Solutions to Chapter 2

Exercises

2.1 Simplify the following Boolean functions

a) $\overline{\overline{(A \cap B \cup C)} \cap \bar{B}}$

$= (A \cap B \cup C) \cup B$

$= AB \cup B \cup C$

$= \boldsymbol{B} \cup \boldsymbol{C}$

b) $(A \cup B) \cap (\bar{A} \cup \bar{B} \cap \bar{A})$

$= (A \cup B) \cap (\bar{A})$

$= (A\bar{A} \cup B\bar{A})$

$= \boldsymbol{B} \cap \bar{\boldsymbol{A}}$

c) $\overline{A \cap B \cap B \cap C} \cap \bar{B}$

$= \overline{(A \cap B \cap B \cap C)} \cap \bar{B}$

$= (\bar{A} \cup \bar{B} \cup \bar{B} \cup \bar{C}) \cap \bar{B}$

$= (\bar{A} \cup \bar{B} \cup \bar{C}) \cap \bar{B}$

$= \bar{A} \cap \bar{B} \cup \bar{B} \cap \bar{B} \cup \bar{C} \cap \bar{B}$

$= \bar{A}\bar{B} \cup \bar{B}\bar{B} \cup \bar{C}\bar{B}$

$= \bar{A}\bar{B} \cup \bar{B} \cup \bar{C}\bar{B}$

$= \bar{\boldsymbol{B}}$

2.2 Reduce the following Boolean function

$A \cap B \cap \left(\overline{C \cup (\bar{C} \cup A) \cup \bar{B}}\right)$

$= A \cap B \cap \left(\bar{C} \cap \left(\overline{\bar{C} \cup A}\right) \cap B\right)$

$= A \cap B \cap (\bar{C} \cap (C \cap \bar{A}) \cap B)$

$= AB\bar{C}C\bar{A}B$

$= \boldsymbol{\phi}$

2.3 Simplify the following Boolean expressions

a) $\overline{\overline{(A \cap B \cup C)} \cap \overline{B}}$

$= (A \cap B \cup C) \cup B$

$= AB \cup B \cup C$

$= \boldsymbol{B} \cup \boldsymbol{C}$

b) $[(A \cup B) \cap \bar{A}] \cup (\bar{B} \cap \bar{A})$

$= [A \cap \bar{A} \cup B \cap \bar{A}] \cup (\bar{B} \cap \bar{A})$

$= [\phi \cup B \cap \bar{A}] \cup (\bar{B} \cap \bar{A})$

$= B\bar{A} \cup \bar{B}\bar{A}$

$= \bar{A}(B \cup \bar{B})$

$= \boldsymbol{\bar{A}}$

2.4 Reduce the following Boolean function

$G = (A \cup B \cup C) \cap \left(\overline{A \cap \bar{B} \cap \bar{C}}\right) \cap \bar{C}$

$G = (A \cup B \cup C) \cap (\bar{A} \cup B \cup C) \cap \bar{C}$

$G = (A \cup B \cup C) \cap (\bar{A}\bar{C} \cup B\bar{C} \cup C\bar{C})$

$G = (A \cup B \cup C) \cap (\bar{A}\bar{C} \cup B\bar{C})$

$G = A\bar{A}\bar{C} \cup B\bar{A}\bar{C} \cup C\bar{A}\bar{C} \cup AB\bar{C} \cup BB\bar{C} \cup CB\bar{C}$

$G = \phi \cup B\bar{A}\bar{C} \cup \phi \cup AB\bar{C} \cup B\bar{C} \cup \phi$

$\boldsymbol{G = B\bar{C}}$

If Pr(A) = Pr(B) = Pr(C) = 0.9, what is Pr(G)?

$\mathbf{Pr}(\boldsymbol{G}) = \mathbf{0.9} \times \mathbf{0.1} = \mathbf{0.09}$

2.5 Simplify the following Boolean equations

a) $(A \cup B \cup C) \cap \left(\overline{A \cap \bar{B} \cap \bar{C}}\right) \cap \bar{C}$

$= (A \cup B \cup C) \cap (\bar{A} \cup B \cup C) \cap \bar{C}$

$= (A \cup B \cup C) \cap (\bar{A}\bar{C} \cup B\bar{C} \cup C\bar{C})$

$= (A \cup B \cup C) \cap (\bar{A}\bar{C} \cup B\bar{C})$

$= A\bar{A}\bar{C} \cup B\bar{A}\bar{C} \cup C\bar{A}\bar{C} \cup AB\bar{C} \cup BB\bar{C} \cup CB\bar{C}$

$= \phi \cup B\bar{A}\bar{C} \cup \phi \cup AB\bar{C} \cup B\bar{C} \cup \phi$

$= \boldsymbol{B\bar{C}}$

b) $(A \cup B) \cap \bar{B}$

$= A\bar{B} \cup B\bar{B}$

$= \boldsymbol{A\bar{B}}$

2.6 Reduce the following Boolean equation

$\left(\overline{\mathbf{A} \cup (\mathbf{B} \cap \mathbf{C})}\right) \cap \left(\overline{\mathbf{B} \cup (\mathbf{D} \cap \mathbf{A})}\right)$

$= \left(\bar{A} \cap (\bar{B} \cup \bar{C})\right) \cap \left(\bar{B} \cap (\bar{D} \cup \bar{A})\right)$

$= (\bar{A}\bar{B} \cup \bar{A}\bar{C}) \cap (\bar{B}\bar{D} \cup \bar{B}\bar{A})$

$= \bar{A}\bar{B}\bar{B}\bar{D} \cup \bar{A}\bar{C}\bar{B}\bar{D} \cup \bar{A}\bar{B}\bar{B}\bar{A} \cup \bar{A}\bar{C}\bar{B}\bar{A}$

$= \bar{A}\bar{B}\bar{D} \cup \bar{A}\bar{C}\bar{B}\bar{D} \cup \bar{A}\bar{B} \cup \bar{A}\bar{C}\bar{B}\bar{A}$

$= \boldsymbol{\bar{A}\bar{B}}$

2.7 Use both Equations (2.25) and (2.29) to find the reliability Pr(s). Which equation is preferred for numerical solution?

$\mathrm{Pr}(s) = \mathrm{Pr}(E_1 \cup E_2 \cup E_3)$

$\mathrm{Pr}(E_1) = 0.8; \quad \mathrm{Pr}(E_2) = 0.9; \quad \mathrm{Pr}(E_3) = 0.95$

Using Equation 2.25:

$\mathrm{Pr}(E_1 \cup E_2 \cup E_3)$
$= [\mathrm{Pr}(E_1) + \mathrm{Pr}(E_2) + \mathrm{Pr}(E_3)] - [\mathrm{Pr}(E_1 \cap E_2) + \mathrm{Pr}(E_1 \cap E_3) + \mathrm{Pr}(E_2 \cap E_3)] + \mathrm{Pr}(E_1 \cap E_2 \cap E_3)$

$= [0.8 + 0.9 + 0.95] - [0.8 \times 0.9 + 0.8 \times 0.95 + 0.9 \times 0.95] + 0.8 \times 0.9 \times 0.95$

$= \mathbf{0.999}$

Using Equation 2.29 (Preferred Equation for Computational Simplicity)

$\Pr(E_1 \cup E_2 \cup E_3)$

$= 1 - [1 - \Pr(E_1)][1 - \Pr(E_2)][1 - \Pr(E_3)]$

$= 1 - [1 - 0.8][1 - 0.9][1 - 0.95]$

$= 1 - 0.001$

$= \mathbf{0.999}$

2.8 A stockpile of 40 relays contain 8 defective relays. If five relays are selected at random and the number of defective relays is known to be greater than two, what is the probability that exactly four relays are defective?

x = number of defective relays in selected sample = 4
n = number of relays in the selected sample = 5
N = number of relays in the stockpile = 40
D = number of defective relays in the stockpile = 8

$$\Pr(X = 4 \mid X > 2) = \frac{\Pr(X > 2 \mid X = 4) \times \Pr(X = 4)}{\Pr(X > 2)} = \frac{\Pr(X = 4)}{\Pr(X > 2)}$$

This problem can be solved using the **Hypergeometric Distribution**

$$\Pr(X = x) = \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}$$

$$\Pr(X = 4) = \frac{\binom{8}{4}\binom{40-8}{5-4}}{\binom{40}{5}} = 0.003404$$

Similarly:
$\Pr(X = 3) = 0.0422$
$\Pr(X = 5) = 8.51 \times 10^{-5}$

$$\Pr(X = 4 \mid X > 2) = \frac{0.003404}{0.0422 + 0.003404 + 8.51 \times 10^{-5}} = \mathbf{0.074488}$$

2.9 Given that P = 0.006 is the probability of an engine failure on a flight between two cities, find the probability of:

a) No engine failure in 1000 flights

b) At least one failure in 1000 flights

c) At least two failures in 1000 flights

Method 1: Using the Binomial Distribution

Given n =1000; p = 0.006

Let X be the r.v. of having an engine failure during a flight

$$\Pr(X = x) = \binom{1000}{x} 0.006^x \times 0.994^{1000-x}$$

a) $\Pr(X = 0) = \binom{1000}{0} 0.006^0 \times 0.994^{1000} = \mathbf{0.00243}$

b) $\Pr(X \geq 1) = 1 - \binom{1000}{0} 0.006^0 \times 0.994^{1000} = \mathbf{0.99757}$

c) $\Pr(X \geq 2) = 1 - \binom{1000}{1} 0.006^1 \times 0.994^{999} - \binom{1000}{0} 0.006^0 \times 0.994^{1000} = \mathbf{0.98287}$

Method 2: Using the Poisson Distribution

If n >> 1 & p << 1, the Poisson distribution can be used to approximate the Binomial distribution.

$$\Pr(X = x) = \frac{\mu^x e^{-\mu}}{x!} \qquad where\ \mu = np = 1000 \times 0.006 = 6$$

a) $\Pr(X = 0) = \frac{6^0 e^{-6}}{0!} = \mathbf{0.00248}$

b) $\Pr(X \geq 1) = 1 - \frac{6^0 e^{-6}}{0!} = \mathbf{0.99752}$

c) $\Pr(X \geq 2) = 1 - \frac{6^1 e^{-6}}{1!} - \frac{6^0 e^{-6}}{0!} = \mathbf{0.98265}$

2.10 A random sample of 10 resistors is to be tested. From past experience, it is known that the probability of a given resistor being defective is 0.08. Let X be the r.v. for number of defective resistors.

a) What kind of distribution function would be recommended for modeling the r.v.?

This problem is best modeled using the Binomial distribution

b) According to the distribution function in (a), what is the probability that in the sample of 10 resistors, there are more than 1 defective resistors in the sample?

Given n =10; p = 0.08

Let X be the r.v. of being given a defective resistor

$$\Pr(X > 1) = 1 - \binom{10}{1} 0.08^1 \times 0.92^9 - \binom{10}{0} 0.08^0 \times 0.92^{10} = \mathbf{0.1879}$$

2.11 How many different license plates can be made if each consists of three numbers and three letters, and no number or letter can appear more than once on a single plate?

This problem is a question of determining the number of possible permutations. Assuming that the digits 0 through to 9 can be used and that all 26 letters of the alphabet can be used, then the solution is given by:

$$Let\ L\ =\ the\ r.v.for\ the\ letters$$

$$Let\ N\ =\ the\ r.v.for\ the\ numbers$$

$$No.of\ Different\ Plates\ =\ L \times L \times L \times N \times N \times N$$

$$No.of\ Different\ Plates\ =\ 26 \times 25 \times 24 \times 10 \times 9 \times 8 = \mathbf{11,232,000}$$

2.12 The consumption of maneuvering jet fuel in a satellite is known to be normally distributed with a mean of 10,000 hours and a standard deviation of 1,000 hours. What is the probability of being able to maneuver the satellite for the duration of a 1-year mission?

Let T be the r.v. for the consumption life of the satellite jet fuel

$$\Pr(T = t) \sim Norm(\mu = 10{,}000; \sigma = 1{,}000)$$

$$T = 1yr =\ 8760\ hrs$$

$$\Pr(T > 8760) = \Pr\left(Z > \frac{T - \mu}{\sigma}\right) = \Pr\left(Z > \frac{8760 - 10000}{1000}\right)$$

$$\Pr(Z > -1.24) = \mathbf{0.8925}$$

2.13 Suppose a process produces electronic components, 20% of which are defective. Find the distribution of x, the number of defective components in a sample size of five. Given that the sample contains at least three defective components, find the probability that four components are defective.

This is a conditional probability with the Binomial distribution with parameters N = 5 and p = 0.2

$$\Pr(X = 4\ |X \geq 3) = \frac{\Pr(X \geq 3\ |X = 4) \times \Pr(X = 4)}{\Pr(X \geq 3)} = \frac{\Pr(X = 4)}{\Pr(X \geq 3)}$$

$$\Pr(X = 4\ |X \geq 3) = \frac{\binom{5}{4} 0.2^4 \times 0.8^1}{\binom{5}{3} 0.2^3 \times 0.8^2 + \binom{5}{4} 0.2^4 \times 0.8^1 + \binom{5}{5} 0.2^5 \times 0.8^0}$$

$$\Pr(X = 4\ |X \geq 3) = \frac{0.0064}{0.0512 + 0.0064 + 0.00032} = \mathbf{0.1105}$$

2.14 If the heights of 300 students are normally distributed, with a mean of 68 inches and standard deviation of 3 inches, how many students have:

a) heights of more than 70 inches?

$$\Pr(X > 70) = \Pr\left(Z > \frac{X-\mu}{\sigma}\right) = \Pr\left(Z > \frac{70-68}{3}\right) = \Pr\left(Z > \frac{2}{3}\right) = \mathbf{0.2525}$$

Therefore, the number of students that are greater than 70 inches in height is given by:

$$300 \times 0.2525 = \mathbf{76}$$

b) heights between 67 and 68 inches?

$$\Pr(67 < X > 68) = \Pr\left(\frac{67-68}{3} < Z < \frac{68-68}{3}\right) = \Pr\left(\frac{-1}{3} < Z < 0\right) = \mathbf{0.13056}$$

Therefore, the number of students with heights between 67 and 68 inches is given by:

$$300 \times 0.13056 = \mathbf{39}$$

2.15 Assume that for a certain type of resistor, 1% are bad when purchased. What is the probability that a circuit with 10 resistors has exactly 1 bad resistor?

Let X be the r.v. of finding a bad resistor

This is a binomial distribution with parameters N = 10 and p = 0.01

$$\Pr(X = 1) = \binom{10}{1} 0.01^1 \times 0.99^9 = \mathbf{0.0914}$$

2.16 Between the hours of 2 and 4 p.m. the average number of phone calls per minute coming into an office is two and one-half. Find the probability that during a particular minute, there will be more than five phone calls.

This is a Poisson Distribution

$$\Pr(X = x) = \frac{\mu^x e^{-\mu}}{x!} \qquad where\ \mu = \lambda t$$

$$Given\ \lambda\ =\ 2.5\ min^{-1}\ \&\ t = 1\ min;\ \mu = 2.5$$

$$\Pr(X > 5) = 1 - \sum_{i=1}^{5} \frac{\mu^x e^{-\mu}}{x!}$$

$$\Pr(X > 5) = 1 - 0.95798 = \mathbf{0.04202}$$

2.17 A guard works between 5 p.m. and 12 midnight; he sleeps an average of 1 hour before 9 p.m., and 1.5 hours between 9 and 12. An inspector finds him asleep, what is the probability that this happens before 9 p.m.?

Let A_1 be the fraction of time between 5-9pm

Let A_2 be the fraction of time between 9-12pm

Let E be the event of the guard sleeping on shift

$\Pr(A_1) = {}^4/_7 \quad \Pr(A_2) = {}^3/_7$

$\Pr(E \mid A_1) = {}^1/_4 \quad \Pr(E \mid A_2) = {}^{1.5}/_3$

Using Bayes Theorem

$$Pr(A_j|E) = \frac{Pr(E|A_j)\Pr(A_j)}{\sum_{j=1}^{n} Pr(E|A_j)\Pr(A_j)}$$

$$Pr(A_1|E) = \frac{Pr(E|A_1)\Pr(A_1)}{Pr(E|A_1)\Pr(A_1) + Pr(E|A_2)\Pr(A_2)} = \frac{{}^1/_4 \times {}^4/_7}{{}^1/_4 \times {}^4/_7 + {}^{1.5}/_3 \times {}^3/_7} = \mathbf{0.4}$$

2.18 The number of system breakdowns occurring with a constant rate in a given length of time has a mean value of two breakdowns. What is the probability that in the same length of time, two breakdowns will occur?

This is a Poisson Distribution

$$\Pr(X = x) = \frac{\mu^x e^{-\mu}}{x!} \qquad where\ \mu = \lambda t$$

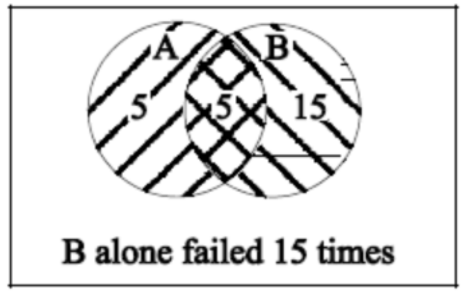
$Given\ \ \mu = 2$

$$\Pr(X = 2) = \frac{2^2 e^{-2}}{2!} = \mathbf{0.271}$$

2.19 An electronic assembly consists of two subsystems, A and B. Each assembly is given one preliminary checkout test. Records on 100 preliminary checkout tests show that subsystem A failed 10 times. Subsystem B alone failed 15 times. Both subsystems A and B failed together five times.

a) What is the probability of A failing, given that B has failed.

b) What is the probability that A alone fails.



Total failures of subsystem A = 10
Simultaneous failures of subsystems A & B = 5
Total failures of subsystem B = 15+5

$\Pr(B) = 0.2$
$\Pr(A \cap B) = 0.05$

Using Law of Conditional Probability:

$$a)\ Pr(A|B) = \frac{Pr(A \cap B)}{\Pr(B)} = \frac{0.05}{0.2} = \mathbf{0.25}$$

Using Law of Total Probability:

$$b)\ Pr(A\ alone) = \sum_{i=1}^{n} Pr(A|B_i)\Pr(B_i) = 0.25 \times 0.2 = \mathbf{0.05}$$

2.20 A presidential election poll shows one candidate leading with 60% of the vote. If the poll is taken from 200 random voters throughout the U.S., what is the probability that the candidate will get less than 50% of the votes in the election? (Assume the 200 voters sampled are true representatives of the voting profile.)

Using a Binomial approximation for a Normal Distribution we find the solution as follows:

Given n = 200 & p = 0.6
$\mu = np = 200 \times 0.6 = 120$
$\sigma^2 = npq = 200 \times 0.6 \times 0.4 = 48$

$$Pr(X < 100) = \Pr\left(Z < \frac{100 - 120}{\sqrt{48}}\right) = \Pr(Z < -2.887) = \mathbf{0.001946}$$

2.21 A newspaper article reports that a New York medical team has introduced a new male contraceptive method. The effectiveness of this method was tested using a number of couples over a period of 5 years. The following statistics are obtained:

Year	Times Employed	Unwanted Pregnancies (X)
1	8200	19
2	10100	18
3	2120	1
4	6120	9
5	18130	30

a) Estimate the mean probability of an unwanted pregnancy per use. What is the standard deviation of the estimate?

Year	Times Employed	Unwanted Pregnancies (X)	Pr(X)	(xi-x_bar)^2
1	8200	19	0.00232	6.05E-07
2	10100	18	0.00178	5.90E-08
3	2120	1	0.00047	1.14E-06
4	6120	9	0.00147	4.71E-09
5	18130	30	0.00165	1.33E-08

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{5}(0.00232 + 0.00178 + \cdots + 0.00165) = \mathbf{0.00154}$$

$$S^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{4}(6.05 \times 10^{-7} + 5.9 \times 10^{-8} + \cdots + 1.33 \times 10^{-8}) = \mathbf{4.55 \times 10^{-7}}$$

$$S = \sqrt{4.55 \times 10^{-7}} = \mathbf{6.75 \times 10^{-4}}$$

b) What are the 95% upper and lower confidence limits of the mean and standard deviation?

$$\left[\bar{x} - t_{\alpha/2}\frac{S}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2}\frac{S}{\sqrt{n}}\right] = 0.95$$

$$\left[0.00154 - t_{0.05/2}\frac{6.75 \times 10^{-4}}{\sqrt{5}} \le \mu \le 0.00154 + t_{0.05/2}\frac{6.75 \times 10^{-4}}{\sqrt{5}}\right]$$

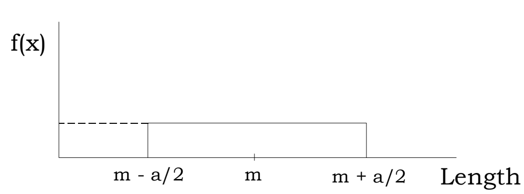
$$[\mathbf{7.01 \times 10^{-4} \le \mu \le 2.38 \times 10^{-3}}]$$

$$\left[\sqrt{\frac{(n-1)S^2}{\chi^2_{1-a/2}(n-1)}} \le \sigma \le \sqrt{\frac{(n-1)S^2}{\chi^2_{a/2}(n-1)}}\right] = 0.95$$

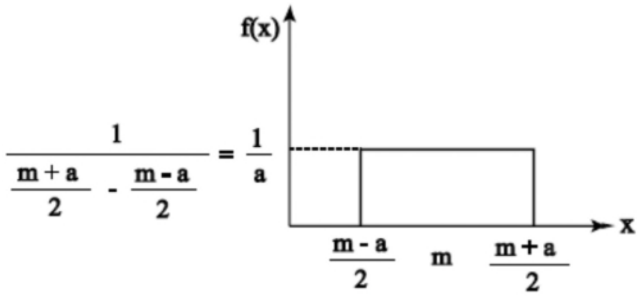
$$\left[\sqrt{\frac{(5-1)4.55 \times 10^{-7}}{\chi^2_{0.975}(5-1)}} \le \sigma \le \sqrt{\frac{(5-1)4.55 \times 10^{-7}}{\chi^2_{0.025}(5-1)}}\right]$$

$$[\mathbf{404 \times 10^{-4} \le \sigma \le 1.94 \times 10^{-3}}]$$

2.22 Suppose the lengths of the individual links of a chain distribute themselves with a uniform distribution, shown below.



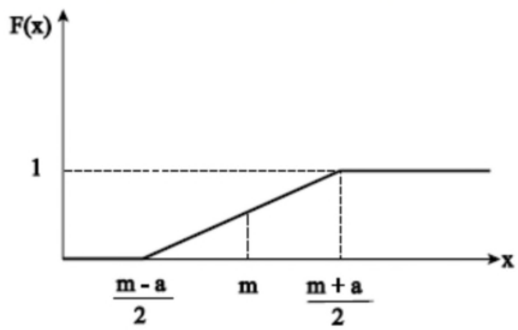
a) What is the height of the rectangle?



The height of this rectangle is 1/a

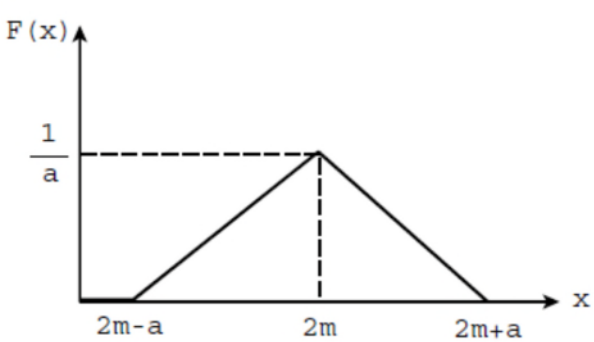
b) Find the cumulative PDF for the above distribution. Make a sketch of the distribution and label the axes.

The cumulative probability density function (cdf) is shown below



$$F(x) = \begin{cases} 0 & x < m - a/2 \\ \dfrac{x - (m - a/2)}{a} & m - a/2 \le x \le m + a/2 \\ 1 & x \ge m + a/2 \end{cases}$$

c) If numerous chains are made from two such links hooked together, what is the PDF of two-link chains?



$$F(x) = \iint f(s)f(t)dtds$$

$$for\ x \leq 2m$$

$$F(x) = \frac{1}{2}bh = \frac{1}{2}(x - 2m + a)\left(\frac{x - 2m + a}{a^2}\right) = \frac{1}{2a^2}(x - 2m + a)^2$$

$$f(x) = \frac{d}{dx}[F(x)] = \frac{d}{dx}\left[\frac{1}{2a^2}(x - 2m + a)^2\right]$$

$$\boldsymbol{f(x) = \frac{(x - 2m + a)}{a^2}}$$

$$for\ 2m \leq x \leq 2m + a$$

$$F(x) = 1 - \frac{1}{2}bh = 1 - \frac{1}{2}(2m + a - x)\left(\frac{2m + a - x}{a^2}\right) = 1 - \frac{1}{2a^2}(2m + a - x)^2$$

$$f(x) = \frac{d}{dx}[F(x)] = \frac{d}{dx}\left[1 - \frac{1}{2a^2}(2m + a - x)^2\right]$$

$$\boldsymbol{f(x) = \frac{(2m + a - x)}{a^2}}$$

d) Consider a 100-link chain. What is the probability that the length of the chain will be less than 100.5 m if a = 0.1m?

Using the Central Limit Theorem, this problem will approximation a normal; distribution with $\mu = 100m$ & $\sigma^2 = {100a^2}/{12}$

$$\Pr(X < 100.5m) = \Pr\left(Z < \frac{100.5m - 100m}{10a/\sqrt{12}}\right) = \Pr(Z < 1.732) = \mathbf{0.9583}$$

2.23 If f(x, y) = 1/2xy^2 + 1/2yx^2, 0 < x < 1, 0 < y < 2:

a) Show that f(x, y) is a joint probability density function.

For f(x,y) to be a joint PDF, it must satisfy the following conditions.

1. $f(x,y) \geq 0 \qquad -\infty < x, y < \infty$

2. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = 1$

Condition 1 is met by observation; the PDF is always greater than or equal to 0.

Condition 2 is proven as follows:

$$\int_0^2\int_0^1 (0.5xy^2 + 0.5x^2y)dxdy$$

$$\int_0^2 \left[\frac{x^2y^2}{4} + \frac{yx^3}{6}\right]_0^1 dy \;=\; \int_0^2 \left(\frac{y^2}{4} + \frac{y}{6}\right)dy \;=\; \left[\frac{y^3}{12} + \frac{y^2}{12}\right]_0^2 \;=\; 1$$

b) Find Pr(x > y), Pr(y > x), Pr(x = y).

(i) Find Pr(x > y) the limits for this problem will be 0<x<1 & 0<y<x

$$\int_0^1\int_0^x (0.5xy^2 + 0.5x^2y)dydx$$

$$\int_0^1 \left[\frac{xy^3}{6} + \frac{y^2x^2}{4}\right]_0^x dx \;=\; \int_0^1 \left(\frac{x^4}{6} + \frac{x^4}{4}\right)dx \;=\; \left[\frac{x^5}{30} + \frac{x^5}{20}\right]_0^1 \;=\; \frac{5}{60}$$

(ii) Pr(y > x) the limits for this problem will be 0<x<1 & x<y<2

$$\int_0^1\int_x^2 (0.5xy^2 + 0.5x^2y)dydx$$

$$\int_0^1 \left[\frac{xy^3}{6} + \frac{y^2x^2}{4}\right]_x^2 dx \;=\; \int_0^1 \left(\left(\frac{8x}{6} + \frac{4x^2}{4}\right) - \left(\frac{x^4}{6} + \frac{x^4}{4}\right)\right)dx$$

$$= \left[\frac{8x^2}{12} + \frac{4x^3}{12} - \frac{x^5}{30} - \frac{x^5}{20}\right]_0^1 \;=\; \frac{55}{60}$$

(iii) Pr(x = y) does not exist as this is a continuous RV and x=y would not form a volume under the integral. Therefore, Pr(x = y) = 0

2.24 A company is studying the feasibility of buying an elevator for a building under construction. One proposal is a 10-passenger elevator that, on average, would arrive in the lobby once per minute. The company rejects this proposal because it expects an average of five passengers per minute to use the elevator.

a) Support the proposal by calculating the probability that in any given minute, the elevator does not show up, and 10 or more passengers arrive.

There are two Poisson Distributions in this problem; one for the elevator and one for the passengers.

Elevator - $\Pr(X = x) = \frac{\mu^x e^{-\mu}}{x!}$ $\quad$ $where\ \mu = \lambda t = \frac{1}{min} \times 1\,\text{min} = 1$

$$\Pr(X = 0) = \frac{1^0 e^{-1}}{0!} = \mathbf{0.369}$$

Passengers - $\Pr(N = n) = \frac{\mu^n e^{-\mu}}{n!}$ $\quad$ $where\ \mu = \lambda t = \frac{5}{min} \times 1\,\text{min} = 5$

$$\Pr(N \geq 10) = 1 - \sum_{1}^{9} \frac{\mu^n e^{-\mu}}{n!} = \mathbf{0.032}$$

Therefore, the probability that an elevator does not show up and 10 or more passengers arrive is given by:

$Pr(X = 0\ \cap N \geq 10) = 0.369 \times 0.032 = \mathbf{0.0117}$

b) Determine the probability that the elevator arrives only once in a 5-minute period.

Elevator - $\Pr(X = x) = \frac{\mu^x e^{-\mu}}{x!}$ $\quad$ $where\ \mu = \lambda t = \frac{1}{min} \times 5\,\text{min} = 5$

$$\Pr(X = 1) = \frac{5^1 e^{-5}}{1!} = \mathbf{0.0337}$$

2.25 The frequency distribution of time to establish the root causes of a failure by a group of experts is observed and given below.

Time Range (hrs)		Obs Freq
45	55	7
55	65	18
65	75	35
75	85	28
85	95	12

Test whether a normal distribution with known **σ = 10** is an appropriate model for these data.

In order to test whether the normal distribution is an adequate fit in this case, we use a Chi Squared Goodness-of-Fit test where

a) H_o: the r.v (X) follows a Normal Distribution with $\sigma = 10$; or

b) H_1: the r.v (X) does not follow a Normal Distribution with $\sigma = 10$

The Chi Squared Statistic is calculated for each row of the data given above using the following process. The complete solution is shown in the table below.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{100}(350 + 1080 + 2450 + 2240 + 1080) = \mathbf{72}$$

$$\Pr(45 < X < 55) = \Pr\left(\frac{45-72}{10} < Z < \frac{55-72}{10}\right) = \Pr(< -2.7 < Z < -1.7) = \mathbf{0.0411}$$

$$Freq_{exp}(45 < X < 55) = \mathrm{N} \times \Pr(45 < X < 55) = 100 \times 0.0411 = \mathbf{4.11}$$

$$\text{Chisq Stat} = \frac{\left(Freq_{obs} - Freq_{exp}\right)^2}{Freq_{exp}} = \frac{(7-4.11)^2}{4.11} = \mathbf{2.032}$$

Time Range (hrs)		Mid Range	Obs Freq	F*Mid	Pr(x)	Ex(F)	Chisq Stat
45	55	50	7	350	0.0411	4.110	2.0324
55	65	60	18	1080	0.1974	19.740	0.1533
65	75	70	35	2450	0.3759	37.595	0.1791
75	85	80	28	2240	0.2853	28.529	0.0098
85	95	90	12	1080	0.0861	8.608	1.3370
		Total	100	7200		**W**	**3.7116**
			Sample Mean	72		R	7.815
			Sigma (Known)	10			

$R \geq \chi^2_{1-\alpha}(k - m - 1)$ where k = # of intervals & m = number of unknown parameters

Using a significance level of $\alpha = 5\%$

$$R \geq \chi^2_{0.95}(5 - 1 - 1)$$

$$R \geq \chi^2_{0.95}(3) = \mathbf{7.815}$$

Given that W < R (ie: 3.712 < 7.815) we **do not reject** the null hypothesis at the 5% significance level.

2.26 A random number generator yields the following sample of 50 digits:

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	4	8	8	4	10	3	2	2	4	5

Is there any reason to doubt the digits are uniformly distributed? (Use the Chi-square goodness-of-fit test.)

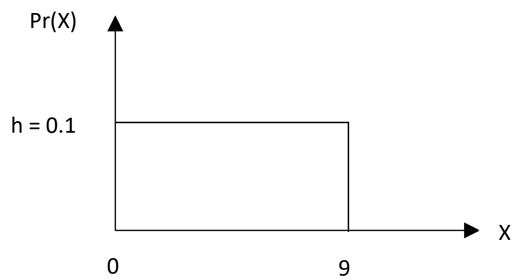
In order to test whether the uniform distribution is an adequate fit in this case, we use a Chi Squared Goodness-of-Fit test where

a) H_o: the r.v (X) follows a Uniform Distribution over the range of data; or

b) H_1: the r.v (X) does not follow a Uniform Distribution.

The Chi Squared Statistic is calculated for each data set given above using the following process. The complete solution is shown in the table below.

$$\Pr(X) = \frac{1}{10} = 0.1$$



$$Freq_{exp}(X = 0) = \mathrm{N} \times \Pr(X = 0) = 50 \times 0.1 = \mathbf{5}$$

$$\text{Chisq Stat} = \frac{\left(Freq_{obs} - Freq_{exp}\right)^2}{Freq_{exp}} = \frac{(4-5)^2}{5} = \mathbf{0.2}$$

Digit	Freq	Pr(x)	Ex(F)	Chisq Stat
0	4	0.10	5.0	0.20
1	8	0.1	5.0	1.80
2	8	0.1	5.0	1.80
3	4	0.1	5.0	0.20
4	10	0.1	5.0	5.00
5	3	0.1	5.0	0.80
6	2	0.1	5.0	1.80
7	2	0.1	5.0	1.80
8	4	0.1	5.0	0.20
9	5	0.1	5.0	0.00
Total	50		**W**	**13.60**
			R	16.92

$R \geq \chi^2_{1-\alpha}(k-m-1)$ where k = # of intervals & m = number of unknown parameters

Using a significance level of $\alpha = 5\%$

$R \geq \chi^2_{0.95}(10-0-1)$

$R \geq \chi^2_{0.95}(9) = \mathbf{16.92}$

Given that W < R (ie: 13.6 < 16.92) we **do not reject** the null hypothesis at the 5% significance level.

2.27 A set of 40 high-efficiency pumps is tested, all of the pumps fail (F = 40) after 400 pump-hours (T = 400). It is believed that the time to failure of the pumps follows an exponential distribution. Using the following table and the goodness-of-fit method, determine if the exponential distribution is a good choice.

Time Interval (hrs)		Observed Failures
0	2	6
2	6	12
6	10	7
10	15	6
15	25	7
25	100	2

In order to test whether the Exponential distribution is an adequate fit in this case, we use a Chi Squared Goodness-of-Fit test where

a) H_0: the r.v (T) follows an Exponential Distribution; or

b) H_1: the r.v (T) does not follow an Exponential Distribution.

The Chi Squared Statistic is calculated for each data set given above using the following process. The complete solution is shown in the table below.

$$\lambda = \frac{n}{T} = \frac{40}{400} = \mathbf{0.1\ hr^{-1}}$$

$$\Pr(0 < T < 2) = e^{-\lambda T_1} - e^{-\lambda T_2} = e^{-\lambda 0} - e^{-\lambda 2} = \mathbf{0.1812}$$

$$Freq_{exp}(0 < T < 2) = \mathrm{N} \times \Pr(0 < T < 2) = 40 \times 0.1812 = \mathbf{7.25}$$

$$\text{Chisq Stat} = \frac{\left(Freq_{obs} - Freq_{exp}\right)^2}{Freq_{exp}} = \frac{(6 - 7.25)^2}{7.25} = \mathbf{0.2158}$$

Time Range f(x)		Observed Freq	Ex(F)	Chisq Stat
0	2	6	7.25	0.2158
2	6	12	10.80	0.1341
6	10	7	7.24	0.0078
10	15	6	5.79	0.0076
15	25	7	5.64	0.3270
25	100	2	3.28	0.5005
	Total	40	**W**	**1.193**
	lambda	0.1	**R**	9.488

$R \geq \chi^2_{1-\alpha}(k - m - 1)$ where k = # of intervals & m = number of unknown parameters

Using a significance level of $\alpha = 5\%$

$R \geq \chi^2_{0.95}(6 - 1 - 1)$

$R \geq \chi^2_{0.95}(4) = \mathbf{9.49}$

Given that W < R (ie: 1.193 < 9.49) we **do not reject** the null hypothesis at the 5% significance level.

2.28 Use Eqn 2.74 and calculate mean and variance of a Weibull distribution.

The Weibull distribution is

$$f(t) = \frac{\beta\, t^{\beta-1}}{\alpha^{\beta}}\, e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

Since

$$m_t(s) = \int_{-\infty}^{\infty} e^{st}\, f(t)\, dt$$

$$\frac{dm_t(s)}{ds} = \int_{-\infty}^{\infty} \frac{d}{ds}\left\{e^{-st}\right\} f(t)\, dt$$

$$-\frac{dm_t(s)}{ds} = \int_{-\infty}^{\infty} e^{-st}\left[t\, f(t)\right] dt = L\{t\, f(t)\}\Bigg|_{s=0}$$

and

$$\frac{dm_t(s)}{ds}\Bigg|_{s=0} = E(t) = \mu$$

The expected value of the Weibull distribution is

$$\mu = E(t) = \int_{0}^{\infty} t\, \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}} dt$$

Making a change of variable, we get:

$u = \left(\frac{t}{\alpha}\right)^{\beta}$ => $t = \alpha u^{\frac{1}{\beta}}$ $\frac{du}{dt} = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}$

$$\mu = E(t) = \int_{0}^{\infty} \alpha u^{\frac{1}{\beta}}\, \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-u}\, \frac{\alpha}{\beta}\left(\frac{\alpha}{t}\right)^{\beta-1} du$$

From the Gamma function,

$$\int_0^\infty u^n e^{-u}\, du = \Gamma(n+1)$$

and

$$\Gamma\left(\frac{1}{\beta}+1\right)$$

where

$$n = \frac{1}{\beta}$$

the mean time to failure for the Weibull model is

$$\boxed{\mu = E(t) = \alpha \Gamma\left(\frac{1}{\beta}+1\right)}$$

and

$$\frac{d^2 m_t(s)}{ds^2} = \int_0^\infty \frac{d^2}{ds^2}\left\{e^{-st}\right\} f(t)\, dt$$

then

$$\frac{d^2 m_t(s)}{ds^2} = \int_0^\infty e^{-st}\left[t^2 f(t)\right] dt = \mathcal{L}\left\{t^2 f(t)\right\}\Big|_{s=0}$$

and

$$\frac{d^2 m_t(s)}{ds^2}\Big|_{s=0} = E^2(t) = \sigma^2 - \mu^2$$

The standard deviation of the Weibull distribution is

$$\sigma^2 = E^2(t) = \int_0^\infty t^2 \frac{\beta t^{\beta-1}}{\alpha^\beta} e^{-\left(\frac{t}{\alpha}\right)^\beta} dt - \left[E(t)\right]^2$$

making a change of variable, one get

$$E^2(t) = \alpha^2\left[\int_0^\infty \left(u^{\frac{1}{\beta}}\right)^2 e^{-u}\, du\right] - \left[E(t)\right]^2$$

Therefore, the variance is

$$\boxed{\sigma^2 = E^2(t) = \alpha^2\left\{\Gamma\left(\frac{2}{\beta}+1\right) - \left[\Gamma\left(\frac{1}{\beta}+1\right)\right]^2\right\}}$$

2.29 Consider the following repair times

Given Data		
Time Range (hrs)		Freq
0	4	17
4	24	41
24	72	12
72	300	7
300	5400	9

Use the Chi-square goodness-of-fit test to determine the adequacy of a lognormal distribution with known parameters $\mu_t = 2.986$ & $\sigma_t = 1.837$

a) For 5% level of significance

b) For 1% level of significance

In order to test whether the Lognormal distribution is an adequate fit in this case, we use a Chi Squared Goodness-of-Fit test where

a) H_o: the r.v (T) follows a Lognormal distribution with parameters $\mu_t = 2.986$ & $\sigma_t = 1.837$; or

b) H_1: the r.v (T) does not follow a Lognormal distribution.

The Chi Squared Statistic is calculated for each data set given above using the following process. The complete solution is shown in the table below.

$$\Pr(0 < T < 4) = \Pr\left(\frac{\ln(0) - 2.986}{1.837} < Z < \frac{\ln(4) - 2.986}{1.837}\right) = \mathbf{0.1919}$$

$$Freq_{exp}(0 < T < 4) = \mathrm{N} \times \Pr(0 < T < 4) = 86 \times 0.1919 = \mathbf{16.5}$$

$$\text{Chisq Stat} = \frac{\left(Freq_{obs} - Freq_{exp}\right)^2}{Freq_{exp}} = \frac{(17 - 16.5)^2}{16.5} = \mathbf{0.01}$$

Given Data		Logarithmic of Given Data					
Time Range f(x)		Time Range f(x)		**Freq**	**Pr(x)**	**Ex(F)**	**Chisq Stat**
0	4	0	1.39	17	0.1919	16.5	0.01
4	24	1.39	3.18	41	0.3497	30	3.97
24	72	3.18	4.28	12	0.2172	19	2.39
72	300	4.28	5.70	7	0.1716	15	4.08
300	5400	5.70	8.59	9	0.0695	6	1.53
			Total	86		**W**	**11.98**
				mu_t	2.986	R(0.95)	9.49
				sig_t	1.837	R(0.99)	13.28

$R \geq \chi^2_{1-\alpha}(k - m - 1)$ where k = # of intervals & m = number of unknown parameters

$R \geq \chi^2_{1-\alpha}(5 - 0 - 1)$

$R \geq \chi^2_{1-\alpha}(4)$

a) Using a significance level of $\alpha = 5\%$

$R \geq \chi^2_{0.95}(4) = \mathbf{9.49}$

Given that W > R (ie: 11.98 > 9.49) we **reject** the null hypothesis at the 5% significance level.

b) Using a significance level of $\alpha = 1\%$

$R \geq \chi^2_{0.99}(4) = \mathbf{13.28}$

Given that W < R (ie: 11.98 < 13.28) we **do not reject** the null hypothesis at the 1% significance level.

2.30 Consider the following time to failure data with the ranked value of t_i. Test the hypothesis that the data fit a normal distribution. (Use the Kolmogorov test for this purpose.)

Event	Time to Failure
1	10.3
2	12.4
3	13.7
4	13.9
5	14.1
6	14.2
7	14.4
8	15.0
9	15.9
10	16.1

This problem was solved using the following assumptions:

a. H_o: the r.v (T) follows a Normal Distribution

H_1: the r.v (T) does not follow a Normal Distribution

b. A K-S goodness to fit test was used where the rejection region (R) was defined as $R \geq D_n(\alpha)$ with a 5% significance level (ie: $\alpha = 0.05$)

c. n = 10 (the number of failure times recorded)

Therefore $D_n(\alpha)$ was found to be **0.409** from tables of Critical value for the Kolmogonov-Smirnov statistic and the Maximum K-S statistic for the data set was calculated as shown in the table below to be **Max K-S = 0.2294**

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{10}(10.3 + 12.4 + 13.7 + \cdots + 16.1) = \mathbf{14}$$

$$S^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{9}[(10.3-14)^2 + (12.4-14)^2 + \cdots + (16.1-14)^2] = \mathbf{2.84}$$

$$S_n(t) = \begin{cases} 0 & -\infty < t < t_{(1)} \\ i/n & t_{(i)} \le t < t_{(i+1)} \\ 1 & t_{(n)} \le t < \infty \end{cases}$$

$$K-S = max[|F(t_i) - S_n(t_i)|;\ |F(t_i) - S_n(t_{i-1})|\,]$$

	Observed CDF				Fitted CDF	K-S Statistic	
Time to Failure	i	Sn(ti)	Sn(ti-1)	Phi(ti)	Fn(ti)	| Fn(ti) - Sn(ti) |	| Fn(ti) - Sn(ti-1) |
10.3	1	0.1	0	-2.1947	0.0141	0.0859	0.0141
12.4	2	0.2	0.1	-0.9491	0.1713	0.0287	0.0713
13.7	3	0.3	0.2	-0.1779	0.4294	0.1294	**0.2294**
13.9	4	0.4	0.3	-0.0593	0.4764	0.0764	0.1764
14.1	5	0.5	0.4	0.0593	0.5236	0.0236	0.1236
14.2	6	0.6	0.5	0.1186	0.5472	0.0528	0.0472
14.4	7	0.7	0.6	0.2373	0.5938	0.1062	0.0062
15.0	8	0.8	0.7	0.5932	0.7235	0.0765	0.0235
15.9	9	0.9	0.8	1.1270	0.8701	0.0299	0.0701
16.1	10	1	0.9	1.2456	0.8936	0.1064	0.0064
						Max K-S Value	**0.2294**

As the **Max (K-S) Value** < **R**, **(ie: 0.2294 < 0.409)**, we ***do not reject*** the H_o: Hypothesis that a Normal Distribution is an appropriate model for these data at the 95% confidence limit.

2.31 If a device has a cycle-to-failure, t, which follows an exponential distribution with λ = 0.003 failures/cycle.

a) Determine the mean-cycle-to-failure for this device.

$$MCTF = \frac{1}{\lambda} = \frac{1}{0.003} = \mathbf{333.3} \text{ cycles to failure}$$

b) If the device is used in a space experiment and is known to have survived for 300 cycles, what is the probability that it will fail sometimes after 1000 cycles?

$$\Pr(T > 1000\,|T > 300) = \frac{\Pr(T > 300\,|T > 1000\,) \times \Pr(T > 1000\,)}{\Pr(T > 300\,)}$$

$$\Pr(T > 1000\,|T > 300) = \frac{\Pr(T > 1000\,)}{\Pr(T > 300\,)} = \frac{e^{-1000\times 0.003}}{e^{-300\times 0.003}} = \mathbf{0.122}$$

2.32 A highly stressed machine part's time to failure is normally distributed with 90% of the failures occurring between 200 and 270 hrs of use (ie: 5% below 200 hrs and 5% above 270 hrs)

a) Find the mean and standard deviation of time to failure.

$$\bar{x} = \frac{1}{2}(270 + 200) = \mathbf{235}\ \boldsymbol{hrs}$$

$$Pr(T < 200) = 0.05$$

$$Pr\left(Z < \frac{T - \bar{x}}{\sigma}\right) = Pr\left(Z < \frac{200 - 235}{\sigma}\right)$$

$$\Phi_Z^{-1}(0.05) = Z_{0.05} = -1.645$$

$$-1.645 = \frac{200 - 235}{\sigma} \qquad => \sigma = \frac{200 - 235}{-1.645} = \mathbf{21.28}$$

b) Determine the part's life if no more than 1% replacement probability can be tolerated.

$$Pr(T < t) = 0.01$$

$$\Phi_Z^{-1}(0.01) = Z_{0.01} = -2.326$$

$$-2.326 = \frac{T_{0.01} - 235}{21.28} \qquad => T_{0.01} = -2.326 \times 21.28 + 235 = \mathbf{185.5}\ \boldsymbol{hrs}$$

c) Compute the probability that the part functions for an additional 50 hrs, if the part has been operating for 200 hrs without failure.

$$\Pr(T > 250\ |T > 200) = \frac{\Pr(T > 200\ |T > 250\ ) \times \Pr(T > 250\ )}{\Pr(T > 200\ )}$$

$$\Pr(T > 250\ |T > 200) = \frac{\Pr(T > 250\ )}{\Pr(T > 200\ )} = \frac{1 - \Pr(T < 250\ )}{1 - \Pr(T < 200\ )}$$

$$\frac{1 - \Pr(T < 250\ )}{1 - \Pr(T < 200\ )} = \frac{1 - \Pr\left(Z < \frac{250 - 235}{21.28}\right)}{1 - \Pr\left(Z < \frac{200 - 235}{21.28}\right)} = \mathbf{0.2531}$$

2.33 Fifty compressors were stress tested for 200 days and the following failure times grouped were regarded. Perform a χ^2 goodness-of-fit test at the 5% significance level to determine the adequacy of an exponential distribution

Interval (days)	**Number of Failures**
0 < *T* < 50	9
50 < *T* < 75	11
75 < *T* < 100	9
100 < *T* < 150	12
150 < *T* < 200	9

Firstly, the mean and standard deviation of the sample needs to be generated:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{50} \times (9 \times 25 + 11 \times 62.5 + 9 \times 87.5 + 12 \times 125 + 9 \times 175) = 95.5$$

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{1}{49} \times [9 \times (25 - 95.5)^2 + 11 \times (62.5 - 95.5)^2 \ldots]$$

$$\therefore \sigma = 50.429$$

Then find Pr(T) for each of the intervals (shown in table below) using the normal distribution table.

Then find e_i as follows:

$$e_i = n \times p_i$$

The Chi-Squared statistic is then:

$$W = \chi^2 = \sum_{i=1}^{k} \frac{(o_i - e_i)^2}{e_i}$$

This information is shown in the following table:

Interval					
Lower	Upper	# Failures	Pr (T)	e_i	Chi^2
0	50	9	0.1543	1.3890	41.7051
50	75	11	0.1587	1.7460	49.0491
75	100	9	0.1934	1.7403	30.2837
100	150	12	0.3245	3.8944	16.8701
150	200	9	0.1208	1.0871	57.5981
	Total	50		9.8568	195.5061

To find R from the chi squared table, we need the number of intervals (k) and the number of unknowns (m). In this problem, k = 5, and m = 2.

$$\therefore R = \chi^2_{1-\alpha}(k - m - 1) = \chi^2_{0.95}(2) = 5.99$$

In this case, clearly W (195.5061) > R (5.99). Therefore we reject the normal distribution as a good fit for this data.

2.34 Perform a χ^2 goodness-of-fit test at the 10% significance level on the following grouped data to fit a uniform PDF of time to failure between two limits of a = 0 and b = 1000 hrs.

Interval (hrs)	Observed Failures
0 -200	16
200 - 400	18
400 - 600	25
600 - 800	23
800 - 1000	24

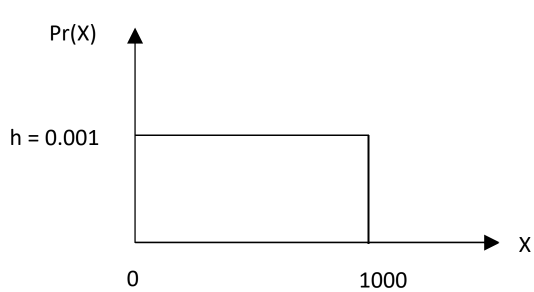
In order to test whether the uniform distribution is an adequate fit in this case, we use a Chi Squared Goodness-of-Fit test where

a) H_o: the r.v (X) follows a Uniform Distribution from 0 to 1000; or

b) H_1: the r.v (X) does not follow a Uniform Distribution.

The Chi Squared Statistic is calculated for each data set given above using the following process. The complete solution is shown in the table below.

$$\Pr(X) = \frac{1}{1000} = 0.001$$



$$Freq_{exp}(0 < X < 200) = \mathrm{N} \times \Pr(0 < X < 200) = 106 \times 0.001 \times (200 - 0) = \mathbf{21.2}$$

$$\text{Chisq Stat} = \frac{\left(Freq_{obs} - Freq_{exp}\right)^2}{Freq_{exp}} = \frac{(16 - 21.2)^2}{21.2} = \mathbf{1.28}$$

Interval		Freq	Pr(x)	Ex(F)	Chisq Stat
0	200	16	0.001	21.200	1.28
200	400	18	0.001	21.200	0.48
400	600	25	0.001	21.200	0.68
600	800	23	0.001	21.200	0.15
800	1000	24	0.001	21.200	0.37
Total		106		**W**	**2.96**
				R	**7.78**

$R \geq \chi^2_{1-\alpha}(k - m - 1)$ where k = # of intervals & m = number of unknown parameters

Using a significance level of $\alpha = 10\%$

$R \geq \chi^2_{0.9}(5 - 0 - 1)$

$R \geq \chi^2_{0.9}(4) = \mathbf{7.78}$

Given that W < R (ie: 2.96 < 7.78) we **do not reject** the null hypothesis at the 10% significance level.

2.35 A turbine in a generator can cause a great deal of damage if it fails critically. The manager of the plant can choose to either replace the old turbine with a new one, or to leave it in place.

The state of the turbine is categorized as either 'good,' 'acceptable' or 'poor.' The probability of critical failure per quarter depends on the state of the turbine:

$Pr(failure \mid good) = 0.0001$

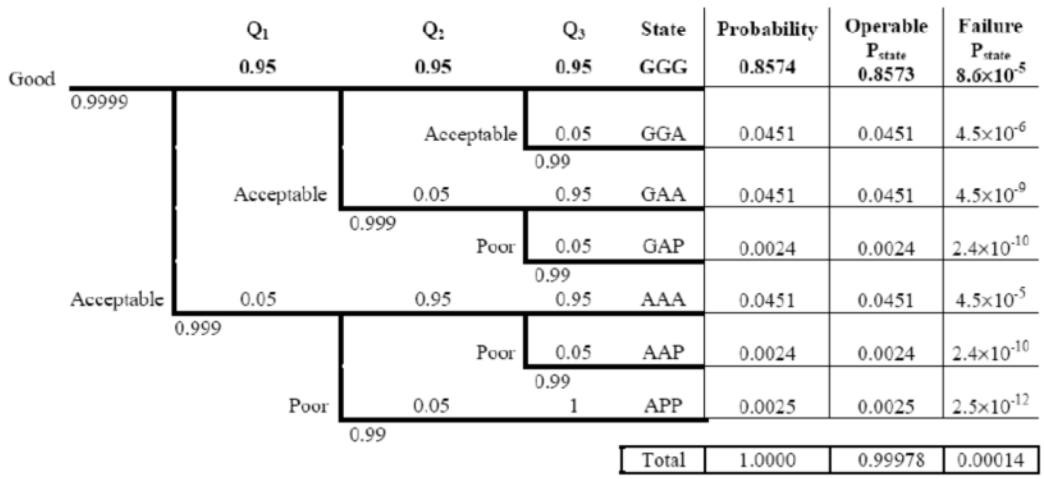
$Pr(failure \mid acceptable) = 0.001$

$Pr(failure \mid poor) = 0.01$

The technical department has made a model for the degradation of the turbine. In this model it is assumed that, if the state is 'good' or 'acceptable,' then the at the end of the quarter it stays the same with probability 0.95 or degrades with probability 0.05 (ie: 'good' becomes 'acceptable' and 'acceptable' becomes 'poor'). If the state is 'poor' then it stays 'poor.'

Determine the probability that a turbine which is 'good' becomes 'good,' 'acceptable' and 'poor' in the next three quarters and does not fail.

State	Failure	Work
Pr(Failure|Good)	0.0001	0.9999
Pr(Failure|Acceptable)	0.001	0.999
Pr(Failure|Poor)	0.01	0.99
End of Quarter Degradation Model Multiplier	0.05	0.95



Answer

State	Probability Does Not Fail
Good	0.8573
Acceptable	0.1353
Poor	0.0072

2.36 Assume there are, on the average, 3 random intermittent interruptions in a network service per week. What is the probability of at least one interruption in a two-week period?

Solution:

This problem can be solved using the Poisson distribution

$$\Pr(X = x) = \frac{\mu^x e^{-\mu}}{x!}$$

And

$\mu = \lambda t$

Given:

$\lambda = 3$

$t = 2$

$\mu = 6$

Find:

$\Pr(X \geq 1)$

Therefore:

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) = 1 - \frac{6^0 e^{-6}}{0!} = 1 - e^{-6} = 1 - 0.002479 = 0.9975$$

2.37 The life of a light bulb is log-normally distributed with mean of 12,000 hr. and standard deviation of 2000 hr. Compute the probability that:

a. A light bulb will last 12,000 hrs

b. A light that has already worked for 10,000 hrs, will last 12,000 hrs.

Solution:

a. Given:

$\mu_y = 12{,}000$

$\sigma_y = 2{,}000$

$$\mu_t = \ln\left[\frac{\mu_y}{\sqrt{1+\left(\frac{\sigma_y^2}{\mu_y^2}\right)}}\right] = \ln\left[\frac{12000}{\sqrt{1+\left(\frac{2000^2}{12000^2}\right)}}\right] = 9.3790$$

$$\sigma_t = \sqrt{\ln\left(1+\frac{\sigma_y^2}{\mu_y^2}\right)} = \sqrt{\ln\left(1+\frac{2000^2}{12000^2}\right)} = 0.1655$$

Find:

$\Pr(T > 12{,}000)$

$$z = \frac{\ln(T) - \mu_t}{\sigma_t} = \frac{\ln(12000) - 9.3790}{0.1655} = 0.082763$$

From the tables this equates to:

$\Pr(T < 12{,}000) \approx 0.53188$

$\therefore \Pr(T > 12{,}000) = 1 - 0.53188 = 0.46812$

b. Find:

$$\Pr(T = 12{,}000|T > 10{,}000) = \frac{\Pr(T = 12{,}000) \times \Pr(\mathrm{T} > 10{,}000|\mathrm{T} = 12{,}000)}{\Pr(T > 10000)}$$

Note, logically:

$\Pr(T > 10{,}000|T = 12{,}000) = 1$

Find:

$\Pr(T > 10{,}000)$

$$z = \frac{\ln(T) - \mu_t}{\sigma_t} = \frac{\ln(10000) - 9.3790}{0.1655} = -1.0187$$

From the tables this equates to:

$\Pr(T < 10{,}000) \approx 0.15386$

$\therefore \Pr(T > 10{,}000) = 1 - 0.15386 = 0.84614$

$$\therefore \Pr(T = 12{,}000|T > 10{,}000) = \frac{0.46812}{0.84614} = 0.55324$$