23. $n = -3, -2, 0, 1$
25. (a) 2, (b) 1, (c) 5
27. (a) 12, (b) 3, (c) 1
29. Let $d = \gcd(2n-1, 2n+1)$. Then $d \mid (2n+1) - (2n-1)$.
31. Let $d = \gcd(b, c)$. Then $d \mid c$, so $d \mid a$. Also, $d \mid b$.
33. $(n+1)(n!+1) - ((n+1)!+1) = n$, so $\gcd(n!+1, (n+1)!+1)$ divides n and $n!+1$.
35. (a) $2 = 14 \cdot (-7) + 100$, (b) $6 = 6 \cdot 1 + 84 \cdot 0$, (c) $14 = 630 \cdot (-2) + 182 \cdot 7$, (d) $24 = 1848 \cdot 25 + 1776 \cdot (-26)$
37. Let $d = \gcd(n^2, n^2+n+1)$. Then $d \mid (n^2+n+1)_n^2$, so $d \mid n+1$ and therefore $d \mid (n+1)(n-1) = n^2 - 1$.
39. (a) $13 = 2 \cdot 5 + 3, \quad 5 = 1 \cdot 3 + 2, \quad 3 = 1 \cdot 2 + 1, \quad 2 = 2 \cdot 1 + 0$
(b) $1111111111111 = 100001000 \cdot 11111 + 111, \quad 11111 = 100 \cdot 111 + 11, \quad 111 = 10 \cdot 11 + 1, \quad 11 = 11 \cdot 1 + 0$
(c) If $a = bq + r$ with $r > 0$, then $c = dQ + R$, where Q is q ones, each separated by $b-1$ zeros, and the last 1 is followed by r zeros; and R is r ones.
41. (a) 194, (b) 21, (c) 133
43. Expand the final number in base 20: $a_0 + a_1 20 + a_2 20^2 + \cdots$. Then a_i is the number of people worth i billion.
45. If $n = rs$ with an odd number $r > 1$, then $a^n + 1$ has $a^s + 1$ as a factor.

Chapter 2

1. (a) $x = 2 + 4t, y = 1 - 3t$, (b) $x = 6 - 7t, y = 3 - 5t$, (c) $x = -5 + 23t, y = 2 - 9t$, (d) No solutions
3. (horses, oxen) = (51, 9), or (30, 40), or (9, 71)
5. Because $\gcd(a, b) = 1$, there is a solution x_0, y_0 with integers that are not necessarily positive. The general solution of $ax - by = c$ is $x = x_0 - bt, y = y_0 - at$. Let t be a large negative number.
7. Largest = 38, smallest = 34.
9. (a) (i) Use the solution for 10 and then add thirty 3-cent stamps
(ii) Use the solution for 8 and then add eighty-four 3-cent stamps.
(iii) Use the solution for 8 and then add ninety-eight 3-cent stamps.
(b) Every number greater than $k + a - 1$ differs from a number on the list by a positive multiple of a.

Chapter 3

1. (a) $3^2 \cdot 5^4$, (b) 5625 is a square
3. Use Theorem 2.2 with $a = b$.
5. (a) Write $a = 2^{a_2} 3^{a_3} \cdots$ and $b = 2^{b_2} 3^{b_3} \cdots$. By Proposition 2.6, $na_p \le nb_p$ for each p. Use Proposition 2.6 again to get $a \mid b$.
(b) Write $a = 2^{a_2} 3^{a_3} \cdots$ and $b = 2^{b_2} 3^{b_3} \cdots$. Proposition 2.6 says that $ma_p \le nb_p$ for each p. Use Proposition 2.6 again to get $a \mid b$.
(c) Let $a = 4, b = 2, m = 1, n = 2$.