**Chapter 2**

2.1 What are the dimensions of the correlation function and the spectral density  if the random function  is

(A) Displacement (in.)

(B) Acceleration ()

(C) Force (lb.)

(D) Stress (psi)

**Solution**

|  |  |  |  |
| --- | --- | --- | --- |
| Part | Function |  |  |
| A | Displacement (in.) |  |  |
| B | Acceleration () |  |  |
| C | Force (lb.) |  |  |
| D | Stress (psi) |  |  |

2.2. What are the dimensions of the probability function and the probability density function  if the random function  is

(A) Displacement (in.)

(B) Acceleration ()

(C) Force (lb.)

(D) Stress (psi)

**Solution**

|  |  |  |  |
| --- | --- | --- | --- |
| Part | Function |  |  |
| A | Displacement (in.) | Dimensionless |  |
| B | Acceleration () | Dimensionless |  |
| C | Force (lb.) | Dimensionless |  |
| D | Stress (psi) | Dimensionless |  |

2.3. Define a stationary random process.

**Solution**

A stationary random process is defined by the correlations function of a random variable being constant regardless of the time at which it is evaluated, so long as the period over which each evaluation is taken is unchanged in duration.

2.4. Consider a stationary ergodic process ; express , , and as the time averages.

**Solution**

For a stationary ergodic process  we have in terms of time averages







2.5. Prove the relations between the spectral density  and the correlation function :





**Solution**





Since 



Take the limit of both sides as 







Thus,  and  



2.6. Assuming that the input-output relation is. Show that

 and 

**Solution**

Given we know that the correlation function is



where









 so 

therefore











therefore



2.7. Derive the expression for the average number of crossings of the value assuming that the joint distribution function  of and is

(A) two-dimensional normal

(B) arbitrary

**Solution**

(a) The joint distribution function  of and is given by



where 

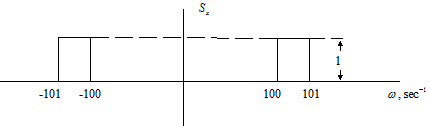
The probability that  is . The duration of the passage from is . Thus the passages form  at a velocity per unit time is



The passages from at any rate is equal to the sum of the passages at each value of , or



2.8. The spectral density of the loading (input) is given in the form shown in Figure 2.29



***Figure 2.29Loading input***

Find the spectral density’s of the outputs (displacement, bending moment, etc.) of the systems described in problem 1(a) of Chapter 1.

**Solution**

From Problem 3(a) of Chapter 1 we know that

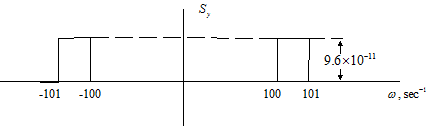
 or 



At 



 so 



Also, the mean square value of is



2.9. A random function has its spectral density as in Problem 2.8. Find the mean square value of . Find the average number of crossings of the values , , and .

**Solution**

From Problem 8 we have



For 



For 

 where 



For 

