Lead	4.84×10^4	20.65	3.68×10^{-3}
Magnesium	22.4×10^4	4.45	3.7×10^{-3}
Mercury	1.0×10^4 (at 50°C)	98.4 (at 50°C)	0.97×10^{-3}
Molybdenum	19.2×10^4 (at 0°C)	5.2 (at 0°C)	5.3×10^{-3}
Nickel (magnetic)	14.6×10^4	6.84	6.92×10^{-3}
Palladium	9.253×10^4	10.8	3.78×10^{-3}
Platinum	10.15×10^4	9.85	3.93×10^{-3}
Rhodium	22.2×10^4	4.51	4.3×10^{-3}
Silver	62.9×10^4	1.59	4.1×10^{-3}
Tantalum	0.74×10^4	13.5	3.83×10^{-3}
Tin	0.90×10^4	11.0 (at 0°C)	3.64×10^{-3}
Titanium	2.38×10^4	42.0	3.5×10^{-3}
Tungsten	18.8×10^4	5.3	4.5×10^{-3}
Zinc	16.9×10^4	5.9	4.1×10^{-3}

Source: Webster, J.G., *Wiley Encyclopedia of Electrical and Electronics Engineering*, Vol. 4, Wiley, New York, 2002; Kasap, S.O., *Principles of Electronic Materials and Devices*, McGraw Hill, New York, 2002.

Note: Unless stated otherwise, values reported are at 300 K. For some materials, the values of the coefficient of resistivity (α_R) are shown using 300 K as the reference temperature, unless stated otherwise. To convert α_R into ppm/ $^\circ\text{C}$, multiply the values listed by 10^6 .



FIGURE 2.4 Typical passive radio frequency–identification (RFID) tag showing the antenna made from copper. (Courtesy of Pavel Nikitin, Intermec Technologies Corporation, Everett, WA.)

TABLE 2.2
American Wire Gauge Diameter Conversion

American Wire Gauge Number	Conductor Diameter (inches)	Conductor Diameter (mm)
20	0.03196118	0.811814
18	0.040303	1.023696
16	0.0508214	1.290864
14	0.064084	1.627734
12	0.0808081	2.052526
10	0.10189	2.588006
8	0.128496	3.263798
6	0.16202	4.115308
5	0.18194	4.621276
4	0.20431	5.189474
3	0.22942	5.827268
2	0.25763	6.543802
1	0.2893	7.34822
0	0.32486	8.251444
00	0.3648	9.26592
000	0.4096	10.40384
0000	0.46	11.684

Source: Powerstream, http://www.powerstream.com/Wire_Size.htm

Note: The diameter stated is that of the conductor and does not include insulation dimensions.

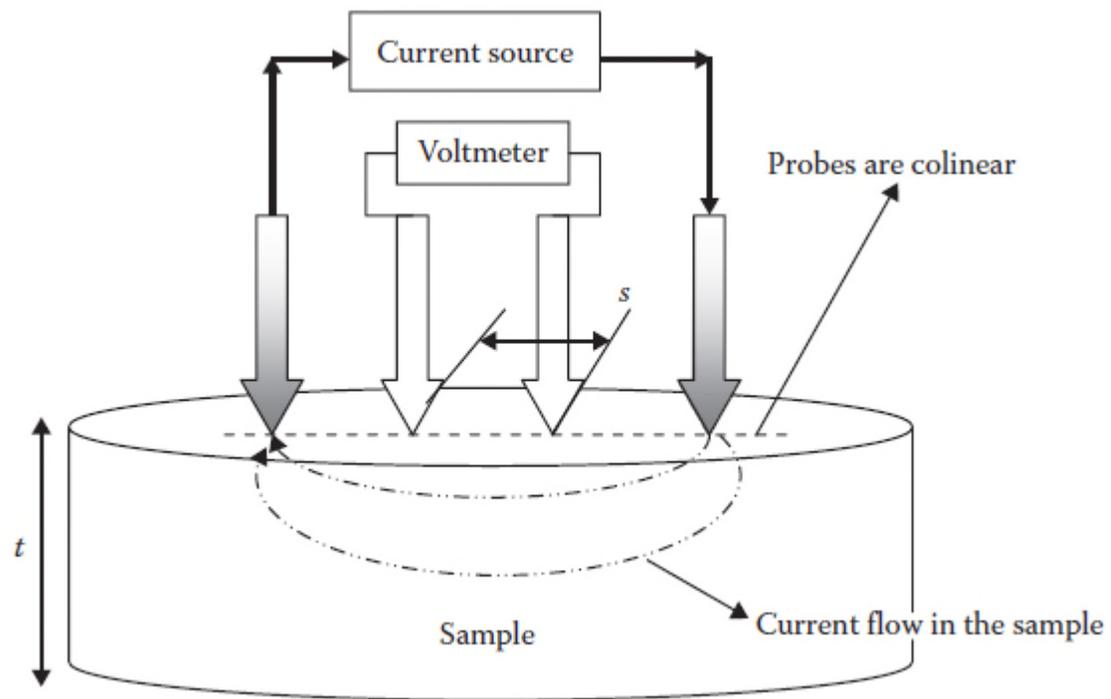


FIGURE 2.5 A schematic of the four-point probe used to measure resistivity. The spacing between probes is s , and the sample thickness is t (not to scale).

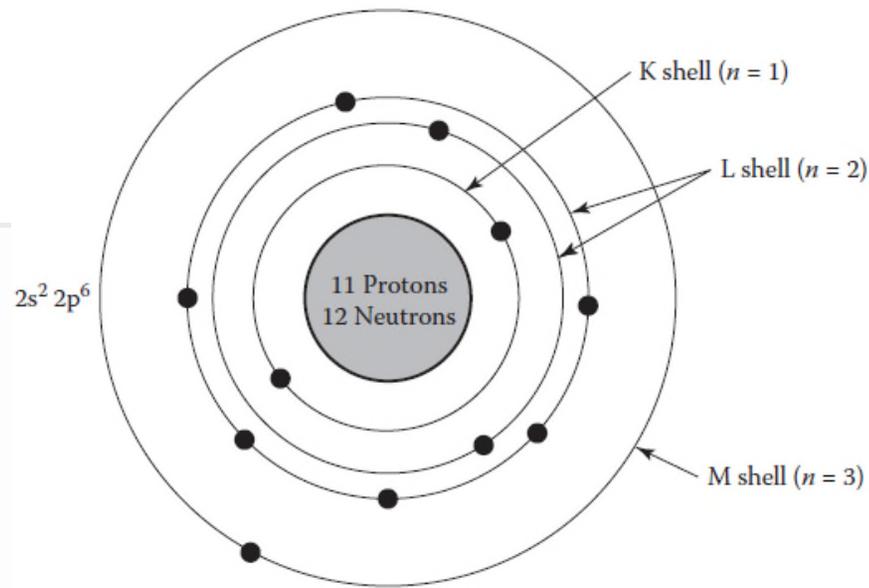


FIGURE 2.6 The structure of a sodium atom (atomic number 11), showing the nucleus surrounded by core electrons and valence electrons.

In metals, a large “sea of electrons” (unbound valence electrons) are the origin of free electrons. They move around within a metallic material under electric field, leading to a higher level of conductivity in metals.

In most solids, the atomic density ranges from $1 \times 10^{22}/\text{cm}^3$ to $1 \times 10^{23}/\text{cm}^3$. For example, for sodium (Na), there is one valence electron per atom, and each sodium atom donates this valence electron to the sea of electrons. Therefore, the free electron density of Na is the same as the atomic density of Na metal (= density ÷ [atomic mass] × [Avogadro’s number]) is $2.54 \times 10^{22}/\text{cm}^3$.

Example) Calculate the concentration of conduction electrons in Al. Assume that the density of Al is 2.73 g/cm³.

Answer)

Because Al has a valence of +3, we assume that each Al atom donates three conduction electrons.

The atomic mass of Al is 27 g; this means that the mass of 6.023×10^{23} atoms (Avogadro's number) is 27 g.

A volume of 1 cm³ is 2.7 g of Al. The number of atoms in this volume will be

$$\begin{aligned} &= \frac{6.023 \times 10^{23} \times \text{density}}{\text{atomic mass}} = \frac{6.023 \times 10^{23} \text{ atoms/mole} \times 2.7 \text{ g/cm}^3}{27 \text{ g/mole}} \\ &= 6.023 \times 10^{22} \text{ atoms/cm}^3 \end{aligned}$$

The concentration of conduction electrons will be expected to be three times the concentration of atoms because each Al atom is assumed to donate three electrons.

Thus, the concentration of conduction electrons in Al will be

$$\begin{aligned} n_{\text{Al}} &= 6.023 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3} \times 3 \frac{\text{electrons}}{\text{atom}} \\ &= 1.807 \times 10^{23} \text{ electron/cm}^3 \end{aligned}$$

Example) The experimentally measured mobility of electrons in copper is $32 \text{ cm}^2/\text{V}\cdot\text{s}$. Calculate the carrier concentration (n) for Cu. Compare this with the values listed in Table 2.3. What does this show about the number of conduction electrons contributed per Cu atom?

Answer)

We use the value of Cu conductivity (σ) provided in Table 2.1.

From Equation 2.22,

$$\sigma_{\text{Cu}} = 59.7 \times 10^4 \text{ S/cm} = (n) \times \left(32 \frac{\text{cm}^2}{\text{V}\cdot\text{s}} \right) \times (1.6 \times 10^{-19} \text{ C})$$

This gives us a concentration of conduction electrons $n = 1.16 \times 10^{23} \text{ electrons/cm}^3$.

In Table 2.3, the value listed for the concentration of electrons for Cu is $= 8.43 \times 10^{22} \text{ electrons/cm}^3$. In arriving at this value in Table 2.3, we had assumed that each Cu atom donates one conduction electron. Because we have now estimated that the actual value of conduction electrons is $1.16 \times 10^{23} \text{ electrons/cm}^3$, we know that each Cu atom must be donating more than one electron. The average number of electrons donated per Cu atom will be

$$\frac{1.16 \times 10^{23} \text{ electrons/cm}^3}{8.43 \times 10^{22} \text{ electrons/cm}^3} = 1.38$$

Thus, in Cu, the average number of electrons donated per atom is 1.38. This is expected because Cu does exhibit valences of +1 and +2.

cf) electron configuration of Cu: $[\text{Ar}] 3d^{10} 4s^1$

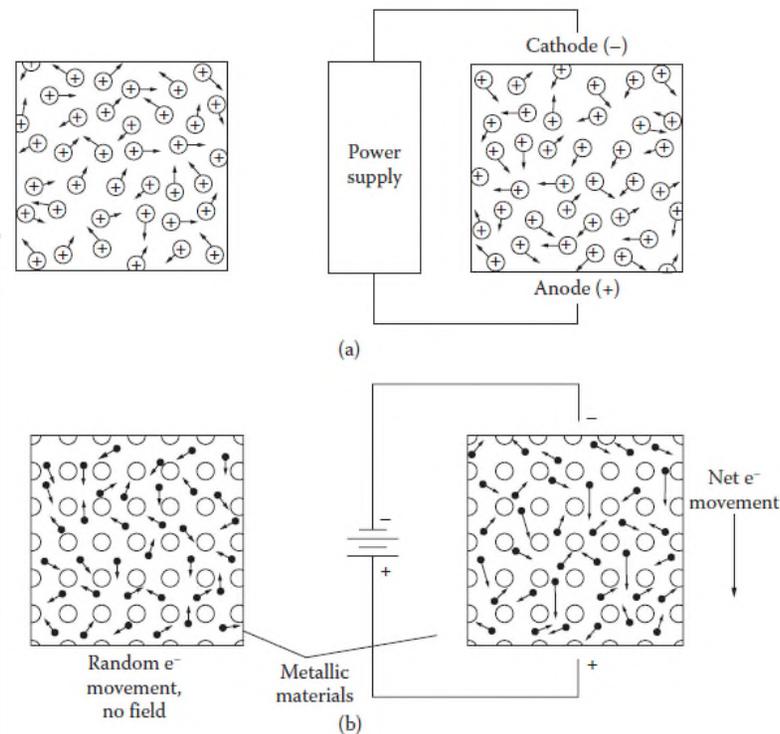


FIGURE 2.7 (a) Random motion of electrons due to thermal energy is similar to the movement of ions in an electroplating solution. (b) The drift of conduction electrons in a pure metal; note that the applied electric field (E) and drift are opposite, and overall, the electrons move within the material toward the anode.

Classical view of electric conduction is that 1) E -field causes a net movement of a small portion of charge carriers (free electrons in metals) and 2) charge carrier motion is limited by collisions with nucleus (similar to friction).

How to describe the motion of carriers (i.e. what is μ)?

In classical Drude's model, a sea of electrons in metals is called as "electron gas" or "plasma". Let's think about how free electrons in metals move under E-field and their motion is quantitatively estimated.

Drift of the electrons (motion induced by electric field):

The force (F) exerted on an electron by the electric field (E) is given by

$$F = qE$$

The force results in acceleration a , given by

$$F = ma$$

Thus, the acceleration experienced by the electrons is given by

$$a = qE/m$$

Because of this externally applied driving force (i.e., qE), conduction electrons begin to show a net movement. This motion of charge carriers (electrons, in this case) due to an applied electric field is known as *drift*. The drift of the carriers is characterized by drift velocity (\mathbf{v}). Note that the electrical current and free electron drift are in opposite directions, because electrons are negatively charged.

Drift velocity

If an electron accelerates and then collides with an atom after time τ , it stops. From Newton's Laws of Motion, we know that $v = u + a\tau$, where v is final velocity, u is initial velocity, a is acceleration, and τ is time. Applying this law here, assuming the initial speed is zero, we get $v = a\tau$. Thus, the average velocity of an electron between collisions is drift velocity (v_{drift}).

If the velocity is linearly proportional to time,

$$v_{\text{avg}} = (1/2)a\tau$$

We should not, however, consider the average time between collisions to calculate the average velocity, because electron velocity gradually decreases. For convenience, we can show that the average velocity or drift velocity (v_{drift}) is given by

$$v_{\text{drift}} = v_{\text{avg}} = a\tau$$

Since $a = qE/m$, $v_{\text{drift}} = v_{\text{avg}} = (qE/m)\tau = (q\tau/m)E$

From $\sigma = Ne\mu$ (N: carrier concentration, μ : carrier mobility),

$$j = \sigma E = Ne\mu E = Ne v \text{ (v: velocity of carriers) } \& \mu = v/E$$

Mobility is a **proportionality constant** between electron velocity and E-field.

The *mean free-path length* (λ) of conduction electrons is defined as the average distance that the electrons travel before colliding again, and it is obtained by multiplying the average speed (v_{avg}) by the time between collisions (τ) :

$$\lambda = v_{\text{avg}} \times \tau \quad (\text{in metals, } \lambda \sim 100\text{nm})$$

Thus, the current density (J) is given by

$$J = n \times v_{\text{drift}} \times q$$

If velocity is replaced,

$$J = n \times \left(\frac{q\tau}{m} \right) E \times q$$

or

$$J = \left(\frac{nq^2\tau}{m} \right) E$$

Then, the electric conductivity becomes

$$\sigma = \left(\frac{nq^2\tau}{m} \right)$$

TABLE 2.3**Atomic Mass, Density, Experimentally Measured Electron Mobility, and Electron Concentration for Selected High-Purity Metals**

Metal	Atomic Mass (g/mol)	Density (g/cm ³)	Mobility of Electrons (μ n) (cm ² /V · s)	Calculated Number of Carrier Particles (n) # (electrons/cm ³)
Silver	107.868	10.5	57	5.86×10^{22}
Copper	63.546	8.92	32	8.43×10^{22} (assumes one electron per atom)
Gold	196.965	19.32	31	5.91×10^{22}
Aluminum	26.981	2.7	13	1.807×10^{23} (assumes three electrons per atom)

Source: Data adapted from Webster, J. G., *Wiley Encyclopedia of Electrical and Electronics Engineering*, Vol. 4. New York: Wiley, 2002; and other sources.

Example) Assume that the mobility of electron in Al is 13 cm²/V·s. Calculate the expected conductivity of Al.

Answer) The conductivity of Al is given by the expression

$$\begin{aligned}\sigma &= \left(1.807 \times 10^{23} \frac{\text{electrons}}{\text{cm}^3} \right) \times \left(13 \frac{\text{cm}^2}{\text{V} \cdot \text{s}} \right) \times \left(1.6 \times 10^{-19} \frac{\text{C}}{\text{electron}} \right) \\ &= 37.9 \times 10^4 \text{ S/cm}\end{aligned}$$

cf) Real conductivity of Al: $37.7 \times 10^4 \text{ S/cm}$

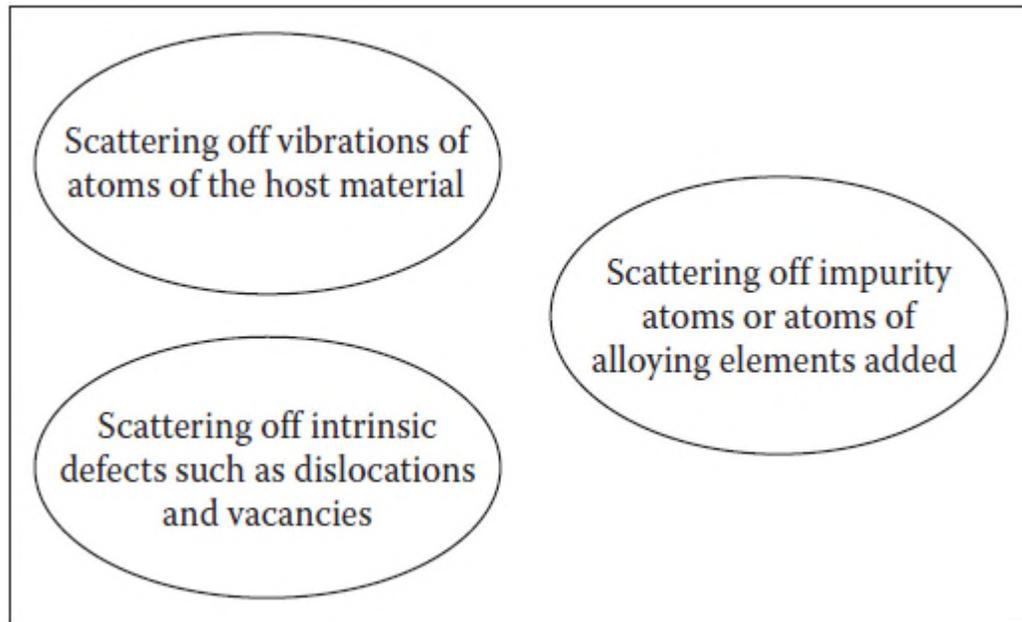


FIGURE 2.8 Schematic showing the sources of the scattering of conduction electrons in a metallic material.

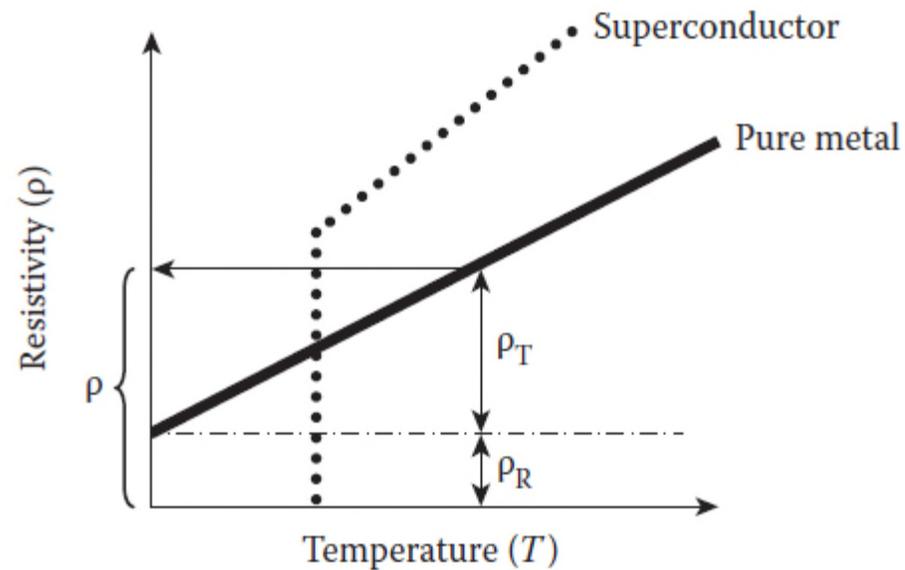


FIGURE 2.9 The temperature dependence of the conductivity of a typical metal and a superconductor.

First, atoms in a metallic material vibrate. We refer to the vibrations of atoms in a material as phonons. The conduction electrons carrying the electrical current bump into or scatter off vibrations of these atoms. The process of electron-scattering of a phonon is temperature-dependent. As the temperature increases, the number of electrons scattering off due to the vibrations of atoms also increases. Thus, the resistivity of a pure metal increases with increasing temperature. This increase in resistivity due to the scattering of electrons by lattice vibrations is linear in general.

The values of a parameter known as the *temperature coefficient of resistivity* (α_R ; TCR).

$$\rho = AT + B$$

$$\alpha_R = \frac{1}{\rho_0} \left(\frac{\rho - \rho_0}{T - T_0} \right)$$

Material	Conductivity (σ) S/cm	Resistivity (ρ) $\mu\Omega \cdot \text{cm}$	Temperature Coefficient of Resistivity (α_R) ($\Omega/\Omega \cdot ^\circ\text{C}$)
Aluminum	37.7×10^4	2.65	4.3×10^{-3}
Beryllium	25×10^4	4.0	25×10^{-3}
Cadmium	14.6×10^4	6.83	4.2×10^{-3}
Chromium	7.75×10^4 (at 0°C)	12.9 (at 0°C)	3.0×10^{-3}
Cobalt (magnetic)	16.0×10^4	6.24	5.30×10^{-3}
Copper	59.7×10^4	1.673	4.3×10^{-3}

TABLE 2.4
Properties of Some Heating-Element Materials

Material	Resistivity at 20°C ($\mu\Omega \cdot \text{cm}$)	α_R at 20°C ($\Omega/\Omega \cdot ^\circ\text{C}$)	Maximum Operating Temperature ($^\circ\text{C}$)	Main Applications
Nickel 80/chromium 20 (Nikorthal™-type alloys)	108	$+14 \times 10^{-3}$	1200	Furnaces, heating elements for domestic appliances
Chromium 22/aluminum 5.8/ balance iron (Kanthal™-type alloys)	145	$+3.2 \times 10^{-3}$	1400	Furnaces for heat treatment
Platinum 90/rhodium 10	18.7	–	1550	Laboratory furnaces
Platinum 60/rhodium 40	17.4	–	1800	Laboratory furnaces
Molybdenum	5.7	4.35×10^{-3}	1750	Vacuum furnaces, inert atmosphere
Tantalum	13.5	3.5×10^{-3}	2500	Vacuum furnaces
Graphite	1000	-26.6×10^{-3}	3000	Furnaces requiring nonoxidizing atmosphere
Molybdenum disilicide (MoSi_2)	40	1200×10^{-3}	1900	Glass industry, laboratory furnaces, ceramic processing
Lanthanum chromite (LaCrO_3)	2100	–	1800	Laboratory furnaces
Silicon carbide (SiC), also known as glowbar	1.1×10^5	–	1650	Industrial furnaces
Zirconia	–	–	2200	Laboratory furnaces, becomes an ionic conductor above $\sim 1000^\circ\text{C}$

Source: Reprinted from Laughton, M.A. and Warne, D.F., *Electrical Engineer's Reference Book*, Copyright (2003), with permission from Elsevier.

TABLE 2.5
Resistivity and Temperature Coefficient Values for Selected Materials

Material	Typical Composition (wt%)	Resistivity ($\mu\Omega \cdot \text{cm}$) at 20°C	Temperature Coefficient of Resistivity (α_R) $\times 10^{-6}$ $\Omega/\Omega \cdot ^\circ\text{C}$ ($T_0 = 300 \text{ K}$)
Lead (Pb)–tin (Sn) solder	Sn: 63, Pb: 37	14.7	
Brass	Cu: 60, Zn: 40	6.4	1000
	Cu: 70, Zn: 30	8.4	2000
Nichrome	Ni: 58.5, Fe: 22.5, Cr: 16, Mn: 3	100	400
Constantan	Cu: 60, Ni: 40	44.1	+2/+33
Manganin	Cu: 84, Mn: 12, Ni: 4	45	+6/–42 (12°C–100°C)
	Cu: 83, Mn: 13, Ni: 4 (wire alloy)	48.2	+15/–15 (15°C–35°C)
	Cu: 86, Mn: 10, Ni: 4 (shunt alloy)	38.3	+15/–15 (40°C–60°C)
Palladium–silver alloy	Pd: 60, Ag: 40	42–44	

Source: Webster, J. G. 2002. *Wiley Encyclopedia of Electrical and Electronics Engineering*, Vol. 4. New York: Wiley; Kasap, S. O. 2002. *Principles of Electronic Materials and Devices*. New York: McGraw Hill; Laughton, M. A. and D. F. Warne, eds. 2003. *Electrical Engineer's Reference Book*. Amsterdam: Elsevier; Askeland, D. and P. Fulay. 2006. *The Science and Engineering of Materials*. Washington, DC: Thomson; ASM International. (1990). *Properties and Selection: Nonferrous Alloys and Special Purpose Materials*. vol. 2. Materials Park, OH: ASM; Davis, J. R. 1997. *Concise Metals Engineering Data Book*. Materials Park, OH: ASM.

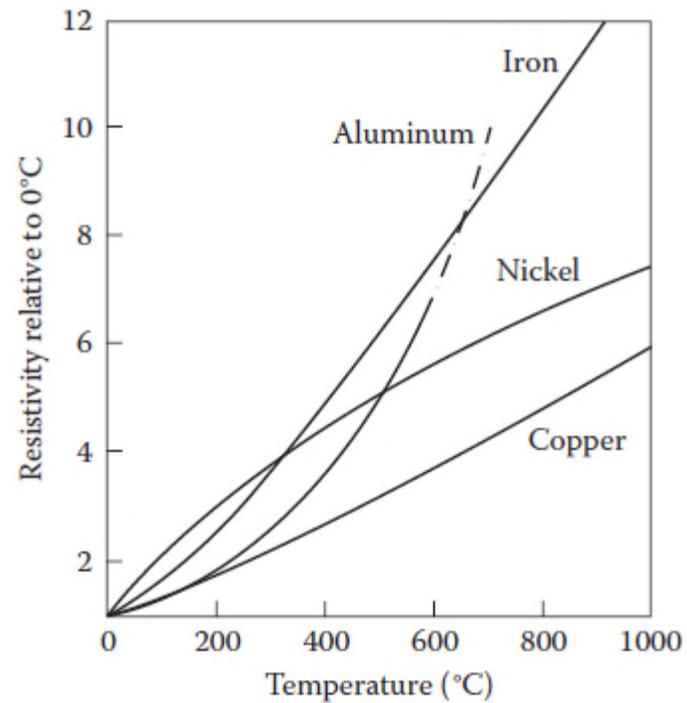


FIGURE 2.10 Relative change in resistivity with temperature (relative to 0°C) for Al, Cu, Ni, and Fe. (Reprinted from *Electrical Engineer's Reference Book*, Laughton, M. A., and D. F. Warne, Copyright 2003, with permission from Elsevier.)

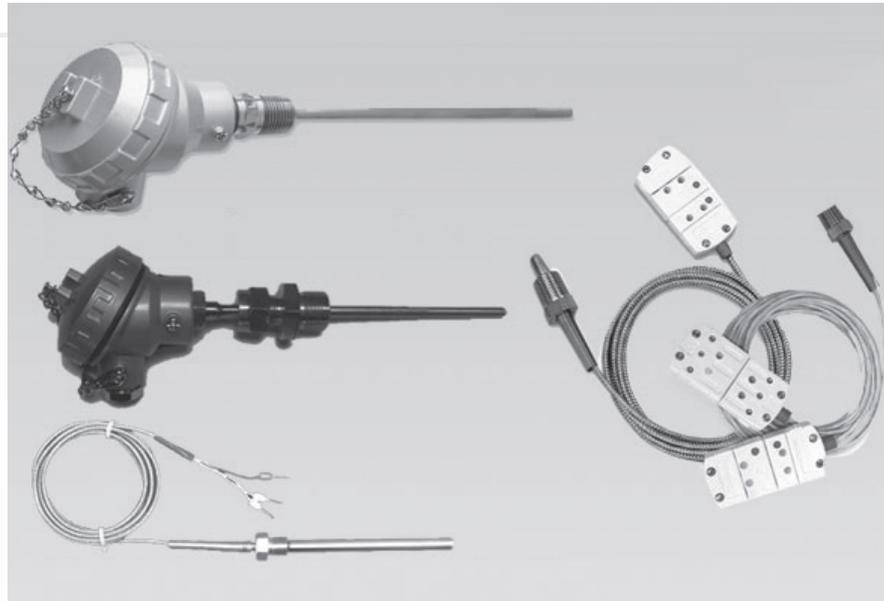


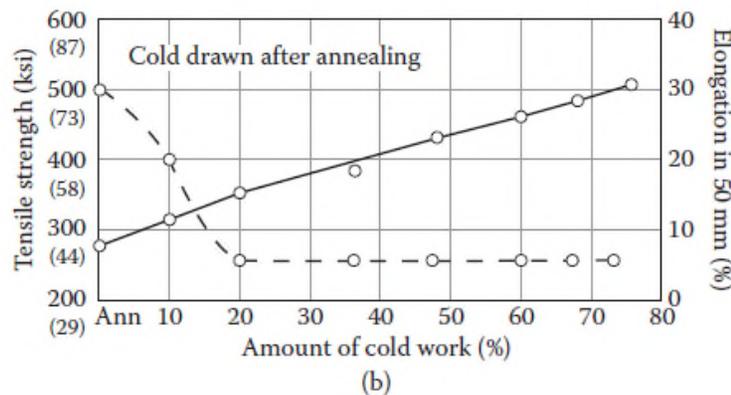
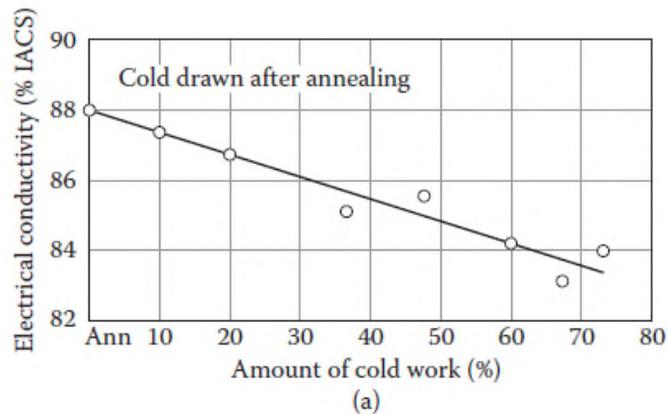
FIGURE 2.11 Commercially available resistance–temperature detector devices. (Courtesy of Omega Corporation, San Diego, CA.)

TABLE 2.6**Conductivity of Different Materials (at 20°C) Based on the IACS Scale**

Material	% IACS Conductivity	Material	% IACS Conductivity
Annealed copper	100	Nickel	25
99.999% Copper	102.5	Iron	17
Electrolytic tough-pitch copper ~99.0%	100.2–101.5	Platinum	16
Oxygen-free high-conductivity copper	101	Tin	13
Silver	104	Lead	8
Aluminum	60		

Source: Webster, J. G. 2002. *Wiley Encyclopedia of Electrical and Electronics Engineering*, Vol. 4. New York: Wiley; Kasap, S. O. 2002. *Principles of Electronic Materials and Devices*. New York: McGraw Hill; Laughton, M. A., and D. F. Warne, eds. 2003. *Electrical Engineer's Reference Book*. Amsterdam: Elsevier; Askeland, D., and P. Fulay. 2006. *The Science and Engineering of Materials*. Washington, DC: Thomson; ASM International. (1990). *Properties and Selection: Nonferrous Alloys and Special Purpose Materials*. vol. 2. Materials Park, OH: ASM; Davis, J. R. 1997. *Concise Metals Engineering Data Book*. Materials Park, OH: ASM.

Note: IACS = International Annealed Copper Standard.



The second factor that affects the mobility of metals is the scattering of electrons off defects such as vacancies, dislocations, and grain boundaries. The concentrations of different defects (i.e. scattering centers) will depend on the microstructure of the material.

FIGURE 2.12 (a) Electrical conductivity of sterling silver (92.5% Ag–7.5% Cu), shown as %IACS, and (b) increase in tensile strength (solid line) and decrease in percentage of elongation (dotted line) as a function of % cold work. The data are for a 2.3-mm wire, which was cold-drawn after annealing. (From ASM International (1990). *Properties and Selection: Nonferrous Alloys and Special Purpose Materials*, Vol. 2, ASM, Materials Park, OH. With permission.)

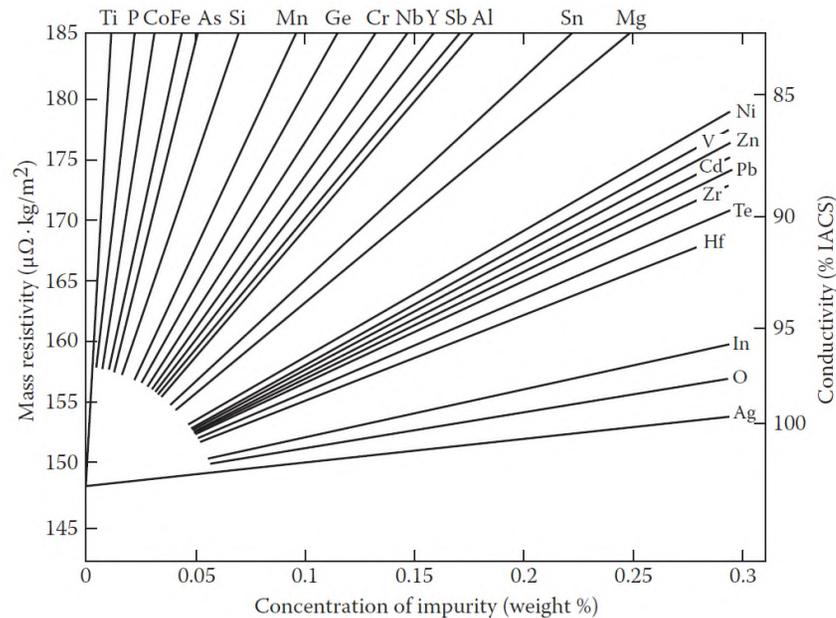


FIGURE 2.13 Effects of different impurities on the conductivity of copper. (Courtesy of Copper Development Association, New York, NY.)

The third factor that influences the resistivity of metallic materials is the presence of impurities or other elements deliberately or accidentally added. Impurities cause an increase in electrical resistivity because the atoms of the added elements act as scattering centers. The effect due to the presence of impurities is generally *not* very sensitive to temperature changes.

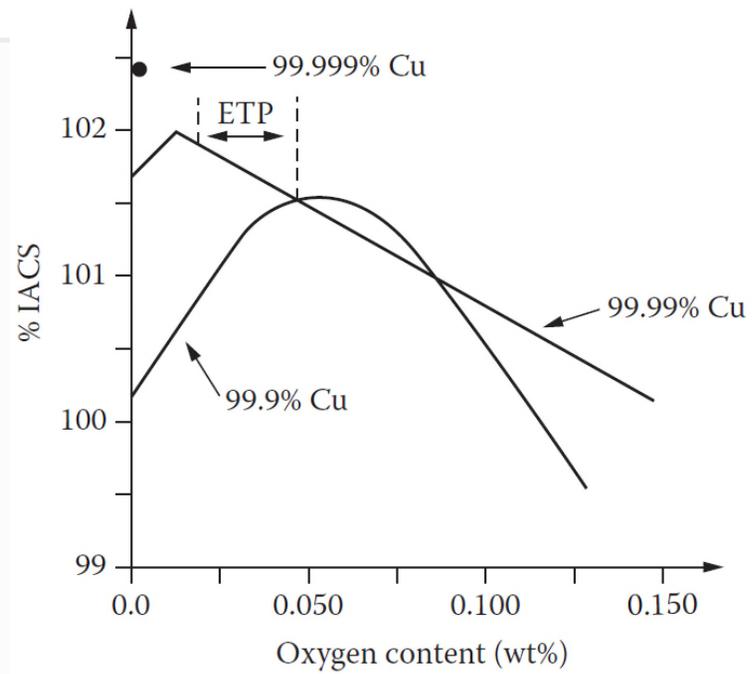


FIGURE 2.14 Effect of oxygen concentration on the conductivity of copper. (Courtesy of Copper Development Association, New York, NY.)

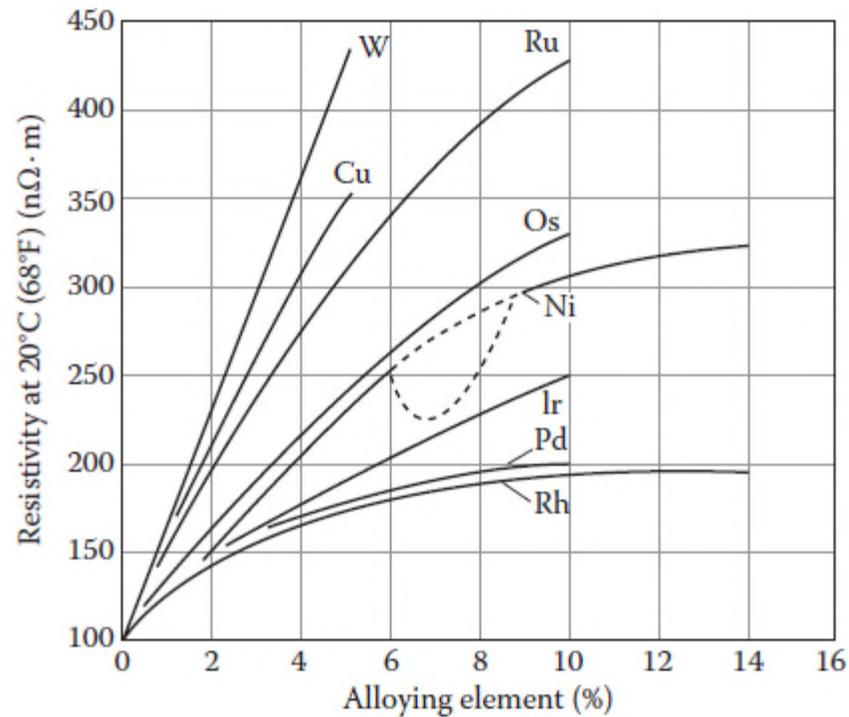


FIGURE 2.15 Effects of additions of various alloying elements on the resistivity of platinum. (From Vines, R.F. and Wise, E.M., *The Platinum Metals and Their Alloys*, International Nickel Co., New York, 1941. With permission.)

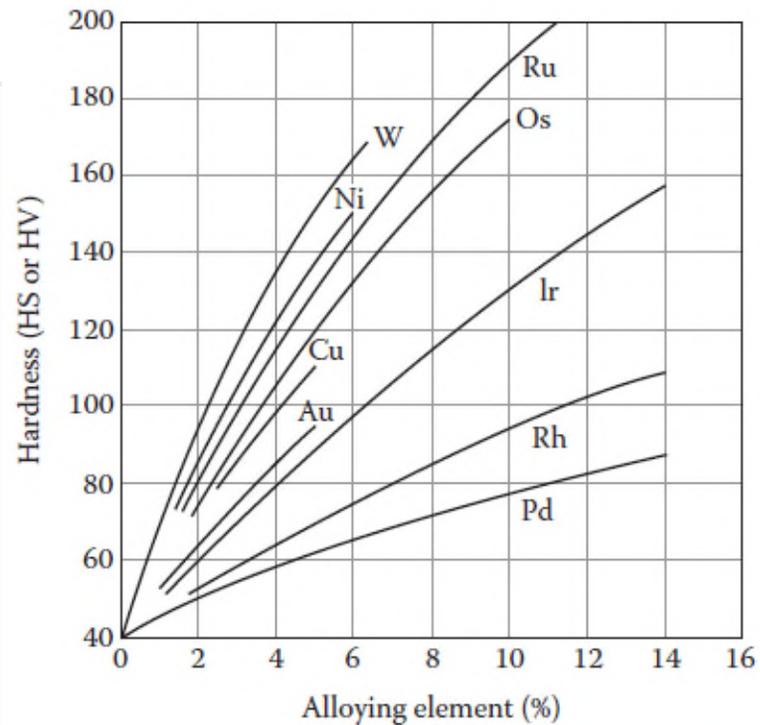


FIGURE 2.16 Increase in the hardness of platinum due to the addition of various alloying elements. (From Lampman, S.R. and Zorc, T.B., eds., *Metals Handbook: Properties and Selection: Nonferrous Alloys and Special Purpose Materials*, ASM, Materials Park, OH, 1990. With permission.)

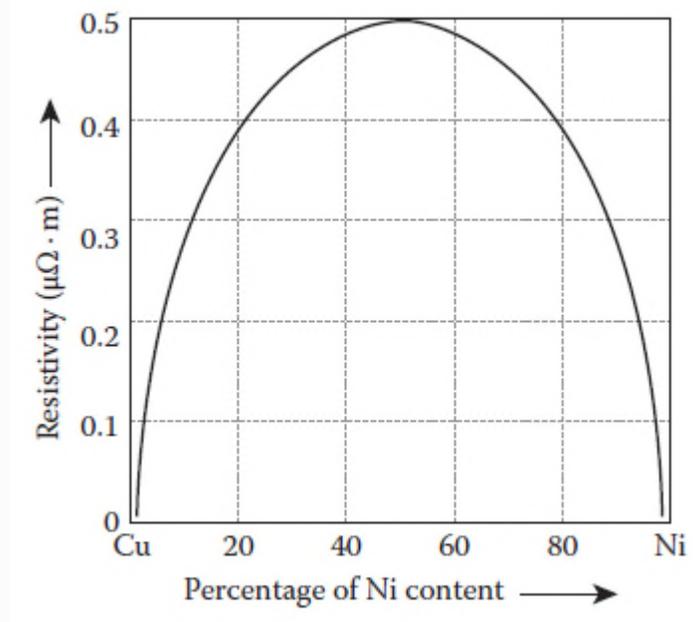


FIGURE 2.17 Variation in the resistivity of copper as a function of nickel concentration. (From Neelkanta, P., *Handbook of Electromagnetic Materials*, CRC Press, Boca Raton, FL, 1995. With permission.)

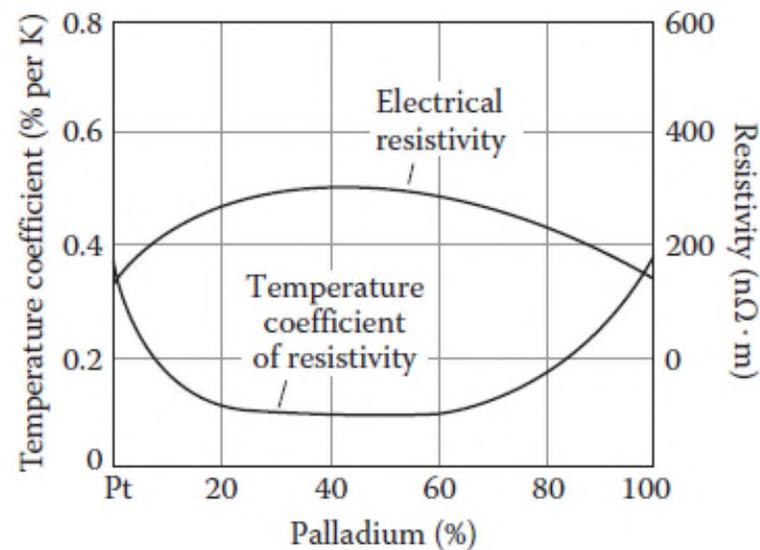


FIGURE 2.18 Electrical resistivity (in $\text{n}\Omega \cdot \text{m}$) and temperature coefficient of resistivity ($\%/K$) for platinum–palladium (Pt–Pd) alloys. (From Lampman, S.R. and Zorc, T.B., eds., *Metals Handbook: Properties and Selection: Nonferrous Alloys and Special Purpose Materials*, ASM, Materials Park, OH, 1990. With permission.)

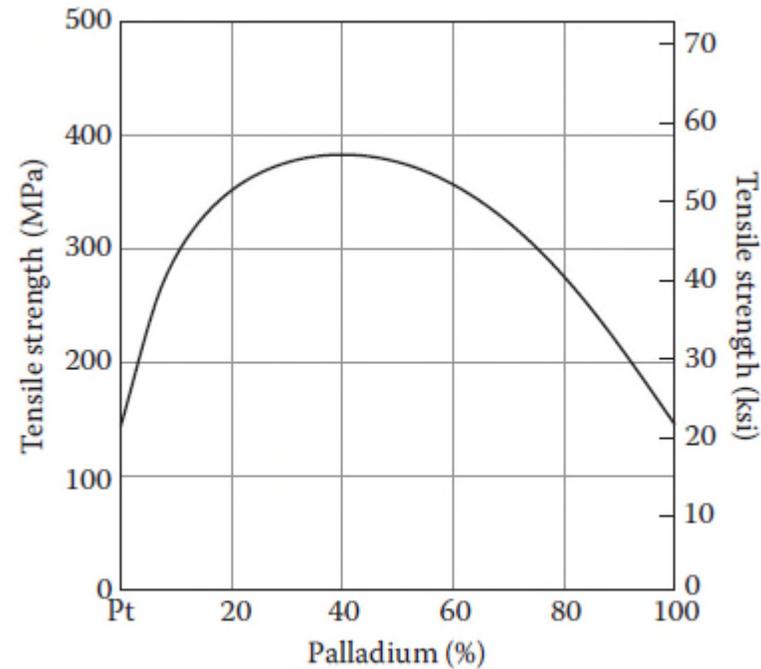


FIGURE 2.19 The change in tensile strength (in MPa on the left y-axis and in ksi on the right y-axis) of annealed platinum–palladium alloys as a function of palladium concentration. (From Lampman, S.R. and Zorc, T.B., eds., *Metals Handbook: Properties and Selection: Nonferrous Alloys and Special Purpose Materials*, ASM, Materials Park, OH, 1990. With permission.)

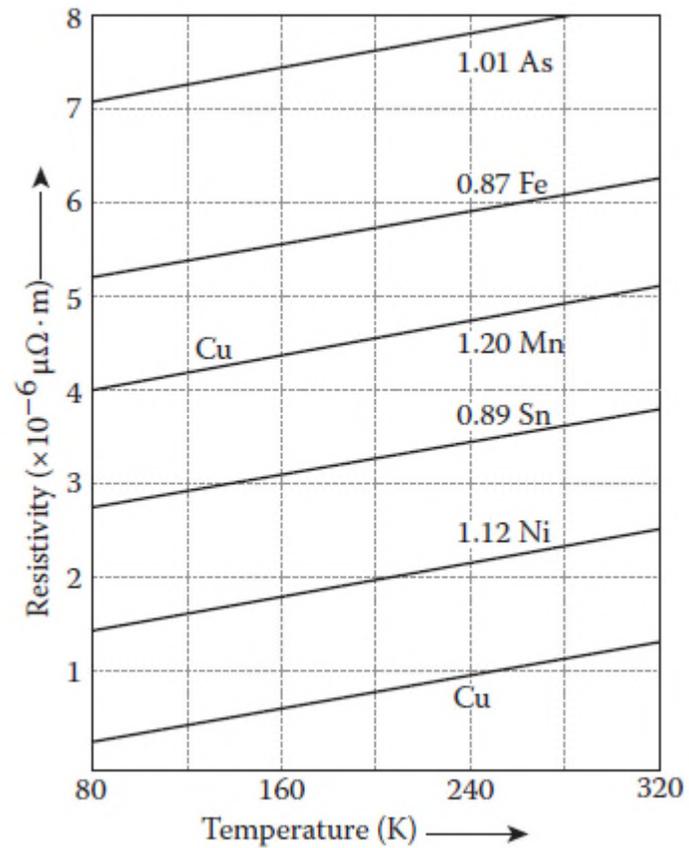


FIGURE 2.20 The resistivity of some copper alloys as a function of temperature. The data for resistivity change of copper is also shown. (From Neelkanta, P., *Handbook of Electromagnetic Materials*, CRC Press, Boca Raton, FL, 1995. With permission.)

Limitation of the Classical Theory of Conductivity

The classical theory based on the delocalization of valence electrons, although useful, does not explain several key features associated with the electrical conductivity of metals. For example, in silicon or diamond (a form of carbon [C]), each atom has four valence electrons, their free charge carrier concentration is much smaller than the valence electron concentration.

Classical theory cannot accurately predict the relationship between the *thermal conductivity* and electrical conductivity of metals. If all electrons in the “sea of electrons” move freely and are responsible for the high electric conductivity of metals, these free electrons should also contribute to the thermal conductivity. However, succeeding quantitative studies show that the thermal conductivity predicted by classical theory is two orders of magnitude lower than the experimentally observed thermal conductivity. Unless the heat capacity of free electrons is two orders of magnitude higher than the measured value, classical theory cannot explain a correlation between the thermal conductivity and the electric conductivity.

Classical model needs to be changed.

TABLE 2.7
Quantum Numbers for Electrons

Principal quantum number (n)

$$n = 1, 2, 3,$$

$n = 1$ is the K shell, $n = 2$ is the L shell, and so on.

The maximum number of electrons for a shell with given n is $2n^2$.

Orbital angular momentum quantum number (l)
(also known as the azimuthal quantum number)

$$l = 0, 1, \dots (n - 1)$$

$l = 0$ indicates the s subshell, $l = 1$ indicates the p subshell, and so on. The maximum number of electrons in the various subshells are s = 2, p = 6, d = 10, f = 14, and g = 18.

Magnetic quantum number (m or m_l)

$$m = -l, -(l-1), \dots, 0, (l-1), l$$

Spin (s or m_s)

$$s = \pm 1/2$$

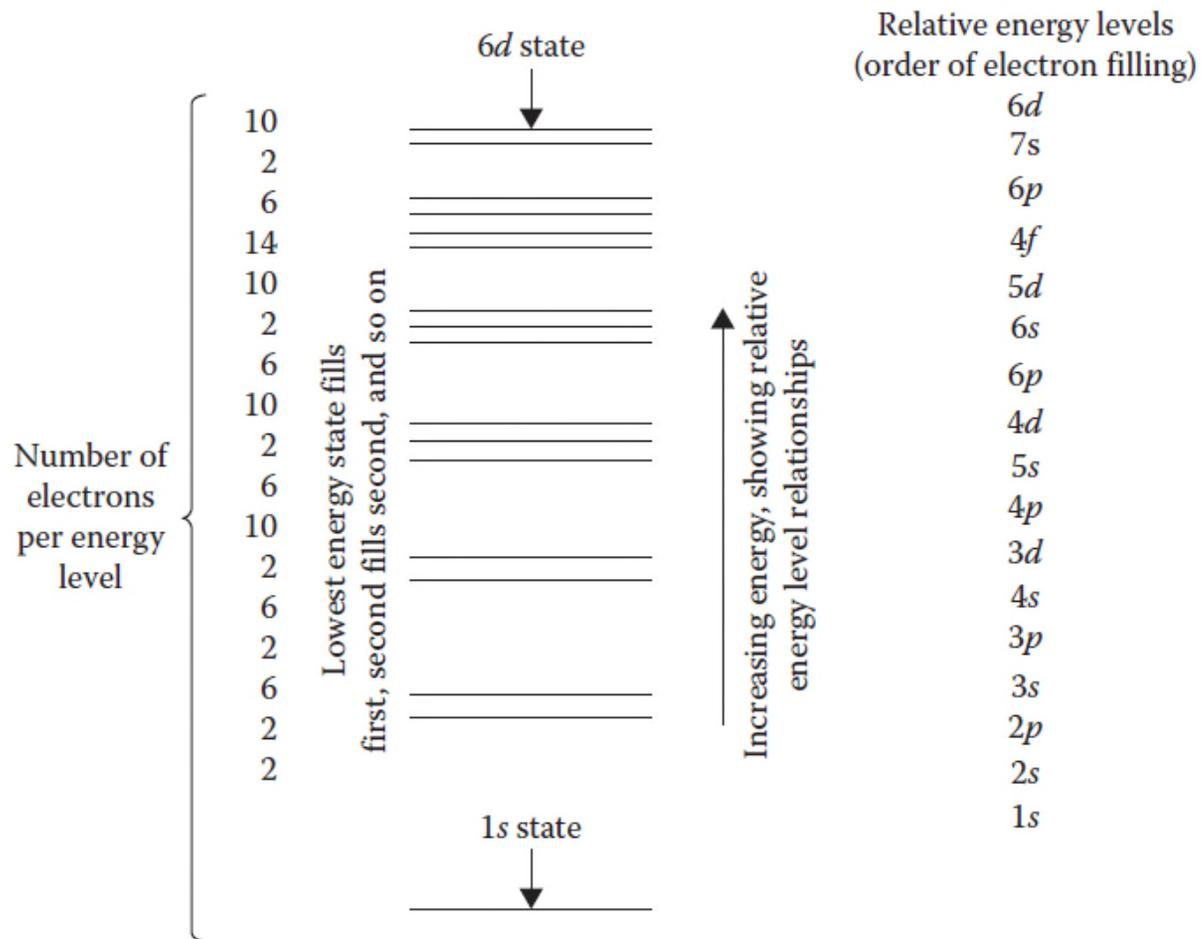


FIGURE 2.21 The order and number of electrons for different elements. (From Mingos, M.L., *Electronic Materials Handbook*, Vol. 1, ASM, Materials Park, OH, 1989. With permission.)

Electron configuration of isolated atoms

The electronic configuration of Al (13 electrons): $1s^2 2s^2 2p^6 3s^2 3p^1$
partially empty 3p shell

The electronic configuration of Fe (26 electrons): $1s^2 2s^2 2p^6 3s^2 3p^6 3d^8$
partially empty 3p shell

TABLE 2.8
Electron Spin States in Iron (Fe)

$n = 1$	$n = 2$		$n = 3$		$n = 4$	
1s	2s	2p	3s	3p	3d	4s
↑	↑	↑↑↑	↑	↑↑↑	↑↑↑↑↑	↑
↓	↓	↓↓↓	↓	↓↓↓	↓	↓

Source: Edwards-Shea, L., *The Essence of Solid-State Electronics*, Prentice Hall, Upper Saddle River, NJ, 1996. With permission.

Note: An arrow pointing up (↑) means $s = +1/2$, or spin up; an arrow pointing down (↓) means $s = -1/2$, or spin down. Note that the 4s level is filled before the 3d level and two of the 3d electrons are spin-paired.

A solid material can have billions of atoms. It is important to know not just the electronic structure of individual atoms but also how different atoms interact with one another when they are in close proximity to one another. When individual atoms are brought together to form a solid material, the atomic distance decreases and the electron orbitals begin to overlap.

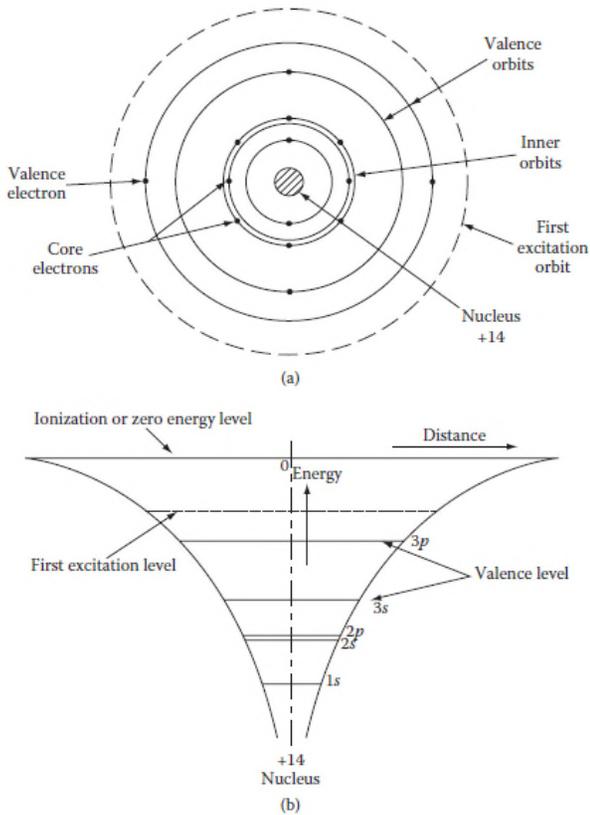


FIGURE 2.22 Electronic structure and energy levels in a Si atom: (a) The orbital model of a Si atom showing the ten core electrons ($n = 1$ and 2) and the four valence electrons ($n = 3$); (b) energy levels in the Coulombic potential of the nucleus are also shown schematically. (From Streetman, B.G. and Banerjee, S., *Solid State Electronic Devices*, 5th ed., Prentice Hall, Upper Saddle River, NJ, 2000. With permission.)

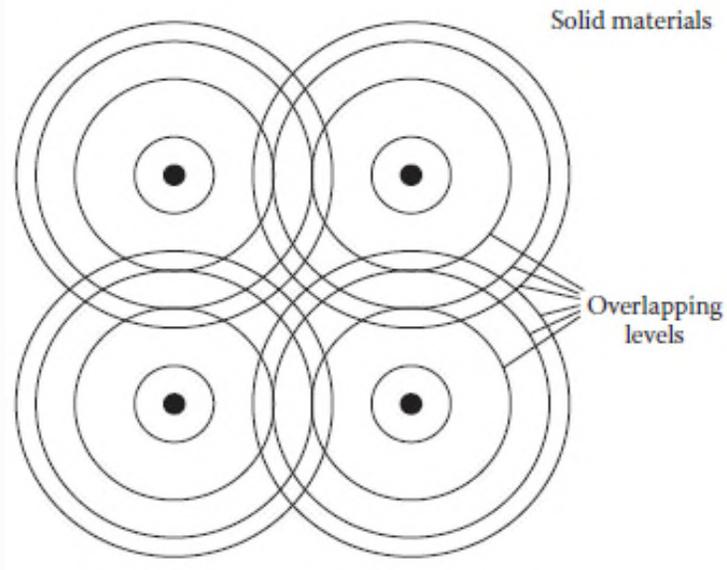


FIGURE 2.23 Overlap of the electron energy levels or wave functions as the atoms come closer. (Adapted from Edwards-Shea, L., *The Essence of Solid-State Electronics*, Prentice Hall, Upper Saddle River, NJ, 1996. With permission.)

Band Structure of Si solid

The overlapping of energy levels of different atoms and the subsequent formation of *energy bands*, which occur in silicon. As the interatomic distance (or lattice spacing) decreases, two discrete energy levels become two separate bands.

Because of the partially filled tetrahedral sites, there are two different valence electron locations.

1) Valence electrons stay near the center of Si–Si bonds, and the Coulombic interactions between valence electrons and the nucleus consequently contributes to strengthening Si–Si bonds. This energy state of valence electrons participating in Si–Si covalent bonding is called the bound state; most valence electrons are in the bound state.

2) The second type of electron state is known as “free state” and only a small portion of the valence electrons take the energy levels in free states. Electrons in free states are found around the empty tetrahedral sites of an Si lattice. Due to the lack of nearby Si atoms, the electrons in the free states are not bound to the atom.

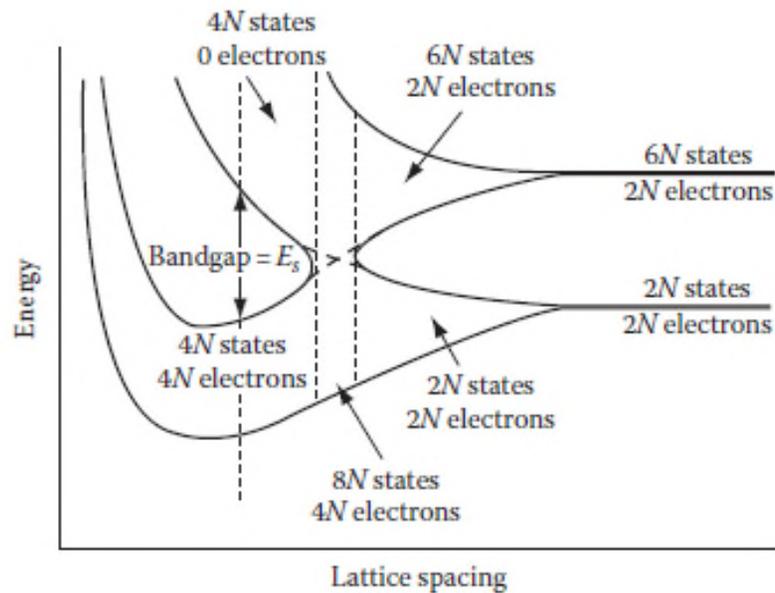


FIGURE 2.24 Formation of energy bands in silicon. (From Streetman, B.G. and Banerjee, S., *Solid State Electronic Devices*, 5th ed., Prentice Hall, Upper Saddle River, NJ, 2000. With permission.)

In Si, the electron energy band corresponding to the bound states is called the valence band, and the electron energy band related to the free states is called the conduction band. The energy gap between the bottom of the conduction band and the top of the valence band is called band gap.

Band Structure of metal solid

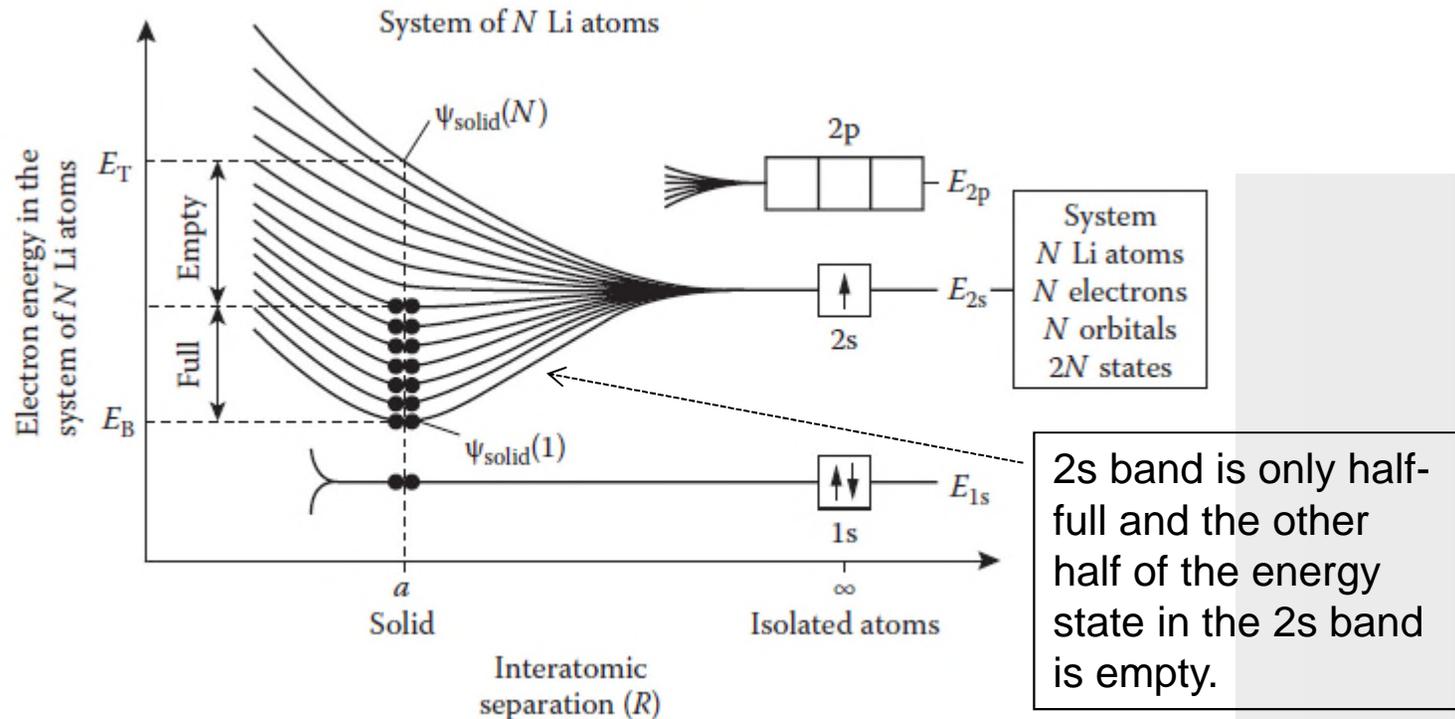


FIGURE 2.25 Formation of energy bands in lithium metal. The 2s band is only half-filled. Note that the 1s level shows very little splitting. (From Kasap, S.O., *Principles of Electronic Materials and Devices*, McGraw Hill, New York, 2002. With permission.)

The band width for electrons of the 1s level does not increase, though the interatomic distance decreases to the equilibrium position. Since the 2s level electrons shield the 1s electrons from outside electrons, the 1s level electrons in the solid are less affected by the presence of electrons of neighboring atoms.

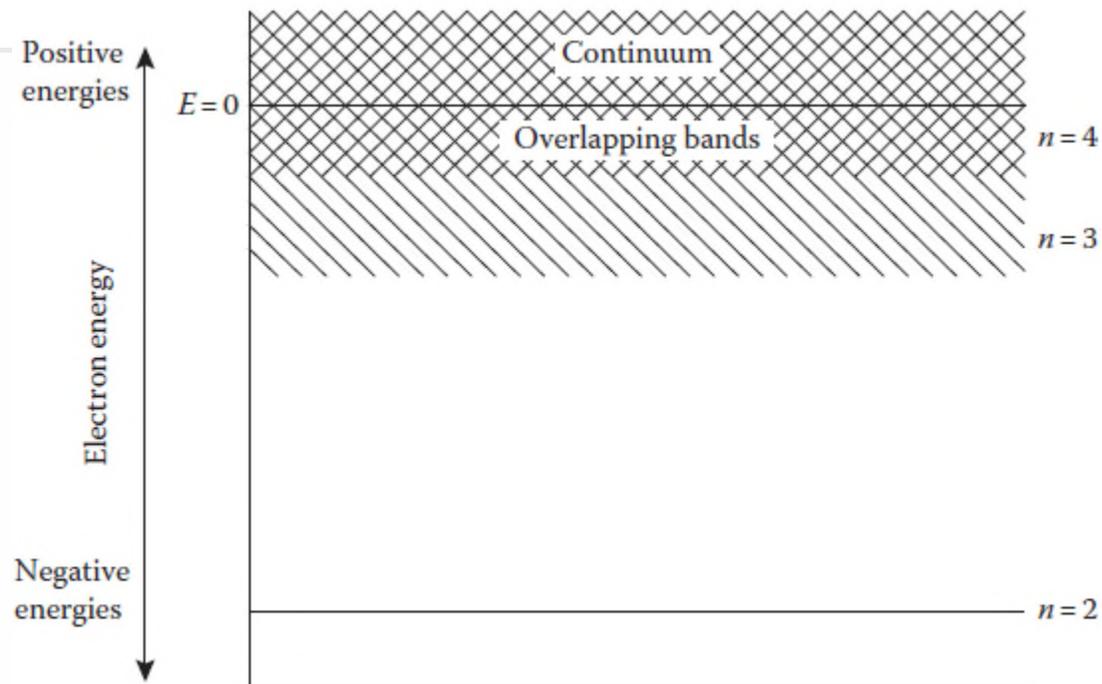


FIGURE 2.26 Schematic of a band diagram for a metal. (From Edwards-Shea, L., *The Essence of Solid-State Electronics*, Prentice Hall, Upper Saddle River, NJ, 1996. With permission.)

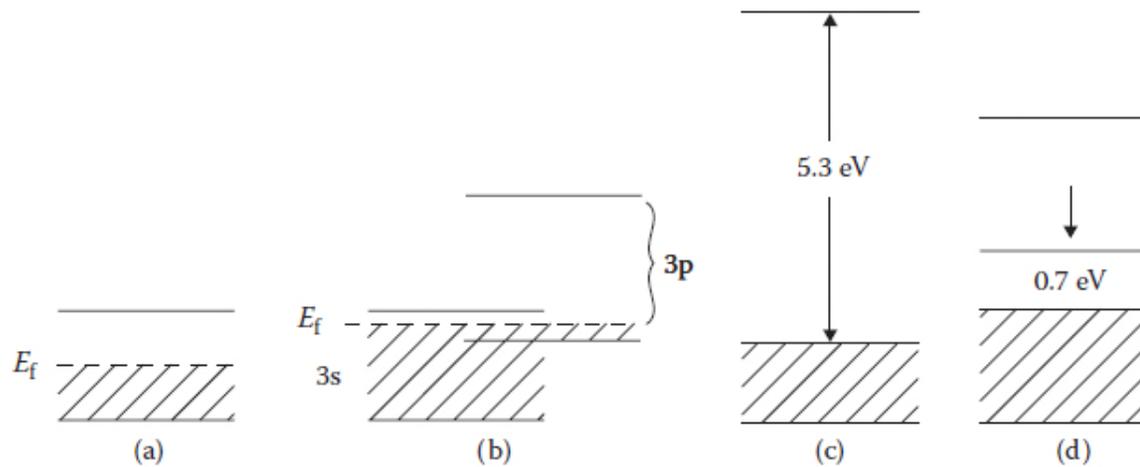


FIGURE 2.27 Band diagrams for conductors: (a) an alkali metal; (b) magnesium (Mg), a bivalent metal; (c) diamond, an insulator; and (d) germanium (Ge), a semiconductor. (From Mahajan, S. and Sree Harsha, K.S., *Principles of Growth and Processing of Semiconductors*, McGraw Hill, New York, 1998. With permission.)

A partially filled valence band is an important feature of metals. In some metals such as magnesium (Mg), the valence band (3s level for Mg) is almost filled, and there is no apparent bandgap. However, the empty 3p bands are overlapped with the almost filled 3s in Mg (Figure 2.27). In this band structure, when an electrical field is applied, the electrons in the almost 3s band accelerate and occupy the empty states in the 3p band and possibly higher bands.

The conducting ability of some materials (e.g., silicon [Si], germanium [Ge], and gallium arsenide [GaAs]) is in between that of insulators and conductors. These materials are known as *semiconductors*.

As shown below, their band gap is marginal and **a considerable number of electrons can jump** from the highest filled band (valence band) to the lowest unfilled band (conduction band) **using thermal energy**.

All the atoms in the crystal vibrates with a distribution of energies (refer to Debye model). As the T increases, the distribution spreads to higher energies. Statistically, **some of the atomic vibrations will be sufficiently energetic to rupture a bond**.

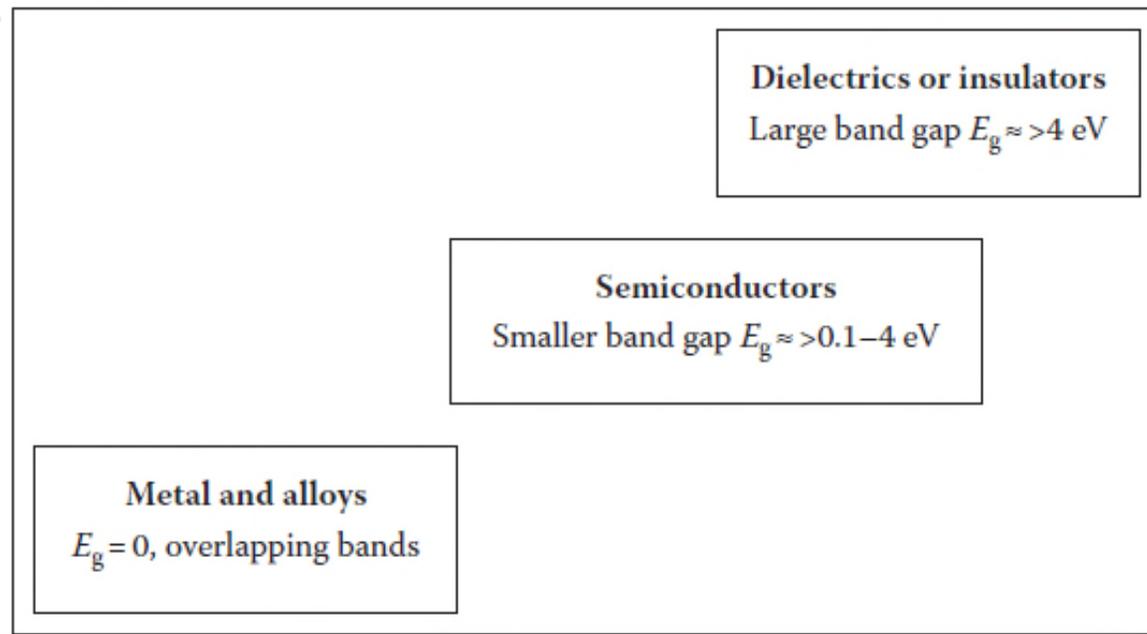


FIGURE 2.28 Classification of materials based on the values of band gap (E_g) in electron volts (eV).

In contrast to metals and semiconductors, most ceramic materials (e.g., silica [SiO₂], zirconia [ZrO₂], alumina [Al₂O₃], and silicon carbide [SiC]) exhibit strong ionic or covalent bonds (or both).

Many polymers (polyethylene, polystyrene, epoxies, etc.) also primarily exhibit covalent bonds *within* the chains of atoms and van der Waals bonds *among* the chains of atoms.

Therefore, ceramics and polymers are usually, but not always, electrical *insulators* and are also referred to as *dielectrics*. The use of the term “insulator” is sometimes preferred when emphasizing the ability of a material to withstand a strong electric field (i.e. *protection of the circuit*) - as opposed to offering only a high electrical resistance. For example, porcelain can be described more appropriately as an insulator rather than as a dielectric.

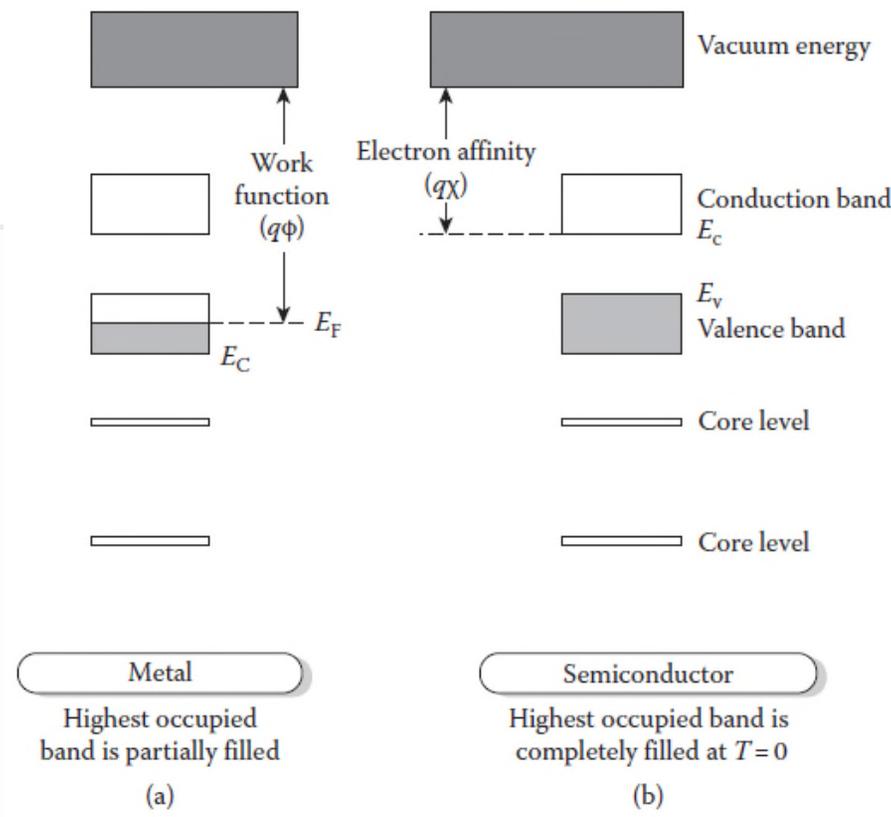


FIGURE 2.29 A schematic of band structure for (a) a typical metal and (b) a semiconductor.

Work function (ϕ) is the energy required to remove the electrons at the Fermi energy level (E_F) to the vacuum level. In addition, the energy needed to free the electron at the bottom of the conduction band to the vacuum level is called electron affinity (χ).

Fermi Energy and Fermi Level

Fermi level - the energy of an electron that is the least tightly bound to the lattice.

Strictly speaking, the Fermi level at 0K is called the Fermi energy; however, Fermi level and Fermi energy are used interchangeably in many cases. For convenience, we will use the term Fermi energy level and display it as E_F .

The Fermi energy level (E_F) can be explained in two different ways.

1) E_F means the highest energy level that electrons occupy at $T = 0K$. Thus, energy levels below E_F are fully taken, and those above E_F are fully empty at $T = 0K$.

2) E_F is the energy level that is occupied with a chance of 50% at $T > 0K$.

(When the temperature is higher than 0K, E_F is not the highest energy level that the electrons occupy. The energy level E_F is partially empty at $T > 0K$. Since electrons jump sequentially from the top energy levels, the distribution of the partially filled energy states above E_F and that of partially empty energy states below E_F is symmetric.)

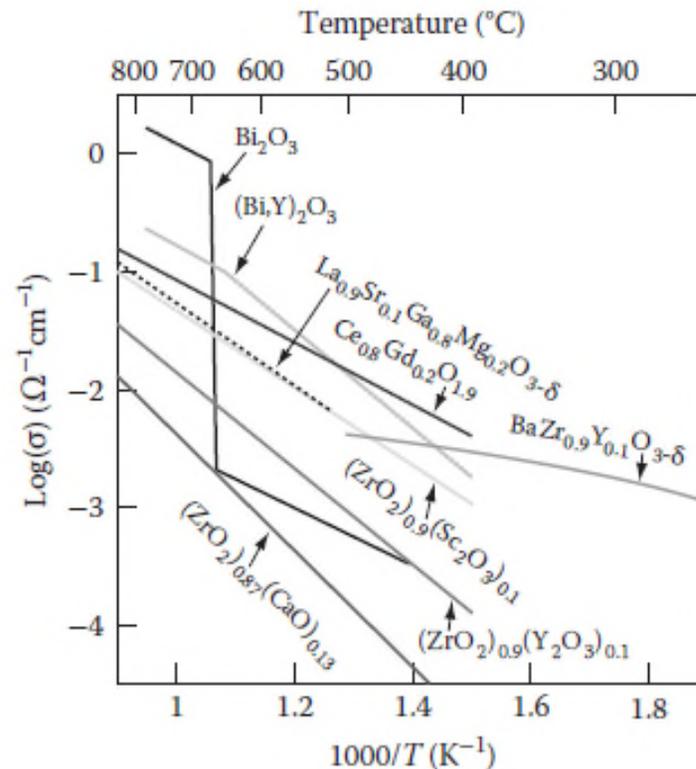


FIGURE 2.30 The conductivity of different materials used as electrolytes in the development of solid oxide fuel cells. (From Haile, S.M., *Acta Mater.*, 51, 5981–6000, 2003. With permission.)