1.1 Both have dimensions 1/E

1.2 This question applies only to the first three lines, since lines four and five are merely definitions. To verify the first line, we should first use line four to convert GeV to Joules, and then use the value of $\hbar$ given in Table A.2. This gives

$$1\,\mathrm{m} = 5.068 \times 10^{15} \times 6.242 * 10^9 \times 1.055 \times 10^{-34} \times 2.998 \times 10^8 = 1.0006\,\mathrm{m}$$

which is in adequate agreement as we work only to four significant figures. The proof for the second and third lines is similar.

1.3 Six quarks, six leptons, four gauge bosons and the Higgs making seventeen in all.

1.4 Charge conservation forbids electron decay because lighter particles have zero charge.

1.5 According to Table 5 and its caption, the nucleon mass is $m = 939 \times 1.78 \times 10^{-30} = 1.67 \times 10^{-27}\,\mathrm{kg}$. Number density of nucleons is $n = M/mV$ where M is the Earth mass and V is its volume. The Earth's radius is 6400 km (not 6400 m as stated in the question). This gives $n = 4.7 \times 10^{30}\,\mathrm{m}^{-3}$. Going to natural units using the first line of Table A.3 gives $n = 3.5 \times 10^{-17}\,\mathrm{GeV}^3$. As seen around Eq. (8.17), working in natural units,

$$\Gamma = \sigma n \sim G_\mathrm{F}^2 E^2 n \sim 10^{-27} E^2\,\mathrm{GeV}^{-1}.$$

In MKS units, time for passage of a neutrino through the earth is $t = 6.4 \times 10^6 m/c\,\mathrm{s}$. Using the second line of Table A.3, this is in natural units $t \simeq 10^{22}\,\mathrm{GeV}^{-1}$. We are asked to compare this with

$$\Gamma^{-1} = 10^{27} E^{-2}\,\mathrm{GeV} = 10^{45} (\,\mathrm{eV}/E)^2\,\mathrm{GeV}^{-1}.$$

The two would be equal if $E \sim 10^{12}\,\mathrm{eV}$. (The above calculation is an exercise in the use of the expression $\sigma \sim G_\mathrm{F}^2 E^2$ given before Eq. (8.17), that being the only expression given in this book. The expression is actually valid only for $E \ll G_\mathrm{F}^{-1/2} \sim 10^{11}\,\mathrm{eV}$. For bigger E, σ is smaller being $\sigma \sim G_\mathrm{F}$. It follows that a typical neutrino actually passes straight through the earth no matter how high is its energy.)

2.1 We can choose the origins of x and t so that $V = x/t$. Then Eq. (1.3) gives

$$V' = \frac{x'}{t'} = \frac{x + vt}{t + vx} = \frac{V + v}{1 + vV}.$$

The non-relativistic expression $V' = V + v$ is recovered when $vV \ll 1$. Using dimensional analysis to restore c, this criterion becomes $vV \ll c^2$.

2.2 The generic Lorentz transformation for time is

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}.$$

Taking the unprimed frame to be the astronaut's, we can set the position x of the astronaut to zero. Taking t' to be the earth time, the amount by which the astronaut has aged is

$$t = 20\sqrt{1 - v^2/c^2} \simeq 20 \times \left(1 - \frac{1}{2}\frac{v^2}{c^2}\right) = 19.9999995\,\text{yr}.$$

This takes the turnaround of the astronaut to be instantaneous and ignores the effect of the acceleration during the turnaround.

2.3 Since $g_{\mu\nu}$ is symmetric and $g^{\mu\nu}$ is the inverse of $g_{\mu\nu}$ we have

$$g_{\mu\nu}g^{\mu\nu} = g_{\mu\nu}(g^{-1})^{\mu\nu} = g_{\mu\nu}(g^{-1})^{\nu\mu} = \delta^{\mu}_{\mu} = 4.$$

2.4 Writing ds^2 first in generic coordinates and then in locally orthonormal coordinates we have

$$\begin{aligned} ds^2 &= g'_{\alpha\beta}dx'^{\alpha}dx'^{\beta} \\ &= g_{\mu\nu}dx^{\mu}dx^{\nu} \\ &= g_{\mu\nu}\frac{\partial x^{\mu}}{\partial x'^{\alpha}}\frac{\partial x^{\nu}}{\partial x'^{\beta}}dx'^{\alpha}dx'^{\beta} \\ &= \eta_{\mu\nu}\frac{\partial x^{\mu}}{\partial x'^{\alpha}}\frac{\partial x^{\nu}}{\partial x'^{\beta}}dx'^{\alpha}dx'^{\beta}. \end{aligned}$$

Equating the first and last expressions gives the desired result.

2.5 By definition $A'_{\mu} = g'_{\mu\nu}A'^{\nu}$. Using Eq. (2.6) this gives

$$\begin{aligned} A'_{\mu} &= \frac{\partial x^{\alpha}}{\partial x'^{\mu}}g_{\alpha\beta}\frac{\partial x^{\beta}}{\partial x'^{\nu}}A'^{\nu} \\ &= \frac{\partial x^{\alpha}}{\partial x'^{\mu}}g_{\alpha\beta}A^{\beta} \\ &= \frac{\partial x^{\alpha}}{\partial x'^{\mu}}A_{\alpha}. \end{aligned}$$

2.6 Using Eq. (2.7), Eq. (2.12) becomes

$$\begin{aligned} D_{\nu}A^{\mu} &= \frac{\partial x^{\mu}}{\partial x'^{\beta}}\partial_{\nu}\left(\frac{\partial x'^{\beta}}{\partial x^{\alpha}}A^{\alpha}\right) \\ &= \frac{\partial x^{\mu}}{\partial x'^{\beta}}\frac{\partial x'^{\beta}}{\partial x^{\alpha}}\partial_{\nu}A^{\alpha} + \frac{\partial x^{\mu}}{\partial x'^{\beta}}\frac{\partial^2 x'^{\beta}}{\partial x^{\nu}\partial x^{\alpha}}A^{\alpha} \\ &= \delta^{\mu}_{\alpha}\partial_{\nu}A^{\alpha} + \Gamma^{\mu}_{\nu\alpha}A^{\alpha} \\ &= \partial_{\nu}A^{\mu} + \Gamma^{\mu}_{\nu\alpha}A^{\alpha}. \end{aligned}$$

2.7 Only the space-space components of $g_{\mu\nu}$ are affected by the transformation. Denote them by g_{ij}. Cartesian coordinates are given in terms of polar coordinates by $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$. In Cartesian coordinates g_{ij} is the unit matrix. Therefore

$$\begin{aligned} g_{rr} &= \left(\frac{\partial x}{\partial r}\right)^2 + \left(\frac{\partial y}{\partial r}\right)^2 + \left(\frac{\partial z}{\partial r}\right)^2 \\ &= \sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta = 1 \\ g_{r\theta} &= \frac{\partial x}{\partial r}\frac{\partial x}{\partial \theta} + \frac{\partial y}{\partial r}\frac{\partial y}{\partial \theta} + \frac{\partial z}{\partial r}\frac{\partial z}{\partial \theta} \\ &= r\sin\theta\cos\phi\cos\theta\cos\phi + r\sin\theta\sin\phi\cos\theta\sin\phi - r\cos\theta\sin\theta = 0. \end{aligned}$$

Evaluating the other components in the same way, one finds that the other non-zero components are $g_{\theta\theta} = r^2$ and $g_{\phi\phi} = r^2\sin^2\theta$. The corresponding spatial line element is

$$d\ell^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

which is usually derived geometrically.

2.8 Using Eq. (2.19), the nonzero components are $\Gamma^\theta_{r\theta} = 1/r$, $\Gamma^\phi_{\phi r} = 2/r$ and $\Gamma^\phi_{\phi\theta} = 2\cot\theta$ (and the equal quantities that are obtained by swopping the lower indices). The only non-zero components of the curvature tensor have all space indices. Also, from the Γ's one can see that all $R^r{}_{ijk}$ vanish, and so do all $R^\theta{}_{ijk}$ if any lower index is ϕ. Of the remaining components, some have all terms vanishing, and the rest have a cancellation as for instance in

$$R^\theta{}_{\theta\theta r} = \left(\Gamma^\theta_{\theta r}\right)^2 - \left(\Gamma^\theta_{\theta r}\right)^2 = 0$$

.

2.9 On the right hand side of Eq. (2.15), it should be A_μ not A^μ. To answer the question we need the following relation which is derived in the same way as Eq. (2.17):

$$D_\mu(A_\alpha B^\beta) = \partial_\mu(A_\alpha B^\beta) + \Gamma^\nu_{\mu\alpha}A_\nu B^\beta + \Gamma^\beta_{\mu\nu}A_\alpha B^\nu.$$

Replacing B^β by A^μ, A_α by D_β and D_μ by D_α, this gives

$$\begin{aligned} D_\alpha D_\beta A^\mu &= \partial_\alpha\partial_\beta A^\mu + (\partial_\alpha\Gamma^\mu_{\nu\beta})A^\nu + \Gamma^\mu_{\nu\beta}\partial_\alpha A^\nu \\ &+ \Gamma^\nu_{\alpha\beta}\left(\partial_\nu A^\mu + \Gamma^\mu_{\gamma\nu}A^\gamma\right) + \Gamma^\mu_{\alpha\nu}\left(\partial_\beta A^\nu + \Gamma^\nu_{\gamma\beta}A^\gamma\right). \end{aligned}$$

This leads to Eq. (2,41), after noting some cancellations.

2.10 Eq. (2.40) corresponds to Eq. (2.38). In the latter, the symmetric matric $g'_{\mu\nu}$ has 10 independent components and the index τ takes on 4 values, which means that there are $10\times 4 = 40$ independent equations.

2.11 A suspended rod will point to the earth if its suspension is sufficiently free of friction.

3.1 The trace of a matrix A_{ij} is $A_{11} + A_{22} + A_{33}$. In Eq. (3.1), the trace of T_{ij} is $3P$ which means that T_{ij} specifies P. Also, $\Sigma_{ij} = T_{ij} - P\delta_{ij}$ which means that T_{ij} also specifies Σ_{ij}.

3.2 To first order in v, the Lorentz boost Eq. (1.3) becomes

$$x' = x - vt, \qquad t' = t - vx.$$