this case, a variety cannot be *randomly applied* to a plant, but randomization can be used to assign plants to fertilizers, and to locations within a garden bed. Continued experimentation in a second year may be desirable, but the potential for systematic effects due to differences between growing seasons (e.g. climate and pest populations) might suggest treating the two years as *blocks*.

CHAPTER 2

1.

(a) $\mathbf{H}_1 = \begin{pmatrix} \frac{1}{3}\mathbf{J}_{3\times 3} & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 3} & \frac{1}{3}\mathbf{J}_{3\times 3} \end{pmatrix}$

(b) $\mathbf{X}_{2|1} = \mathbf{X}_2 - \frac{1}{3}\mathbf{J}_{6\times 3}$

(c) $\mathbf{c}'\boldsymbol{\theta}_2$ is estimable if $\mathbf{c}'$ can be expressed as a linear combination of:

$$\begin{array}{c} (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}) \\ (-\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}) \\ (-\frac{1}{3}, -\frac{1}{3}, \frac{2}{3}) \end{array}$$

2.

(a) $\mathcal{I} = 2(\mathbf{I}_{3\times 3} - \frac{1}{3}\mathbf{J}_{3\times 3})$

(b) $\mathbf{H}_1 = \frac{1}{3r}\begin{pmatrix} \mathbf{J} & \mathbf{0} & \mathbf{J} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{J} & \mathbf{0} & \dots & \mathbf{J} \\ \mathbf{J} & \mathbf{0} & \mathbf{J} & \dots & \mathbf{0} \\ \dots & \dots & \dots & \dots & \dots \\ \mathbf{0} & \mathbf{J} & \mathbf{0} & \dots & \mathbf{J} \end{pmatrix}$, where each indicated submatrix is of size 3×3. $\mathbf{X}_2 = \begin{pmatrix} \mathbf{I} \\ \mathbf{I} \\ \dots \\ \mathbf{I} \end{pmatrix}$, where each submatrix is of size 3×3. $\mathbf{X}_{2|1} = \mathbf{X}_2 - \frac{1}{3}\mathbf{J}_{6r\times 3}$. $\mathcal{I} = 2r(\mathbf{I}_{3\times 3} - \frac{1}{3}\mathbf{J}_{3\times 3})$.

(c) Let matrices associated with the replicated design be designated by underlines, and those associated with the unreplicated design be given in the usual notation. Then:

$$\underline{\mathbf{H}}_1 = \frac{1}{r}\begin{pmatrix} \mathbf{H}_1 & \mathbf{H}_1 & \dots & \mathbf{H}_1 \\ \mathbf{H}_1 & \mathbf{H}_1 & \dots & \mathbf{H}_1 \\ \dots & \dots & \dots & \dots \\ \mathbf{H}_1 & \mathbf{H}_1 & \dots & \mathbf{H}_1 \end{pmatrix}, \quad \underline{\mathbf{X}}_{2|1} = \underline{\mathbf{X}}_2 - \begin{pmatrix} \mathbf{H}_1\mathbf{X}_2 \\ \mathbf{H}_1\mathbf{X}_2 \\ \dots \\ \mathbf{H}_1\mathbf{X}_2 \end{pmatrix}, \quad \underline{\mathcal{I}} = r\mathcal{I}$$

3.

(a) $\bar{y} = \frac{1}{N}\mathbf{1}'\mathbf{y}$, $s^2 = \frac{1}{N-1}\mathbf{y}'(\mathbf{I} - \frac{1}{N}\mathbf{J})\mathbf{y}$, $(\frac{1}{N}\mathbf{1})'(\sigma^2\mathbf{I})(\frac{1}{N-1}(\mathbf{I} - \frac{1}{N}\mathbf{J})) = \mathbf{0}$.

(b) $(\frac{1}{N}\mathbf{1})'\boldsymbol{\Sigma}(\frac{1}{N-1}(\mathbf{I} - \frac{1}{N}\mathbf{J})) = \frac{1}{N(N-1)}(\mathbf{1}'\boldsymbol{\Sigma} - \frac{1}{N}\mathbf{1}'\boldsymbol{\Sigma}\mathbf{J}) = \frac{\sigma^2}{N(N-1)}((1+(N-1)\rho)\mathbf{1}' + \frac{N}{N}(1-(N-1)\rho)\mathbf{1}') = \mathbf{0}'$, yes.

4.

(a) $\boldsymbol{\theta}_2' = (1, 0, 0)$

(b) $\boldsymbol{\theta}_2' = (0, 0, 1)$

(c) σ^2, rank($\mathbf{X}$), and N

5. $(\mathcal{I}^A)^- = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{8} & 0 \\ 0 & 0 & \frac{1}{4} \end{pmatrix}, \quad (\mathcal{I}^B)^{-1} = \begin{pmatrix} \frac{1}{6} & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{6} \end{pmatrix}.$

(a) $\mathbf{c}' = (1, 1, 0)$, $\mathbf{c}'(\mathcal{I}^A)^-\mathbf{c} = \frac{1}{8}$, $\mathbf{c}'(\mathcal{I}^B)^{-1}\mathbf{c} = \frac{1}{3}$

(b) $\mathbf{c}' = (\frac{1}{2}, \frac{1}{2}, -1)$, $\mathbf{c}'(\mathcal{I}^A)^-\mathbf{c} = \frac{9}{32}$, $\mathbf{c}'(\mathcal{I}^B)^{-1}\mathbf{c} = \frac{1}{4}$

6.

(a) $\mathbf{y} = \mathbf{X}_1\boldsymbol{\alpha} + \mathbf{X}_2\boldsymbol{\beta} + \boldsymbol{\epsilon}$

(b) $\mathbf{y} = \begin{pmatrix} 1 & 0 & ... & 0 \\ 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ 0 & 1 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & 1 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ ... \\ \alpha_p \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ ... & \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \boldsymbol{\epsilon}$

(c) $\mathbf{H}_1 = \frac{1}{2} \begin{pmatrix} \mathbf{J}_{2\times2} & \mathbf{0}_{2\times2} & ... & \mathbf{0}_{2\times2} \\ \mathbf{0}_{2\times2} & \mathbf{J}_{2\times2} & ... & \mathbf{0}_{2\times2} \\ ... & ... & ... & ... \\ \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} & ... & \mathbf{J}_{2\times2} \end{pmatrix}, \mathbf{X}_{2|1} = \frac{1}{2} \begin{pmatrix} +1 & -1 \\ -1 & +1 \\ ... & \\ -1 & +1 \end{pmatrix}, \mathcal{I} = p(\mathbf{I}_{2\times2} - \frac{1}{2}\mathbf{J}_{2\times2})$

7.

(a) $Var(\widehat{\mathbf{c}'\boldsymbol{\theta}_2})/\sigma^2 = \mathbf{c}'\mathcal{I}^-\mathbf{c} = \mathbf{c}'(\sum_i \lambda_i^{-1}\mathbf{e}_i\mathbf{e}_i')\mathbf{c} = \sum_i \lambda_i^{-1}(\mathbf{c}'\mathbf{e}_i)^2$, so every λ_i^{-1} must be at least as small for Design A as for Design B, or every λ_i must be at least as large for Design A as for Design B.

(b) $\lambda\sigma^2 = \boldsymbol{\theta}_2'\mathcal{I}\boldsymbol{\theta}_2 = \boldsymbol{\theta}_2'(\sum_i \lambda_i\mathbf{e}_i\mathbf{e}_i')\boldsymbol{\theta}_2 = \sum_i \lambda_i(\boldsymbol{\theta}_2'\mathbf{e}_I)^2$, so every λ_i must be larger for Design A than for Design B.

8.

(a) $\mathbf{y} = \mathbf{X}_1\boldsymbol{\alpha} + \mathbf{X}_2\boldsymbol{\beta} + \boldsymbol{\epsilon}$, with

$$\mathbf{X}_1 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}, \mathbf{X}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

(b) $\mathbf{H}_1 = \begin{pmatrix} \frac{1}{4}\mathbf{J}_{4\times 4} & \mathbf{0}_{4\times 4} \\ \mathbf{0}_{4\times 4} & \frac{1}{4}\mathbf{J}_{4\times 4} \end{pmatrix}$.

(c) $\mathbf{X}_{2|1} = \frac{1}{2}\begin{pmatrix} +1 & -1 \\ -1 & +1 \\ \dots & \\ -1 & +1 \end{pmatrix}$, $\mathcal{I} = 4(\mathbf{I} - \frac{1}{2}\mathbf{J})$.

9.

(a) $\mathbf{H}_1 = \begin{pmatrix} \frac{1}{4}\mathbf{J}_{4\times 4} & \mathbf{0}_{4\times 3} \\ \mathbf{0}_{3\times 4} & \frac{1}{3}\mathbf{J}_{3\times 3} \end{pmatrix}$.

(b) $\mathbf{X}_{2|1} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{3} & -\frac{1}{3} \\ -\frac{2}{3} & \frac{2}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{pmatrix}$, $\mathcal{I} = \frac{5}{3}\begin{pmatrix} +1 & -1 \\ -1 & +1 \end{pmatrix}$.

(c) $\sigma^2 t^2_{.95}(4)(1,-1)\mathcal{I}^- \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (where one generalized inverse for $\mathcal{I}$ is $\begin{pmatrix} 0 & 0 \\ 0 & \frac{3}{5} \end{pmatrix}$), $= 2.7269\sigma^2$.

10. For this experiment,

$$\mathbf{X}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{pmatrix}, \mathbf{X}_2 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

This leads to $\mathbf{H}_1 = \begin{pmatrix} 1 & \mathbf{0}_{1\times 2} \\ \mathbf{0}_{2\times 1} & \frac{1}{2}\mathbf{J}_{2\times 2} \end{pmatrix}$ and $\mathbf{X}_{2|1} = \frac{1}{2}\begin{pmatrix} 0 & 0 \\ +1 & -1 \\ -1 & +1 \end{pmatrix}$. But the right side of the reduced normal equations is:

$$\mathbf{X}'_{2|1}\mathbf{y} = \begin{pmatrix} y_2 - y_3 \\ y_3 - y_2 \end{pmatrix},$$

i.e. y_1 is not involved. Since the "china cup" parameter is part of the expectation of only y_1, it cannot be eliminated in the expectation of any linear contrast involving this data value.

CHAPTER 3

1.

(a) Under this model, the unique rows of the model matrix and the parameter vector may be written as: