and condition (P1.1) becomes

$$(\lambda - G)\sin(2a) + 2\sqrt{2}\,\Delta_3\cos(2a) = 0,$$

$$\tan(2a) = \frac{2\Delta_3\sqrt{2}}{G-\lambda},$$

$$b_+ = \sin(a) = \frac{1}{\sqrt{2}}\left(1 + \frac{G-\lambda}{\sqrt{(G-\lambda)^2 + 8\Delta_3^2}}\right)^{1/2},$$

$$b_- = \cos(a) = \frac{1}{\sqrt{2}}\left(1 - \frac{G-\lambda}{(G-\lambda)^2 + 8\Delta_3^2}\right)^{1/2}.$$

1.4 Coefficients $\alpha, \beta, b_\pm$ are defined in Eqs. (1.36) and (1.40):

$$SS^+ = \begin{bmatrix} \alpha^* & 0 & 0 & \alpha & 0 & 0 \\ 0 & \beta b_+ & \beta^* b_- & 0 & \beta^* b_+ & \beta b_- \\ 0 & -\beta b_- & \beta^* b_+ & 0 & -\beta^* b_- & \beta b_+ \\ \alpha^* & 0 & 0 & -\alpha & 0 & 0 \\ 0 & \beta b_+ & -\beta^* b_- & 0 & -\beta^* b_+ & \beta b_- \\ 0 & -\beta b_- & -\beta^* b_+ & 0 & \beta^* b_- & \beta b_+ \end{bmatrix} \times \begin{bmatrix} \alpha & 0 & 0 & \alpha & 0 & 0 \\ 0 & \beta^* b_+ & -\beta^* b_- & 0 & \beta^* b_+ & \beta^* b_- \\ 0 & \beta b_- & \beta b_+ & 0 & -\beta b_- & -\beta b_+ \\ \alpha^* & 0 & 0 & -\alpha^* & 0 & 0 \\ 0 & \beta b_+ & -\beta^* b_- & 0 & -\beta b_+ & \beta b_- \\ 0 & \beta^* b_- & \beta^* b_+ & 0 & \beta^* b_- & \beta^* b_+ \end{bmatrix} =$$

$$\begin{bmatrix} 2|\alpha|^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2|b|2(b_+^2 + b_-^2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 2|b|2(b_+^2 + b_-^2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 2|\alpha|^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2|b|2(b_+^2 + b_-^2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2|b|2(b_+^2 + b_-^2) \end{bmatrix}$$

$$= \mathrm{diag}(1,1,1,1,1,1)$$

Chapter 2

2.1 $$|\theta + 2\pi, \varphi\rangle = \begin{pmatrix} \cos(\theta/2 + \pi) \\ \sin(\theta/2 + \pi)\exp(-i\varphi) \end{pmatrix} = -\begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)\exp(-i\varphi) \end{pmatrix}.$$

After rotation through 2π the spinor changes its sign, in other words, acquires Berry phase π.

2.2 Starting with Hamiltonian (2.14) we add Zeeman spin slitting h into a diagonal part of the Hamiltonian:

$$H = h\sigma_z + \alpha_R(\sigma_x k_y - k_x\sigma_y), \tag{P2.1}$$

Eigenvalues and eigenvectors of 2 × 2 matrix (P2.1) are expressed as

$$E_{1,2} = \pm\sqrt{h^2 + a_R^2 k^2}, \tan(\varphi) = \frac{k_y}{k_x},$$

$$e_+ = \begin{pmatrix} -\dfrac{ie^{-i\varphi}(-h + \sqrt{h^2 + \alpha_R^2\ k^2})}{\alpha_R k} \\ 1 \end{pmatrix},$$

$$e_- = \begin{pmatrix} \dfrac{ie^{-i\varphi}(h + \sqrt{h^2 + \alpha_R^2 k^2})}{\alpha_R k} \\ 1 \end{pmatrix}. \tag{P2.2}$$

Before we rush to calculate average spin components as matrix elements with wave functions (P2.2), we have to make sure that these functions are normalized. Calculating $|e_\pm|^2 = 1$ we find normalization constants that, when substituted into functions $e_\pm$, give wave functions

$$\Psi_+ = \left(2 + \frac{2h(h - \sqrt{h^2 + \alpha_R^2 k^2})}{\alpha_R^2 k^2}\right)^{-1/2} \begin{pmatrix} -\dfrac{ie^{-i\varphi}(-h + \sqrt{h^2 + \alpha_R^2 k^2})}{\alpha_R k} \\ 1 \end{pmatrix}$$

$$\Psi_- = \left(2 + \frac{2h(h + \sqrt{h^2 + \alpha_R^2 k^2})}{\alpha_R^2 k^2}\right)^{-1/2} \begin{pmatrix} \dfrac{ie^{-i\varphi}(h + \sqrt{h^2 + \alpha_R^2 k^2})}{\alpha_R k} \\ 1 \end{pmatrix}. \tag{P2.3}$$

Then the average spin components follow:

$$S_{x\pm} = \frac{1}{2}\langle\Psi_\pm|\sigma_x|\Psi_\pm\rangle = \mp\frac{\alpha_R k}{\sqrt{h^2 + a_R^2 k^2}}\sin(\varphi),$$

$$S_{y\pm} = \frac{1}{2}\langle\Psi_\pm|\sigma_y|\Psi_\pm\rangle = \pm\frac{\alpha_R k}{\sqrt{h^2 + \alpha_R^2 k^2}}\cos(\varphi),$$

$$S_{z\pm} = \frac{1}{2}\langle\Psi_\pm|\sigma_z|\Psi_\pm\rangle = \mp\frac{h}{\sqrt{h^2 + \alpha_R^2 k^2}}. \tag{P2.4}$$

Spectrum $E_{1,2}$ acquires an energy gap proportional to the perpendicular magnetic field. For small electron momentum $\alpha_R k << h$, electron spins are oriented along the z-direction (out of the (x, y) plane). When momentum increases, spins deflect from the z-direction, x,y-components rise, so in the limit of the weak magnetic field, spins tend to lie in the x–y plane as shown in Fig. 2.5.

Chapter 3

3.1 Normalization condition:

$$\int_{-\infty}^{\infty} \Phi^2(z)dz = 1,$$

or

$$\frac{A^2 b^3 z_0^2}{4\kappa_b} + \frac{1}{2} A^2(2 + bz_0(2 + bz_0)) = 1,$$

$$A^2 = \frac{4z_0^{-2}}{(2z_0^{-1} + b)^2 + b^2 + b^3\kappa_b^{-1}}.$$

Continuity of electron current at the interface can be written as

$$\frac{1}{m_{zB}} \frac{\partial \Phi}{\partial z}\bigg|_{z=-0} = \frac{1}{m_{zC}} \frac{\partial \Phi}{\partial z}\bigg|_{z=+0},$$

The solution to this equation gives

$$z_0^{-1} = \frac{\kappa_b m_{zC}}{m_{zB}} + b/2.$$

3.2 Integrating Eq. (3.25) twice we obtain

$$\varphi(z) = -\frac{qN_S}{\epsilon_0 \epsilon_C}\left[\int_0^z dy \int_0^y \Phi^2(x)dx + Az + B\right], \tag{P3.1}$$

where A and B are constants to be determined from boundary conditions.