

Chapter 2

Section 2.1 Practice Exercises

1. $x - 2$ if $x = 5$
 $x - 2 = 5 - 2 = 3$
2. $y(x - 3)$ if $x = 3$ and $y = 7$
 $y(x - 3) = 7(3 - 3) = 3(0) = 0$
3. $\frac{y+6}{x}$ if $x = 2$ and $y = 8$
 $\frac{y+6}{x} = \frac{8+6}{2} = \frac{14}{2} = 7$
4. $25 - z^3 + x$ if $z = 2$ and $x = 1$
 $25 - z^3 + x = 25 - (2)^3 + (1) = 25 - 8 + 1 = 18$
5. $\frac{5(F-32)}{9}$ if $F = 41$
 $\frac{5(F-32)}{9} = \frac{5(41-32)}{9} = \frac{5(9)}{9} = \frac{45}{9} = 5$
6.
 - a. Twice a number is $2x$.
 - b. 8 increased by a number is $8 + x$ or $x + 8$.
 - c. 10 minus a number is $10 - x$.
 - d. 10 subtracted from a number is $x - 10$.
 - e. The quotient of a number and 16 is $x \div 16$ or $\frac{x}{16}$.

Vocabulary, Readiness & Video Check 2.1

1. A combination of operations on letters (variables) and numbers is an expression.
2. A letter that represents a number is a variable.
3. $3x - 2y$ is called an expression and the letters x and y are variables.
4. Replacing a variable in an expression by a number and then finding the value of the expression is called evaluating the expression.
5. When a letter and a variable are next to each other, the operation is an understood multiplication.
6. decreased by

Exercise Set 2.1

2.

a	b	$a + b$	$a - b$	$a \cdot b$	$a \div b$
24	6	$24 + 6 = 30$	$24 - 6 = 18$	$24 \cdot 6 = 144$	$24 \div 6 = 4$

4.

a	b	$a + b$	$a - b$	$a \cdot b$	$a \div b$
298	0	$298 + 0 = 298$	$298 - 0 = 298$	$298 \cdot 0 = 0$	$298 \div 0$ is undefined.

6.

a	b	$a + b$	$a - b$	$a \cdot b$	$a \div b$
82	1	$82 + 1 = 83$	$82 - 1 = 81$	$82 \cdot 1 = 82$	$82 \div 1 = 82$

8. $7 + 3z$ for $x = 2$, $y = 5$, and $z = 3$
 $7 + 3z = 7 + 3(3) = 7 + 9 = 16$

10. $4yz + 2x$ for $x = 2$, $y = 5$, and $z = 3$
 $4yz + 2x = 4(5)(3) + 2(2) = 60 + 4 = 64$

12. $x + 5y - z$ for $x = 2$, $y = 5$, and $z = 3$
 $x + 5y - z = 2 + 5(5) - 3 = 2 + 25 - 3 = 24$

14. $2y + 5z$ for $x = 2$, $y = 5$, and $z = 3$
 $2y + 5z = 2(5) + 5(3) = 10 + 15 = 25$

16. $y^3 - z$ for $x = 2$, $y = 5$, and $z = 3$
 $y^3 - z = (5)^3 - (3) = 125 - 3 = 122$

18. $3yz^2 + 1$ for $x = 2$, $y = 5$, and $z = 3$
 $3yz^2 + 1 = 3(5)(3)^2 + 1$
 $= 3(5)(9) + 1$
 $= 135 + 1$
 $= 136$

20. $5 + (2x - 1)$ for $x = 2$, $y = 5$, and $z = 3$
 $5 + (2x - 1) = 5 + (2(2) - 1) = 5 + (4 - 1) = 5 + 3 = 8$

22. $x^4 - (y - z)$ for $x = 2$, $y = 5$, and $z = 3$
 $x^4 - (y - z) = (2)^4 - (5 - 3) = 16 - (2) = 14$

24. $\frac{8yz}{15}$ for $x = 2$, $y = 5$, and $z = 3$
 $\frac{8yz}{15} = \frac{8(5)(3)}{15} = \frac{120}{15} = 8$

26. $\frac{6+3x}{z}$ for $x = 2$, $y = 5$, and $z = 3$

$$\frac{6+3x}{z} = \frac{6+3(2)}{(3)} = \frac{6+6}{3} = \frac{12}{3} = 4$$

28. $\frac{2z+6}{3}$ for $x = 2$, $y = 5$, and $z = 3$

$$\frac{2z+6}{3} = \frac{2(3)+6}{3} = \frac{6+6}{3} = \frac{12}{3} = 4$$

30. $\frac{70}{2y} - \frac{15}{z}$ for $x = 2$, $y = 5$, and $z = 3$

$$\frac{70}{2y} - \frac{15}{z} = \frac{70}{2(5)} - \frac{15}{(3)} = \frac{70}{10} - \frac{15}{3} = 7 - 5 = 2$$

32. $3z^2 - z + 10$ for $x = 2$, $y = 5$, and $z = 3$

$$\begin{aligned} 3z^2 - z + 10 &= 3(3)^2 - (3) + 10 \\ &= 3(9) - 3 + 10 \\ &= 27 - 3 + 10 \\ &= 24 + 10 \\ &= 34 \end{aligned}$$

34. $(4y+3z)^2$ for $x = 2$, $y = 5$, and $z = 3$

$$\begin{aligned} (4y+3z)^2 &= (4(5)+3(3))^2 \\ &= (20+9)^2 \\ &= 29^2 \\ &= 841 \end{aligned}$$

36. $(xz-5)^4$ for $x = 2$, $y = 5$, and $z = 3$

$$(xz-5)^4 = ((2)(3)-5)^4 = (6-5)^4 = 1^4 = 1$$

38. $3x(y+z)$ for $x = 2$, $y = 5$, and $z = 3$

$$3x(y+z) = 3(2)(5+3) = 6(8) = 48$$

40. $xz(2y+x-z)$ for $x = 2$, $y = 5$, and $z = 3$

$$\begin{aligned} xz(2y+x-z) &= (2)(3)(2(5)+(2)-(3)) \\ &= 6(10+2-3) \\ &= 6(9) \\ &= 54 \end{aligned}$$

42. $\frac{6z+2y}{4}$ for $x = 2$, $y = 5$, and $z = 3$

$$\frac{6z+2y}{4} = \frac{6(3)+2(5)}{4} = \frac{18+10}{4} = \frac{28}{4} = 7$$

44.

F	50	59	68	77
$\frac{5(F-32)}{9}$	$\frac{5(50-32)}{9}$	$\frac{5(59-32)}{9}$	$\frac{5(68-32)}{9}$	$\frac{5(77-32)}{9}$
	$= \frac{5(18)}{9}$	$= \frac{5(27)}{9}$	$= \frac{5(36)}{9}$	$= \frac{5(45)}{9}$
	$= \frac{90}{9}$	$= \frac{135}{9}$	$= \frac{180}{9}$	$= \frac{225}{9}$
	$= 10$	$= 15$	$= 20$	$= 25$

46. Ten plus a number is written $10 + x$.

48. The difference of a number and five hundred is written $x - 500$.

50. A number less thirty is written $x - 30$.

52. A number times twenty is written $20x$.

54. The quotient of seven and a number is written $7 \div x$ or $\frac{7}{x}$.

56. The difference of twice a number, and four is written $2x - 4$.

58. The quotient of twenty and a number, decreased by three is written $\frac{20}{x} - 3$.

60. Twelve subtracted from a number is written $x - 12$.

62. The product of a number and 7 is $7x$.

64. The sum of a number and 7 is written $x + 7$.

66. Twenty decreased by twice a number is written $20 - 2x$.

68. The digit 7 is in the ones place in the number 2307.

70. The digit 7 is in the tens place in the number 179.

72. $2z - 4x$ for $x = 2$, $y = 0$, $z = 7$
 $2z - 4x = 2(7) - 4(2) = 14 - 8 = 6$
 The given answer is correct.

74. $\frac{2xy}{z}$ for $x = 2$, $y = 0$, $z = 7$
 $\frac{2xy}{z} = \frac{2(2)(0)}{(7)} = \frac{0}{7} = 0$
 The given answer is incorrect.

76. $2(x + y)^2$ for $x = 23$, $y = 72$

$$\begin{aligned} 2(x + y)^2 &= 2(23 + 72)^2 \\ &= 2(95)^2 \\ &= 2(9025) \\ &= 18,050 \end{aligned}$$

78. $16y - 20x + x^3$ for $x = 23$, $y = 72$

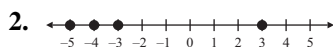
$$\begin{aligned} 16y - 20x + x^3 &= 16(72) - 20(23) + (23)^3 \\ &= 1152 - 460 + 12,167 \\ &= 12,859 \end{aligned}$$

80. If x is a whole number, then either $x = 0$ or $x > 0$. If $x = 0$, all three answers would be 0. However, if $x > 0$, then the smallest of the three numbers would be $\frac{x}{3}$.

82. In Exercise 44, as F gets larger, $\frac{5(F - 32)}{9}$ gets larger.

Section 2.2 Practice Exercises

1. a. If 0 represents the surface of the earth, then 3805 below the surface of the earth is -3805 .
- b. If zero degrees Fahrenheit is represented by 0°F , then 85 degrees below zero, Fahrenheit is represented by -85°F .



3. a. $0 > -3$ since 0 is to the right of -3 on a number line.
- b. $-5 < 5$ since -5 is to the left of 5 on a number line.
- c. $-8 > -12$ since -8 is to the right of -12 on a number line.
4. a. $|-4| = 4$ since -4 is 4 units from 0 on a number line.
- b. $|2| = 2$ since 2 is 2 units from 0 on a number line.
- c. $|-8| = 8$ since -8 is 8 units from 0 on a number line.
5. a. The opposite of 7 is -7 .

- b. The opposite of -17 is $-(-17)$ or 17.

6. a. $-|-2| = -2$

b. $-|5| = -5$

c. $-(-11) = 11$

7. $-|x|$ for $x = -9$
 $-|x| = -|-9| = -9$

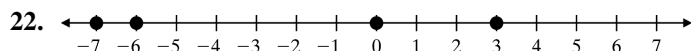
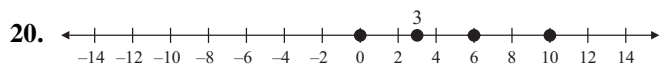
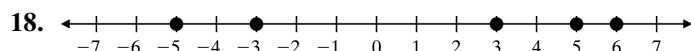
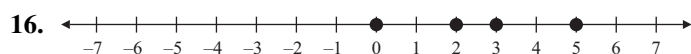
8. The planet with the highest average daytime surface temperature is the one that corresponds to the bar that extends the furthest in the positive direction (upward). Venus has the highest average daytime surface temperature of 867°F .

Vocabulary, Readiness & Video Check 2.2

1. The numbers $\dots -3, -2, -1, 0, 1, 2, 3, \dots$ are called integers.
2. Positive numbers, negative numbers, and zero, together are called signed numbers.
3. The symbols " $<$ " and " $>$ " are called inequality symbols.
4. Numbers greater than 0 are called positive numbers while numbers less than 0 are called negative numbers.
5. The sign " $<$ " means is less than and " $>$ " means is greater than.
6. On a number line, the greater number is to the right of the lesser number.
7. A number's distance from 0 on the number line is the number's absolute value.
8. The numbers -5 and 5 are called opposites.
9. number of feet a miner works underground
10. The tick marks are labeled with the integers.
11. 0 will always be greater than any of the negative integers.
12. 8; $|8| = 8$ also.
13. A negative sign can be translated into the phrase "opposite of."
14. Lake Eyre

Exercise Set 2.2

2. -35 represents 35 feet below the surface.
4. -282 represents 282 feet below sea level.
6. If 0 represents 0 degrees Fahrenheit, then 134 degrees above zero is $+134$.
8. If 0 represents the surface of the ocean, then 14,040 below the surface of the ocean is $-14,040$.
10. If 0 represents a loss of \$0, then a loss of \$555 million is -555 million.
12. If 0 represents 0° Celsius, then 10° below 0° Celsius is -10 . Since 5° below 0° Celsius is -5 and -10 is less than -5 , -10 (or 10° below 0° Celsius) is cooler.
14. If 0 represents a decrease of 0%, then a 23 percent decrease is -23 .



24. $-8 < 0$ since -8 is to the left of 0 on a number line.
26. $-12 < -10$ since -12 is to the left of -10 on a number line.
28. $-27 > -29$ since -27 is to the right of -29 on a number line.
30. $13 > -13$ since 13 is to the right of -13 on a number line.
32. $|7| = 7$ since 7 is 7 units from 0 on a number line.
34. $|-19| = 19$ since -19 is 19 units from 0 on a number line.
36. $|100| = 100$ since 100 is 100 units from 0 on a number line.
38. $|-10| = 10$ since -10 is 10 units from 0 on a number line.
40. The opposite of 8 is negative 8.
 $-(8) = -8$
42. The opposite of negative 6 is 6.
 $-(-6) = 6$
44. The opposite of 123 is negative 123.
 $-(123) = -123$
46. The opposite of negative 13 is 13.
 $-(-13) = 13$

48. $|-11| = 11$

50. $-|43| = -43$

52. $-|-18| = -18$

54. $-(-27) = 27$

56. $-|-29| = -29$

58. $-(-14) = 14$

60. $-|x| = -|-8| = -8$

62. $-|-x| = -|-10| = -10$

64. $|x| = |32| = 32$

66. $|-x| = |-1| = 1$

68. $-4 > -17$ since -4 is to the right of -17 on a number line.

70. $|-8| = 8$
 $|-4| = 4$
Since $8 > 4$, $|-8| > |-4|$.

72. $-|17| = -17$
 $-(-17) = 17$
Since $-17 < 17$, $-|17| < -(-17)$.

74. $|-24| = 24$
 $-(-24) = 24$
Since $24 = 24$, $|-24| = -(-24)$.

76. $-45 < 0$ since -45 is to the left of 0 on a number line.

78. $|-45| = 45$
 $|0| = 0$
Since $45 > 0$, $|-45| > |0|$.

80. $-|-8| = -8$
 $-|-4| = -4$
Since $-8 < -4$, $-|-8| < -|-4|$.

82. $-(-38) = 38$
Since $-22 < 38$, $-22 < -(-38)$.

84. If the number is -13 , then the absolute value of -13 is 13 and the opposite of -13 is 13 .

86. If the opposite of a number is 90 , then the number is -90 and its absolute value is 90 .

88. The 'bar' that is equal to 0 corresponds to Lake Maracaibo, so Lake Maracaibo has an elevation at sea level.

90. The bar that extends second to the farthest in the negative direction corresponds to Lake Eyre, so Lake Eyre has the second lowest elevation.

92. The smallest number on the graph is -269°C , which corresponds to helium.

94. The number on the graph closest to $+300^{\circ}\text{C}$ is 280°C , which corresponds to phosphorus.

96. $9 + 0 = 9$

98.
$$\begin{array}{r} 20 \\ + 15 \\ \hline 35 \end{array}$$

100.
$$\begin{array}{r} 1 \\ 362 \\ 37 \\ + 90 \\ \hline 489 \end{array}$$

102. $|10| = 10$, $2^3 = 8$, $-|-5| = -5$, and $-(-4) = 4$, so the numbers in order from least to greatest are $-|-5|$, $-(-4)$, 2^3 , $|10|$.

104. $1^4 = 1$, $-(-3) = 3$, $-|7| = -7$, and $|-20| = 20$, so the numbers in order from least to greatest are $-|7|$, 1^4 , $-(-3)$, $|-20|$.

106. $3^3 = 27$, $-|-11| = -11$, $-(-10) = 10$, $-4 = -4$, $-|2| = -2$, so the numbers in order from least to greatest are $-|-11|$, -4 , $-|2|$, $-(-10)$, and 3^3 .

108. $|x| > 4$

a. $|0| > 4$
 $0 > 4$ False

b. $|x| > 4$
 $|-4| > 4$
 $4 > 4$ False

c. $|x| > 4$
 $|-1| > 4$
 $1 > 4$ False

d. $|x| > 4$
 $|-100| > 4$
 $100 > 4$ True

d is the only value that makes the statement true.

110. $-(-|(-7)|) = -(-|7|) = |7| = 7$

112. False; consider 0, where $|0| = 0$ and 0 is not positive.

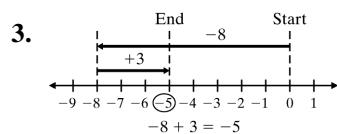
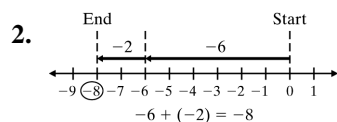
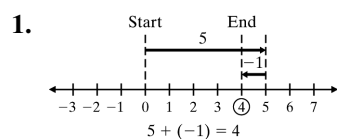
114. True; zero is always less than a positive number since it is to the left of it on a number line.

116. No; $b > a$ because b is to the right of a on the number line.

118. answers may vary

120. no; answers may vary

Section 2.3 Practice Exercises



4. $(-3) + (-9)$
 $|-3| = 3$
 $|-9| = 9$
 $3 + 9 = 12$
 Their common sign is negative, so the sum is negative.
 $(-3) + (-9) = -12$

5. $(-12) + (-3) = -15$

6. $9 + 5 = 14$

7. $(-3) + 59 = ?$
 $|-3| = 3$, $|59| = 59$ and $59 - 3 = 56$. 59 has the larger absolute value and its sign is an understood +, so $(-3) + 59 = 56$.

8. $22 + (-28) = ?$
 $|22| = 22$, $|-28| = 28$ and $28 - 22 = 6$. -28 has the larger absolute value and its sign is -, so
 $22 + (-28) = -6$.

9. $-46 + 20 = -26$

10. $8 + (-6) = 2$

11. $-2 + 0 = -2$

12. $15 + (-15) = 0$

13. $-80 + 80 = 0$

14. $8 + (-3) + (-13) = 5 + (-13) = -8$

15. $5 + (-3) + 12 + (-14) = 2 + 12 + (-14)$
 $= 14 + (-14)$
 $= 0$

16. $x + 3y$ for $x = -4$, $y = 1$
 $x + 3y = (-4) + 3(1) = -4 + 3 = -1$

17. $x + y$ for $x = -11$, $y = -6$
 $x + y = (-11) + (-6) = -17$

18.

Beginning Temperature

 +

Rise of 4°

 +

Rise of 27°

 \downarrow \downarrow \downarrow
 -8 $+4$ $+7$
 $= (-4) + 7$
 $= 3$
 The temperature was 3°F at 8 A.M.

Calculator Explorations

1. $-256 + 97 = -159$

2. $811 + (-1058) = -247$

3. $6(15) + (-46) = 44$

4. $-129 + 10(48) = 351$

5. $-108,650 + (-786,205) = -894,855$

6. $-196,662 + (-129,856) = -326,518$

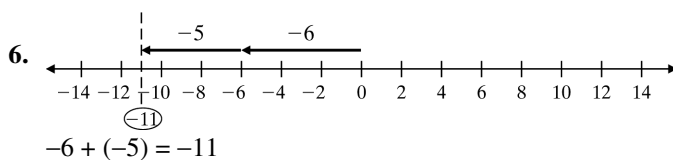
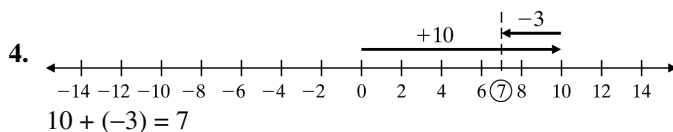
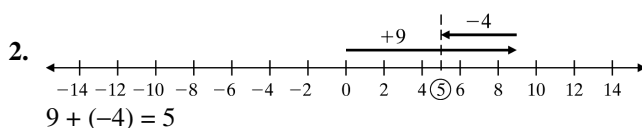
Vocabulary, Readiness & Video Check 2.3

1. If n is a number, then $-n + n = 0$.

2. Since $x + n = n + x$, we say that addition is commutative.

3. If a is a number, then $-(-a) = a$.

4. Since $n + (x + a) = (n + x) + a$, we say that addition is associative.
5. $5 + 0 = 5$
6. $0 + 3 = 3$
7. $0 + (-35) = -35$
8. $(-2) + 0 = -2$
9. $-12 + 12 = 0$
10. $-9 + 9 = 0$
11. $28 + (-28) = 0$
12. $48 + (-48) = 0$
13. Negative; the numbers have different signs and the sign of the sum is the same as the sign of the number with the larger absolute value, -6 .
14. Negative; the numbers have the same sign—both are negative—and we keep this common sign in the sum.
15. The diver's current depth is 231 feet below the surface.

Exercise Set 2.3

8. $15 + 42 = 57$
10. $|-5| + |-4| = 5 + 4 = 9$
 The common sign is negative, so $-5 + (-4) = -9$.
12. $-62 + 62 = 0$
14. $|8| - |-3| = 8 - 3 = 5$
 $8 > 3$, so the answer is positive.
 $8 + (-3) = 5$

16. $|12| - |-8| = 12 - 8 = 4$
 $12 > 8$, so the answer is positive.
 $-8 + 12 = 4$
18. $|-9| - |5| = 9 - 5 = 4$
 $9 > 5$, so the answer is negative.
 $5 + (-9) = -4$
20. $|-6| + |-1| = 6 + 1 = 7$
The common sign is negative, so $-6 + (-1) = -7$.
22. $|-23| + |-23| = 23 + 23 = 46$
The common sign is negative, so $-23 + (-23) = -46$.
24. $|-45| + |-90| = 45 + 90 = 135$
The common sign is negative, so
 $-45 + (-90) = -135$.
26. $|-500| + |-230| = 500 + 230 = 730$
The common sign is negative, so
 $-500 + (-230) = -730$.
28. $-10 + 10 = 0$
30. $|24| - |-10| = 24 - 10 = 14$
 $24 > 10$, so the answer is positive.
 $24 + (-10) = 14$
32. $|-8| - |2| = 8 - 2 = 6$
 $8 > 2$, so the answer is negative.
 $-8 + 2 = -6$
34. $|-15| - |5| = 15 - 5 = 10$
 $15 > 5$, so the answer is negative.
 $-15 + 5 = -10$
36. $|89| - |-37| = 89 - 37 = 52$
 $89 > 37$, so the answer is positive.
 $89 + (-37) = 52$
38. $|65| - |-25| = 65 - 25 = 40$
 $65 > 25$, so the answer is positive.
 $-25 + 65 = 40$
40. $|164| - |-64| = 164 - 64 = 100$
 $164 > 64$, so the answer is positive.
 $-64 + 164 = 100$
42. $|-83| - |42| = 83 - 42 = 41$
 $83 > 42$, so the answer is negative.
 $42 + (-83) = -41$
44. $|-375| - |325| = 375 - 325 = 50$
 $375 > 325$, so the answer is negative.
 $325 + (-375) = -50$
46. $|-56| + |-33| = 56 + 33 = 89$
The common sign is negative, so
 $-56 + (-33) = -89$.
48. $-1 + 5 + (-8) = 4 + (-8) = -4$
50. $-103 + (-32) + (-27) = -135 + (-27) = -162$
52. $18 + (-9) + 5 + (-2) = 9 + 5 + (-2)$
 $= 14 + (-2)$
 $= 12$
54. $34 + (-12) + (-11) + 213 = 22 + (-11) + 213$
 $= 11 + 213$
 $= 224$
56. $-10 + (-6) + (-1) = -16 + (-1) = -17$
58. $-35 + (-12) = -47$
60. $3 + (-23) + 6 = -20 + 6 = -14$
62. $-100 + 70 = -30$
64. $(-45) + 22 + 20 = -23 + 20 = -3$
66. $-87 + 0 = -87$
68. $-16 + 6 + (-14) + (-20) = -10 + (-14) + (-20)$
 $= -24 + (-20)$
 $= -44$
70. $94 + (-94) = 0$
72. $x + y = -1 + (-29) = -30$
74. $3x + y = 3(7) + (-11) = 21 + (-11) = 10$
76. $3x + y = 3(13) + (-17) = 39 + (-17) = 22$
78. The sum of -30 and 10 is $-30 + 10 = -20$.
80. The sum of -49 , -2 , and 40 is
 $-49 + (-2) + 40 = -51 + 40 = -11$.
82. $0 + (-248) + 6 + (-17) + (-24) + 23$
 $= -248 + 6 + (-17) + (-24) + 23$
 $= -242 + (-17) + (-24) + 23$
 $= -259 + (-24) + 23$
 $= -283 + 23$
 $= -260$
The diver's final depth is 260 feet below the surface.

84. Since $-7 < -2$, Dufner won Round 2.

86. The bar for 2001 has a height of -25 , so the net income in 2001 was $-\$25,000,000$.

88. The net incomes for 2007, 2009, and 2011 are 3496, 8240, and 25,922. The total is $3496 + 8240 + 25,922 = 37,658$. The total net income for 2007, 2009, and 2011 was $\$37,658,000,000$.

90. $14 + (-5) + (-8) + 7 = 9 + (-8) + 7 = 1 + 7 = 8$. Her total score was 8.

92. The net income for 2010 was $\$75,083$.
The net income for 2011 was $-\$10,412$.
The net income for 2012 was $-\$1786$.
The net income for 2013 was $\$96,398$.
 $\$75,083 + [-\$10,412] + (-\$1786) + \$96,398$
 $= \$64,671 + (-\$1786) + \$96,398$
 $= \$62,885 + \$96,398$
 $= \$159,283$
The total net income for the four years was $\$159,283$.

94. $-45 + 3 = -42$
The lowest temperature ever recorded in Pennsylvania is -42°F .

96. $-8605 + 1070 = -7535$
The depth of the Cayman Trench is -7535 meters.

98. $91 - 0 = 91$

$$\begin{array}{r} 9 \\ 3 \cancel{0} 10 \\ 100. \quad \cancel{A} \cancel{0} \cancel{0} \\ \quad - 18 \\ \hline \quad 382 \end{array}$$

102. answers may vary; two examples are $-15 + (-15) = -30$ and $-40 + 10 = -30$

104. $(-4) + 14 = 10$

106. $-15 + (-17) = -32$

108. True; the sum of two positive numbers is always a positive number.

110. True; the sum of zero and a negative number is always a negative number.

112. answers may vary

Section 2.4 Practice Exercises

1. $12 - 7 = 12 + (-7) = 5$

2. $-6 - 4 = -6 + (-4) = -10$

3. $11 - (-14) = 11 + 14 = 25$

4. $-9 - (-1) = -9 + 1 = -8$

5. $5 - 9 = 5 + (-9) = -4$

6. $-12 - 4 = -12 + (-4) = -16$

7. $-2 - (-7) = -2 + 7 = 5$

8. Subtract 5 from -10 .
 $-10 - 5 = -10 + (-5) = -15$

9. $-4 - 3 - 7 - (-5) = -4 + (-3) + (-7) + 5$
 $= -7 + (-7) + 5$
 $= -14 + 5$
 $= -9$

10. $3 + (-5) - 6 - (-4) = 3 + (-5) + (-6) + 4$
 $= -2 + (-6) + 4$
 $= -8 + 4$
 $= -4$

11. $x - y$ when $x = -2$, $y = 14$
 $x - y = -2 - 14 = -2 + (-14) = -16$

12. $3y - z$ when $y = 9$, $z = -4$
 $3y - z = 3(9) - (-4) = 27 - (-4) = 27 + 4 = 31$

13.

How much higher is Mount Everest?

 $=$

Height of Mt. Everest

 $-$

Height of Dead Sea

How much higher is Mount Everest?

 $= 29,028 - (-1312)$
 $= 29,028 + 1312$
 $= 30,340$

Mount Everest is 30,340 feet higher than the lowest point in the Dead Sea.

Vocabulary, Readiness & Video Check 2.4

1. It is true that $a - b = a + (-b)$. **b**

2. The opposite of n is $-n$. **a**

3. To evaluate $x - y$ for $x = -10$ and $y = -14$, we replace x with -10 and y with -14 and evaluate $-10 - (-14)$. **d**

4. The expression $-5 - 10$ equals $-5 + (-10)$. c
5. $5 - 5 = 0$
6. $7 - 7 = 0$
7. $8642 - 8642 = 0$
8. $9012 - 9012 = 0$
9. additive inverse
10. $-3 + 4 + (23) + (-10)$; all the subtraction operations are rewritten as additions in one step rather than changing one operation at a time as you work from left to right.
11. to follow the order of operations
12. The warmest temperature is 263° warmer than the coldest temperature.
30. $-50 - (-50) = -50 + 50 = 0$
32. $-35 + (-11) = -46$
34. $7 - 30 = 7 + (-30) = -23$
36. $-105 - 68 = -105 + (-68) = -173$
38. $86 - 98 = 86 + (-98) = -12$
40. $8 - 4 - 1 = 8 + (-4) + (-1) = 4 + (-1) = 3$
42. $30 - 7 - 12 = 30 + (-7) + (-12) = 23 + (-12) = 11$
44. $-10 - 6 - (-9) = -10 + (-6) + 9 = -16 + 9 = -7$
46. $-15 + (-8) - 4 = -15 + (-8) + (-4)$
 $= -23 + (-4)$
 $= -27$
48. $23 - (-17) + (-9) = 23 + 17 + (-9)$
 $= 40 + (-9)$
 $= 31$
50. $-(-9) - 7 + (-23) = 9 + (-7) + (-23)$
 $= 2 + (-23)$
 $= -21$
52. $-6 - (-8) + (-12) - 7 = -6 + 8 + (-12) + (-7)$
 $= 2 + (-12) + (-7)$
 $= -10 + (-7)$
 $= -17$
54. $5 + (-18) - (-21) - 2 = 5 + (-18) + 21 + (-2)$
 $= -13 + 21 + (-2)$
 $= 8 + (-2)$
 $= 6$
56. $x - y$ for $x = -7, y = 1$
 $x - y = -7 - 1 = -7 + (-1) = -8$
58. $x - y$ for $x = 9, y = -2$
 $x - y = 9 - (-2) = 9 + 2 = 11$
60. $2x - y$ for $x = 8, y = -10$
 $2x - y = 2(8) - (-10) = 16 + 10 = 26$
62. $2x - y$ for $x = 14, y = -12$
 $2(14) - (-12) = 28 + 12 = 40$

Exercise Set 2.4

2. $-6 - (-6) = -6 + 6 = 0$
4. $5 - 2 = 5 + (-2) = 3$
6. $2 - 5 = 2 + (-5) = -3$
8. $12 - (-12) = 12 + 12 = 24$
10. $(-25) - (-25) = -25 + 25 = 0$
12. $-2 - 42 = -2 + (-42) = -44$
14. $8 - 9 = 8 + (-9) = -1$
16. $17 - 63 = 17 + (-63) = -46$
18. $844 - (-20) = 844 + 20 = 864$
20. $-5 - 8 = -5 + (-8) = -13$
22. $-12 - (-5) = -12 + 5 = -7$
24. $16 - 45 = 16 + (-45) = -29$
26. $-22 - 10 = -22 + (-10) = -32$
28. $-8 - (-13) = -8 + 13 = 5$

$$\begin{aligned}
 64. \quad \boxed{\text{Difference in Temperature}} &= \boxed{\text{Temperature on Venus}} - \boxed{\text{Temperature on Mars}} \\
 &\quad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \\
 &= \qquad \quad 867 \qquad \quad - \qquad \quad (-24) \\
 &= 867 + 24 \\
 &= 891
 \end{aligned}$$

The difference between the temperatures on Venus and Mars is 891 degrees Fahrenheit.

$$\begin{aligned}
 66. \quad \boxed{\text{Difference in Temperature}} &= \boxed{\text{Temperature on Jupiter}} - \boxed{\text{Temperature on Saturn}} \\
 &\quad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \\
 &= \qquad \quad -162 \qquad \quad - \qquad \quad (-218) \\
 &= -162 + 218 \\
 &= 56
 \end{aligned}$$

The difference between the temperatures for Jupiter and Saturn is 56 degrees Fahrenheit.

$$\begin{aligned}
 68. \quad \boxed{\text{How much warmer is}} &= \boxed{\text{Highest temp } 134^{\circ}\text{F}} - \boxed{\text{Lowest temp } -80^{\circ}} \\
 &\quad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \\
 &= \qquad \quad 134 \qquad \quad - \qquad \quad (-80) \\
 &= 134 + 80 \\
 &= 214
 \end{aligned}$$

134°F is 214°F warmer than -80°F .

$$\begin{aligned}
 70. \quad \boxed{\text{Account}} &= \boxed{\text{Spent}} + \boxed{\text{Top}} + \boxed{\text{Night shirt}} \\
 &\quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 &= -93 + 18 + 26 \\
 &= -75 + 26 \\
 &= -49
 \end{aligned}$$

She owes \$49 on her account.

$$\begin{aligned}
 72. \quad \boxed{\text{Difference in elevation of Mauna Kea}} &= \boxed{\text{Elevation of}} - \boxed{\text{Elevation of Mid-America}} \\
 &\quad \boxed{\text{and Mid-America Trench}} \quad \boxed{\text{Mauna Kea}} \quad \boxed{\text{Trench}} \\
 &= 13,796 - (-21,857) \\
 &= 13,796 + 21,857 \\
 &= 35,653
 \end{aligned}$$

The difference in elevation of Mauna Kea and the Mid-America Trench is 35,653 feet.

$$\begin{aligned}
 74. \quad \boxed{\text{Difference in elevation of Danakil}} &= \boxed{\text{Elevation of}} - \boxed{\text{Elevation of}} \\
 &\quad \boxed{\text{and Turfan Depressions}} \quad \boxed{\text{Danakil}} \quad \boxed{\text{Turfan Depression}} \\
 &= -384 - (-505) \\
 &= -384 + 505 \\
 &= 121
 \end{aligned}$$

The difference in elevation of Danakil and Turfan Depressions is 121 feet.

76. The highest elevation shown is the Salton Sea, and the lowest elevation shown is the Turfan Depression.

$$\begin{aligned}
 \boxed{\text{Difference in elevation of Salton Sea and the Turfan Depression}} &= \boxed{\text{Elevation of Salton Sea}} - \boxed{\text{Elevation of Turfan Depression}} \\
 &= -236 - (-505) \\
 &= -236 + 505 \\
 &= 269
 \end{aligned}$$

The difference between the highest and lowest elevation shown is 269 feet.

78.
$$\begin{aligned}
 \boxed{\text{Difference in elevation of Great Bear Lake and the Caspian Sea}} &= \boxed{\text{Elevation of Great Bear Lake}} - \boxed{\text{Elevation of the Caspian Sea}} \\
 &= 512 - (-92) \\
 &= 512 + 92 \\
 &= 604
 \end{aligned}$$

The difference in elevation of Great Bear Lake and the Caspian Sea is 604 feet.

80.
$$\begin{aligned}
 \boxed{\text{Difference in elevation of Lake Eyre and the Caspian Sea}} &= \boxed{\text{Elevation of Lake Eyre}} - \boxed{\text{Elevation of the Caspian Sea}} \\
 &= -52 - (-92) \\
 &= -52 + 92 \\
 &= 40
 \end{aligned}$$

The difference in elevation of Lake Eyre and the Caspian Sea is 40 feet.

82.
$$\begin{aligned}
 \boxed{\text{Difference in exports and imports}} &= \boxed{\text{Amount of Exports}} - \boxed{\text{Amount of Imports}} \\
 &= 1165 - 3878 \\
 &= 1165 - 3878 \\
 &= -2713
 \end{aligned}$$

The trade balance for petroleum products in 2012 was -2713 million barrels.

84. The difference of -3 and a number is $-3 - x$.

86. Add a number and -36 is $x + (-36)$.

88. $\frac{96}{3} = 32$

$$\begin{array}{r}
 32 \\
 3 \overline{) 96} \\
 \underline{-9} \\
 06 \\
 \underline{-6} \\
 0
 \end{array}$$

90.
$$\begin{array}{r}
 51 \\
 \times 89 \\
 \hline
 459 \\
 4080 \\
 \hline
 4539
 \end{array}$$

92. answers may vary

94. $-4 - 8 = -4 + (-8) = -12$

96. $-3 - (-10) = -3 + 10 = 7$

98. $|-12| - |-5| = 12 - 5 = 12 + (-5) = 7$

100. $|-9| - |9| = 9 - 9 = 9 + (-9) = 0$

102. $|-23| - |-42| = 23 - 42 = 23 + (-42) = -19$

104. $|-2 - (-6)| = |-2 + 6| = |4| = 4$
 $|-2| - |-6| = 2 - 6 = 2 + (-6) = -4$
 Since $4 \neq -4$, the statement is false.

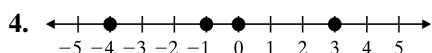
106. no; answers may vary

Integrated Review

1. 29,028 feet above sea level is represented by the integer +29,028.

2. 35,840 feet below sea level is represented by the integer -35,840.

3. 7 miles below sea level is represented by the integer -7.

5. $0 > -3$ since 0 is to the right of -3 on a number line.6. $-15 < -5$ since -15 is to the left of -5 on a number line.7. $-1 < 1$ since -1 is to the left of 1 on a number line.8. $-2 > -7$ since -2 is to the right of -7 on a number line.

9. $|-1| = 1$

10. $-|-4| = -4$

11. $|-8| = 8$

12. $-(-5) = 5$

13. The opposite of 6 is -6.

14. The opposite of -3 is 3.

15. The opposite of 89 is -89.

16. The opposite of 0 is 0.

17. $-7 + 12 = 5$

18. $-9 + (-11) = -20$

19. $25 + (-35) = -10$

20. $1 - 3 = 1 + (-3) = -2$

21. $26 - (-26) = 26 + 26 = 52$

22. $-2 - 1 = -2 + (-1) = -3$

23. $-18 - (-102) = -18 + 102 = 84$

24. $-8 + (-6) + 20 = -14 + 20 = 6$

25. $-11 - 7 - (-19) = -11 + (-7) + 19 = -18 + 19 = 1$

26. $-4 + (-8) - 16 - (-9) = -4 + (-8) + (-16) + 9$
 $= -12 + (-16) + 9$
 $= (-28) + 9$
 $= -19$

27. $26 - 14 = 26 + (-14) = 12$

28. $-12 - (-8) = -12 + 8 = -4$

29. $-17 + (-27) = -44$

30. a. $|x| > 0$
 $|0| > 0$
 $0 > 0$ False

b. $|x| > 0$
 $|18| > 0$
 $18 > 0$ True

c. $|x| > 0$
 $|-3| > 0$
 $3 > 0$ True

d. $|x| > 0$
 $|-21| > 0$
 $21 > 0$ True

The numbers for x that make the statement true are b, c, and d.

31. a. $|x| > -5$
 $|0| > -5$
 $0 > -5$ True

b. $|x| > -5$
 $|3| > -5$
 $3 > -5$ True

$$\begin{aligned} \text{c. } |x| &> -5 \\ |-1| &> -5 \\ 1 &> -5 \quad \text{True} \end{aligned}$$

$$\begin{aligned} \text{d. } |x| &> -5 \\ |-1000| &> -5 \\ 1000 &> -5 \quad \text{True} \end{aligned}$$

The numbers for x that make the statement true are a, b, c, and d.

$$\begin{aligned} 32. \quad x + y \text{ for } x = -1, y = 11 \\ x + y = -1 + 11 = 10 \end{aligned}$$

$$\begin{aligned} 33. \quad x - y \text{ for } x = -1, y = 11 \\ x - y = -1 - 11 = -1 + (-11) = -12 \end{aligned}$$

$$\begin{aligned} 34. \quad y - x \text{ for } x = -1, y = 11 \\ y - x = 11 - (-1) = 11 + 1 = 12 \end{aligned}$$

$$\begin{aligned} 35. \quad y + x \text{ for } x = -1, y = 11 \\ y + x = 11 + -1 = 10 \end{aligned}$$

$$\begin{aligned} 36. \quad 5y - x \text{ for } x = -1, y = 11 \\ 5y - x = 5(11) - (-1) = 55 + 1 = 56 \end{aligned}$$

$$\begin{aligned} 37. \quad x - 3y \text{ for } x = -1, y = 11 \\ x - 3y = -1 - 3(11) = -1 - 33 = -1 + (-33) = -34 \end{aligned}$$

Section 2.5 Practice Exercises

$$1. \quad -3 \cdot 8 = -24$$

$$2. \quad -5(-2) = 10$$

$$3. \quad 0 \cdot (-20) = 0$$

$$4. \quad 10(-5) = -50$$

$$5. \quad 8(-6)(-2) = -48(-2) = 96$$

$$6. \quad (-9)(-2)(-1) = 18(-1) = -18$$

$$7. \quad (-3)(-4)(-5)(-1) = 12(-5)(-1) = -60(-1) = 60$$

$$\begin{aligned} 8. \quad (-2)^4 &= (-2)(-2)(-2)(-2) \\ &= 4(-2)(-2) \\ &= -8(-2) \\ &= 16 \end{aligned}$$

$$9. \quad -8^2 = -(8 \cdot 8) = -64$$

$$10. \quad \frac{42}{-7} = -6$$

$$11. \quad -16 \div (-2) = 8$$

$$12. \quad \frac{-80}{10} = -8$$

$$13. \quad \frac{-6}{0} \text{ is undefined.}$$

$$14. \quad \frac{0}{-7} = 0$$

$$15. \quad xy = 5 \cdot (-8) = -40$$

$$16. \quad \frac{x}{y} = \frac{-12}{-3} = 4$$

$$\begin{aligned} 17. \quad \text{total score} &= 4 \cdot (-13) = -52 \\ \text{The card player's total score was } &-52. \end{aligned}$$

Vocabulary, Readiness & Video Check 2.5

1. The product of a negative number and a positive number is a negative number.
2. The product of two negative numbers is a positive number.
3. The quotient of two negative numbers is a positive number.
4. The quotient of a negative number and a positive number is a negative number.
5. The product of a negative number and zero is 0.
6. The quotient of 0 and a negative number is 0.
7. The quotient of a negative number and 0 is undefined.
8. When a negative sign is involved in an expression with an exponent, parentheses tell you whether or not the exponent applies to the negative sign. In Example 3, $(-3)^2$, the exponent applies to everything within the parentheses, so -3 is squared; in Example 4, -3^2 , the exponent does *not* apply to the sign and only 3 is squared.

9. We can find out about sign rules for division because we know sign rules for multiplication.

10. That ab means $a \cdot b$.

11. The phrase “lost four yards” in the example translates to the negative number -4 .

Exercise Set 2.5

2. $5(-3) = -15$

4. $-7(-2) = 14$

6. $-9(7) = -63$

8. $-6(0) = 0$

10. $-2(3)(-7) = -6(-7) = 42$

12. $-8(-3)(-3) = 24(-3) = -72$

14. $2(-5)(-4) = -10(-4) = 40$

16. $3(0)(-4)(-8) = 0$

18. $-2(-1)(3)(-2) = 2(3)(-2) = 6(-2) = -12$

20. $-2^4 = -(2)(2)(2)(2) = -4(2)(2) = -8(2) = -16$

22. $(-1)^4 = (-1)(-1)(-1)(-1)$
 $= 1(-1)(-1)$
 $= -1(-1)$
 $= 1$

24. $-4^3 = -(4 \cdot 4 \cdot 4) = -64$

26. $(-3)^3 = (-3)(-3)(-3) = 9(-3) = -27$

28. $90 \div (-9) = -10$

30. $\frac{56}{-8} = -7$

32. $\frac{-32}{4} = -8$

34. $\frac{-13}{0}$ is undefined.

36. $\frac{0}{-15} = 0$

38. $\frac{-24}{-12} = 2$

40. $0(-100) = 0$

42. $-6 \cdot 2 = -12$

44. $-12(13) = -156$

46. $-9(-5) = 45$

48. $-7(-5)(-3) = 35(-3) = -105$

50. $(-6)^2 = (-6)(-6) = 36$

52. $-\frac{30}{5} = -6$

54. $-\frac{49}{7} = -7$

56. $-15 \div 3 = -5$

58. $6(-5)(-2) = -30(-2) = 60$

60. $-20 \cdot 5 \cdot (-5) \cdot (-3) = -100 \cdot (-5) \cdot (-3)$
 $= 500 \cdot (-3)$
 $= -1500$

62. $\frac{0}{-14} = 0$

64. $\frac{63}{-9} = -7$

66. $480 \div (-8) = \frac{480}{-8} = -60$

68. $\frac{-36}{-3} = 12$

70. $-2^3 = -(2 \cdot 2 \cdot 2) = -8$

72. $(-11)^2 = (-11)(-11) = 121$

74. $-1(2)(7)(-3) = -2(7)(-3) = -14(-3) = 42$

76. $(-1)^{33} = -1$, since there are an odd number of factors.

78. $-2(-2)(-3)(-2) = 4(-3)(-2) = -12(-2) = 24$

80.
$$\begin{array}{r} 56 \\ \times 43 \\ \hline 168 \\ 2240 \\ \hline 2408 \\ -56 \cdot 43 = -2408 \end{array}$$
82.
$$\begin{array}{r} 23 \\ \times 70 \\ \hline 1610 \\ 70 \cdot (-23) = -1610 \end{array}$$
84. $ab = 5(-1) = -5$
86. $ab = (-8)(8) = -64$
88. $ab = (-9)(-6) = 54$
90. $\frac{x}{y} = \frac{9}{-3} = -3$
92. $\frac{x}{y} = \frac{0}{-5} = 0$
94. $\frac{x}{y} = \frac{-10}{-10} = 1$
96. $xy = 20 \cdot (-5) = -100$
 $\frac{x}{y} = \frac{20}{-5} = -4$
98. $xy = -3 \cdot 0 = 0$
 $\frac{x}{y} = \frac{-3}{0}$ is undefined.
100. $-63 \div (-3) = 21$
 The quotient of -63 and -3 is 21.
102.
$$\begin{array}{r} 49 \\ \times 5 \\ \hline 245 \\ -49(5) = -245 \\ \hline \end{array}$$

 The product of -49 and 5 is -245 .
104. The quotient of -8 and a number is $\frac{-8}{x}$ or $-8 \div x$.
106. The sum of a number and -12 is $x + (-12)$.
108. The difference of a number and -10 is $x - (-10)$.
110. Multiply a number by -17 is $x \cdot (-17)$ or $-17x$.
112. A loss of \$400 is represented by -400 .
 $7 \cdot (-400) = -2800$
 His total loss was \$2800.
114. A drop of 5 degrees is represented by -5 .
 $6 \cdot (-5) = -30$
 The total drop in temperature was 30 degrees.
116. $-1 \cdot (-39) = 39$
 The melting point of rubidium is 39°C .
118. $-11 \cdot (-70) = 770$
 The melting point of strontium is 770°C .
120. $\frac{-30,000}{3} = -10,000$
 The expected loss for each month would be 10,000 shipments or $-10,000$ shipments per month.
122. a. $405 - 30 = 375$
 There were about 375 more California Condors in 2012 than in 1987. This is a change of 375 condors.
- b. This is a period of 25 years.
- $$\frac{375}{25} = 15 \qquad \begin{array}{r} 15 \\ 25 \overline{) 375} \\ \underline{-25} \\ 125 \\ \underline{-125} \\ 0 \end{array}$$
- The average change was 15 California Condors per year.
124. $3 \cdot (7 - 4) + 2 \cdot 5^2 = 3 \cdot 3 + 2 \cdot 5^2$
 $= 3 \cdot 3 + 2 \cdot 25$
 $= 9 + 2 \cdot 25$
 $= 9 + 50$
 $= 59$
126. $12 \div (4 - 2) + 7 = 12 \div 2 + 7 = 6 + 7 = 13$
128. $-9(-11) = 99$
130. $-4 + (-3) + 21 = -7 + 21 = 14$
132. $-16 - (-2) = -16 + 2 = -14$

134. The product of an even number of negative numbers is positive, so the product of ten negative numbers is positive.

136. $(-1)^{50}$ and $(-7)^{20}$ are positive since there are an even number of factors. Note that $(-7)^{20} > (-1)^{50}$ since $(-1)^{50} = 1$.

$(-1)^{55}$ and $(-7)^{23}$ are negative since there are an odd number of factors. Note that $(-7)^{23} < (-1)^{55}$ since $(-1)^{55} = -1$.

$$0^{15} = 0$$

The numbers from least to greatest are

$$(-7)^{23}, (-1)^{55}, 0^{15}, (-1)^{50}, (-7)^{20}.$$

138. answers may vary

Section 2.6 Practice Exercises

- $(-2)^4 = (-2)(-2)(-2)(-2) = 16$
- $-2^4 = -(2)(2)(2)(2) = -16$
- $3 \cdot 6^2 = 3 \cdot (6 \cdot 6) = 3 \cdot 36 = 108$
- $\frac{-25}{5(-1)} = \frac{-25}{-5} = 5$
- $\frac{-18+6}{-3-1} = \frac{-12}{-4} = 3$
- $30+50+(-4)^3 = 30+50+(-64)$
 $= 80+(-64)$
 $= 16$
- $-2^3 + (-4)^2 + 1^5 = -8+16+1 = 8+1 = 9$
- $2(2-9)+(-12)-\sqrt{9} = 2(-7)+(-12)-3$
 $= -14+(-12)-3$
 $= -26-3$
 $= -29$
- $(-5) \cdot |-8| + (-3) + 2^3 = (-5) \cdot 8 + (-3) + 2^3$
 $= (-5) \cdot 8 + (-3) + 8$
 $= -40 + (-3) + 8$
 $= -43 + 8$
 $= -35$

$$\begin{aligned} 10. \quad -4[-6+5(-3+5)]-7 &= -4[-6+5(2)]-7 \\ &= -4[-6+10]-7 \\ &= -4(4)-7 \\ &= -16-7 \\ &= -23 \end{aligned}$$

$$\begin{aligned} 11. \quad x^2 &= (-15)^2 = (-15)(-15) = 225 \\ -x^2 &= -(-15)^2 = -(-15)(-15) = -225 \end{aligned}$$

$$\begin{aligned} 12. \quad 5y^2 &= 5(4)^2 = 5(16) = 80 \\ 5y^2 &= 5(-4)^2 = 5(16) = 80 \end{aligned}$$

$$\begin{aligned} 13. \quad x-y+3z &= -6-(-3)+3(12) \\ &= -6+3+36 \\ &= 33 \end{aligned}$$

$$14. \quad 4-x^2 = 4-(-8)^2 = 4-64 = -60$$

$$\begin{aligned} 15. \quad \text{average} &= \frac{\text{sum of numbers}}{\text{number of numbers}} \\ &= \frac{17+(-1)+(-11)+(-13)+(-16)+(-13)+2}{7} \\ &= \frac{-35}{7} \\ &= -5 \end{aligned}$$

The average of the temperatures is -5°F .

Calculator Explorations

- $\frac{-120-360}{-10} = 48$
- $\frac{4750}{-2+(-17)} = -250$
- $\frac{-316+(-458)}{28+(-25)} = -258$
- $\frac{-234+86}{-18+16} = 74$

Vocabulary, Readiness & Video Check 2.6

- To simplify $-2 \div 2 \cdot (3)$, which operation should be performed first? division
- To simplify $-9 - 3 \cdot 4$, which operation should be performed first? multiplication

3. The average of a list of numbers is $\frac{\text{sum of numbers}}{\text{number of numbers}}$.
4. To simplify $5[-9 + (-3)] \div 4$, which operation should be performed first? addition
5. To simplify $-2 + 3(10 - 12) \cdot (-8)$, which operation would be performed first? subtraction
6. To evaluate $x - 3y$ for $x = -7$ and $y = -1$, replace x with -7 and y with -1 and evaluate $-7 - 3(-1)$.
7. In the expression -3^2 , the base is 3 and the exponent is 2.
8. In the expression $(-3)^2$, the base is -3 and the exponent is 2.
9. In the expression $4 \cdot 2^3$, one base is 2 and its exponent is 3. The other base is 4 and its exponent is 1.
10. In the expression $9 \cdot 5^6$, one base is 5 and its exponent is 6. The other base is 9 and its exponent is 1.
11. In the expression $(-7)^5$, the base is -7 and the exponent is 5.
12. In the expression -9^4 , the base is 9 and the exponent is 4.
13. In the expression $5^7 \cdot 10$, one base is 5 and its exponent is 7. The other base is 10 and its exponent is 1.
14. In the expression $2^8 \cdot 11$, one base is 2 and its exponent is 8. The other base is 11 and its exponent is 1.
15. A fraction bar means “divided by” and is a grouping symbol.
16. To make sure that the entire value of -2 , including the sign, is squared.
17. Finding the average is a good application of both order of operations and adding and dividing integers.

Exercise Set 2.6

2. $-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2) = -16$
4. $(-2)^4 = (-2)(-2)(-2)(-2) = 16$
6. $5 \cdot 2^3 = 5 \cdot 2^3 = 5 \cdot 8 = 40$
8. $(-1)(-2) + 1 = 2 + 1 = 3$
10. $10 - 23 - 12 = 10 + (-23) + (-12)$
 $= -13 + (-12)$
 $= -25$
12. $-8 + 4(3) = -8 + 12 = 4$
14. $7(-6) + 3 = -42 + 3 = -39$
16. $(-12) + 6 \div 3 = (-12) + 2 = -10$
18. $5 + 9 \cdot 4 - 52 = 5 + 36 + (-52) = 41 + (-52) = -11$
20. $\frac{20-15}{-1} = \frac{5}{-1} = -5$
22. $\frac{88}{-8-3} = \frac{88}{-11} = -8$
24. $7(-4) - (-6) = -28 - (-6) = -28 + 6 = -22$
26. $(-24) - 14(2) = -24 - 28 = -52$
28. $-12 + 3^3 = -12 + 27 = 15$
30. $[9 + (-2)]^3 = [7]^3 = 343$
32. $7 \cdot 6 - 6 \cdot 5 + (-10) = 42 - 6 \cdot 5 + (-10)$
 $= 42 - 30 + (-10)$
 $= 12 + (-10)$
 $= 2$
34. $20 - (-5)^2 = 20 - 25 = 20 + (-25) = -5$
36. $|-3 + 7| \cdot 7^2 = |4| \cdot 7^2 = 4 \cdot 7^2 = 4 \cdot 49 = 196$
38. $10 \cdot 5^3 + 7 = 10 \cdot 125 + 7 = 1250 + 7 = 1257$
40. $8^2 - (5 - 2)^4 = 64 - (3)^4 = 64 - 81 = -17$
42. $|12 - 19| \div 7 = |-7| \div 7 = 7 \div 7 = 1$

$$44. -(-2)^3 = -(-8) = 8$$

$$46. (2-7)^2 \div (4-3)^4 = (-5)^2 \div (1)^4 = 25 \div 1 = 25$$

$$\begin{aligned} 48. |3-15| \cdot (-4) \div (-16) &= |-12| \cdot (-4) \div (-16) \\ &= 12 \cdot (-4) \div (-16) \\ &= -48 \div (-16) \\ &= 3 \end{aligned}$$

$$50. (-20-5) \div 5 - 15 = (-25) \div 5 - 15 = -5 - 15 = -20$$

$$\begin{aligned} 52. 3 \cdot (8-3) + (-4) - 10 &= 3 \cdot (5) + (-4) - 10 \\ &= 15 + (-4) - 10 \\ &= 11 - 10 \\ &= 1 \end{aligned}$$

$$54. (4-12) \cdot (8-17) = (-8) \cdot (-9) = 72$$

$$56. 4 - 12 \cdot 8 - 17 = 4 - 96 - 17 = -92 - 17 = -109$$

$$58. (-4 \div 4) - (8 \div 8) = -1 - 1 = -2$$

$$60. -4^4 - 5^4 = -256 - 625 = -881$$

$$62. (-4)^4 - 5^4 = 256 - 625 = -369$$

$$64. (11-3^2)^3 = (11-9)^3 = (2)^3 = 8$$

$$\begin{aligned} 66. -3(4-8)^2 + 5(14-16)^3 &= -3(-4)^2 + 5(-2)^3 \\ &= -3(16) + 5(-8) \\ &= -48 - 40 \\ &= -88 \end{aligned}$$

$$\begin{aligned} 68. 12 - [7 - (3-6)] + (2-3)^3 &= 12 - [7 - (-3)] + (-1)^3 \\ &= 12 - [7+3] + (-1) \\ &= 12 - 10 + (-1) \\ &= 2 + (-1) \\ &= 1 \end{aligned}$$

$$\begin{aligned} 70. \frac{10(-1) - (-2)(-3)}{2[-8 \div (-2-2)]} &= \frac{-10-6}{2[-8 \div (-4)]} \\ &= \frac{-16}{2[2]} \\ &= \frac{-16}{4} \\ &= -4 \end{aligned}$$

$$\begin{aligned} 72. -2[1+3(7-12)] - 35 &= -2[1+3(-5)] - 35 \\ &= -2[1+(-15)] - 35 \\ &= -2[-14] - 35 \\ &= 28 - 35 \\ &= -7 \end{aligned}$$

$$\begin{aligned} 74. -2[6+4(2-8)] - 25 &= -2[6+4(-6)] - 25 \\ &= -2[6+(-24)] - 25 \\ &= -2(-18) - 25 \\ &= 36 - 25 \\ &= 11 \end{aligned}$$

$$\begin{aligned} 76. x - y - z &= -2 - 4 - (-1) \\ &= -2 - 4 + 1 \\ &= -6 + 1 \\ &= -5 \end{aligned}$$

$$\begin{aligned} 78. 5x - y + 4z &= 5(-2) - 4 + 4(-1) \\ &= -10 - 4 + (-4) \\ &= -14 + (-4) \\ &= -18 \end{aligned}$$

$$80. x^2 + z = (-2)^2 + (-1) = 4 + (-1) = 3$$

$$82. \frac{4x}{y} = \frac{4(-2)}{4} = \frac{-8}{4} = -2$$

$$84. z^2 = (-4)^2 = 16$$

$$86. -x^2 = -(-3)^2 = -9$$

$$88. 3 - z^2 = 3 - (-4)^2 = 3 - 16 = -13$$

$$90. 3z^2 - x = 3(-4)^2 - (-3) = 3(16) + 3 = 48 + 3 = 51$$

$$\begin{aligned} 92. \text{average} &= \frac{-18 + (-8) + (-1) + (-1) + 0 + 4}{6} \\ &= \frac{-24}{6} \\ &= -4 \end{aligned}$$

$$\begin{aligned} 94. \text{average} &= \frac{-40 + (-20) + (-10) + (-15) + (-5)}{5} \\ &= \frac{-90}{5} \\ &= -18 \end{aligned}$$

96. The two lowest scores are -12 and -5 .
 $-5 - (-12) = -5 + 12 = 7$
 The difference between the two lowest scores is 7.

98.
$$\text{average} = \frac{-12 + (-5) + (-1) + 6}{4} = \frac{-12}{4} = -3$$

The average of the scores is -3 .

100. no; answers may vary

102. $90 \div 45 = 2$

104. $45 + 90 = 135$

106. $3 + 5 + 3 + 5 = 16$
 The perimeter is 16 centimeters.

108. $17 + 23 + 32 = 72$
 The perimeter is 72 meters.

110. $(7 \cdot 3 - 4) \cdot 2 = (21 - 4) \cdot 2 = 17 \cdot 2 = 34$

112. $2 \cdot (8 \div 4 - 20) = 2 \cdot (2 - 20) = 2 \cdot (-18) = -36$

114. yes; answers may vary

116. answers may vary

118. $(-17)^6 = (-17)(-17)(-17)(-17)(-17)(-17)$
 $= 24,137,569$

120. $3x^2 + 2x - y = 3(-18)^2 + 2(-18) - 2868$
 $= 3(324) + (-36) - 2868$
 $= 972 + (-36) - 2868$
 $= 936 - 2868$
 $= -1932$

122. $5(ab + 3)^b = 5(-2 \cdot 3 + 3)^3$
 $= 5(-6 + 3)^3$
 $= 5(-3)^3$
 $= 5(-27)$
 $= -135$

Chapter 2 Vocabulary Check

- Two numbers that are the same distance from 0 on the number line but are on opposite sides of 0 are called opposites.
- Together, positive numbers, negative numbers, and 0 are called signed numbers.

3. The absolute value of a number is that number's distance from 0 on a number line.

4. The integers are $\dots -3, -2, -1, 0, 1, 2, 3, \dots$

5. A letter used to represent a number is called a variable.

6. The negative numbers are numbers less than zero.

7. The positive numbers are numbers greater than zero.

Chapter 2 Review

1. $\frac{2x}{z} = \frac{2(5)}{2} = \frac{10}{2} = 5$

2. $4x - 3 = 4(5) - 3 = 20 - 3 = 17$

3. $\frac{x+7}{y} = \frac{5+7}{0} = \frac{12}{0}$ is undefined.

4. $\frac{y}{5x} = \frac{0}{5(5)} = \frac{0}{25} = 0$

5. $x^3 - 2z = (5)^3 - 2(2) = 125 - 4 = 121$

6. $\frac{7+x}{3z} = \frac{7+5}{3(2)} = \frac{12}{6} = 2$

7. $(y+z)^2 = (0+2)^2 = (2)^2 = 4$

8. $\frac{100}{x} + \frac{y}{3} = \frac{100}{(5)} + \frac{(0)}{3} = 20 + 0 = 20$

9. Five subtracted from a number is written $x - 5$.

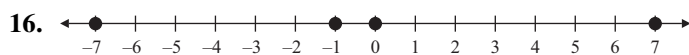
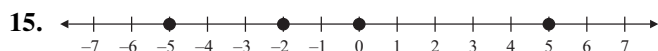
10. Seven more than a number is written as $x + 7$.

11. Ten divided by a number is written $10 \div x$ or $\frac{10}{x}$.

12. The product of 5 and a number is written $5x$.

13. 1435 feet down a mine is written as -1435 .

14. 11,239 feet above sea level is written $+11,239$ or 11,239.



17. $|-12| = 12$

18. $|0| = 0$

19. $-|6| = -6$

20. $-(-9) = 9$

21. $-|-9| = -9$

22. $-(-2) = 2$

23. $-18 > -20$ since -18 is to the right of -20 on a number line.

24. $-5 < 5$ since -5 is to the left of 5 on a number line.

25. Since $|-123| = 123$ and $-|-198| = -198$ and $123 > -198$, then $|-123| > -|-198|$.

26. Since $8 - |-12| = 8 - 12 = -4$ and $-|-16| = -16$ and $-4 > -16$, then $8 - |-12| > -|-16|$.

27. The opposite of -12 is 12 .

28. Since $-(-3) = 3$, then the opposite of $-(-3)$ is -3 .

29. The statement $a < b$ is false. $0 < 15$, and 0 is not a negative number.

30. True; the absolute value of an integer is always 0 or a positive number. Absolute value represents that number's distance from 0 , and distance cannot be negative.

31. True; a negative number is always less than a positive number.

32. True; if a is a negative number, then $-a$ is a positive number.

33. $5 + (-3) = 2$

34. $18 + (-4) = 14$

35. $-12 + 16 = 4$

36. $-23 + 40 = 17$

37. $-8 + (-15) = -23$

38. $-5 + (-17) = -22$

39. $-24 + 3 = -21$

40. $-89 + 19 = -70$

41. $15 + (-15) = 0$

42. $-24 + 24 = 0$

43. $-43 + (-108) = -151$

44. $-100 + (-506) = -606$

45. $-15 - 5 = -20$

The temperature at 6 A.M. was -20° Celsius.

46. $-127 - 23 = -150$

The diver was at -150 ft or 150 feet below sea level.

47. $-3 + 2 + (-4) + (-3) = -1 + (-7) = -8$

Bubba Watson had a total score of -8 for the tournament.

48. $-14 + 3 = -11$

Jason Bohn had a score of -11 .

49. $-56 + 155 = 99$

London's high temperature is 99°F .

50. $-13 + 123 = 110$

The high temperature for Los Angeles is 110°F .

51. $12 - 4 = 12 + (-4) = 8$

52. $-12 - 4 = -12 + (-4) = -16$

53. $8 - 19 = 8 + (-19) = -11$

54. $-8 - 19 = -8 + (-19) = -27$

55. $7 - (-13) = 7 + 13 = 20$

56. $-6 - (-14) = -6 + 14 = 8$

57. $16 - 16 = 0$

58. $-16 - 16 = -16 + (-16) = -32$

59. $-12 - (-12) = -12 + 12 = 0$

60. $|-5| - |-12| = 5 - 12 = 5 + (-12) = -7$

61. $\begin{aligned} -(-5) - 12 + (-3) &= 5 + (-12) + (-3) \\ &= -7 + (-3) \\ &= -10 \end{aligned}$

62. $\begin{aligned} -8 + |-12| - 10 - |-3| &= -8 + 12 - 10 - 3 \\ &= 4 - 10 - 3 \\ &= -6 - 3 \\ &= -9 \end{aligned}$

63. $142 - 125 + 43 - 85 = 17 + 43 - 85 = 60 - 85 = -25$

Josh has a $-\$25$ balance in his checkbook.

$$\begin{array}{rcl}
 \text{64. } \boxed{\begin{array}{l} \text{Difference in elevation} \\ \text{of Lake Superior and} \\ \text{Caspian Sea} \end{array}} & = & \boxed{\begin{array}{l} \text{Elevation of} \\ \text{Lake Superior} \end{array}} - \boxed{\begin{array}{l} \text{Elevation of} \\ \text{Caspian Sea} \end{array}} \\
 & = & 600 - (-92) \\
 & = & 600 + 92 \\
 & = & 692
 \end{array}$$

There is a difference of 692 feet in the elevations of Lake Superior and the Caspian Sea.

$$65. \quad 32 - 35 = -3$$

The low temperature for Reykjavik is -3°F .

$$66. \quad 10 - 14 = -4$$

The low temperature for Berlin, Germany is -4°F .

$$67. \quad |-5| - |-6| = 5 - 6$$

Since $|-5| = 5$ and $|-6| = 6$, then $|-5| - |-6| = 5 - 6$ so the statement is true.

$$68. \quad |-5 - (-6)| = |-5 + 6| = |1| = 1$$

$$|-5 - (-6)| \neq 5 + 6$$

This statement is false.

69. $b > a$, then $b - a$ is a positive number. $6 > 5$, and $6 - 5 = 1$, and 1 is a positive number. This statement is true.

70. If $b < a$, then $b - a$ is a negative number. $5 < 6$ and $5 - 6$ is -1 , and -1 is a negative number. The statement is true.

$$71. \quad -3(-7) = 21$$

$$72. \quad -6(3) = -18$$

$$73. \quad -4(16) = -64$$

$$74. \quad -5(-12) = 60$$

$$75. \quad (-5)^2 = (-5)(-5) = 25$$

$$76. \quad (-1)^5 = -1 \text{ since there are an odd number of factors.}$$

$$77. \quad 12(-3)(0) = 0$$

$$78. \quad -1(6)(2)(-2) = -6(2)(-2) = -12(-2) = 24$$

$$79. \quad -15 \div 3 = \frac{-15}{3} = -5$$

$$80. \quad \frac{-24}{-8} = 3$$

$$81. \quad \frac{0}{-3} = 0$$

82. $\frac{-46}{0}$ is undefined.

83. $\frac{100}{-5} = -20$

84. $\frac{-72}{8} = -9$

85. $\frac{-38}{-1} = 38$

86. $\frac{45}{-9} = -5$

87. $2(-5) = -10$

88. $(-50)(4) = -200$

89. $2(-9) = -18$
The low for Bucharest, Romania is -18°F .

90. $\frac{30}{-10} = -3$
The low temperature for Geneva, Switzerland is -3°F .

91. $\frac{-56}{-2} = 28$
The low temperature for Capetown, South Africa is 28°F .

92. $2(-13) = -26$
The low temperature for Stockholm, Sweden is -26°F .

93. $(-7)^2 = (-7)(-7) = 49$

94. $-7^2 = -(7 \cdot 7) = -49$

95. $-2^5 = -(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) = -32$

96. $(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$

97. $5 - 8 + 3 = 5 + (-8) + 3 = -3 + 3 = 0$

98. $-3 + 12 + (-7) - 10 = 9 + (-7) - 10 = 2 - 10 = -8$

99. $-10 + 3 \cdot (-2) = -10 + (-6) = -16$

100. $5 - 10 \cdot (-3) = 5 - (-30) = 5 + 30 = 35$

101. $16 \cdot (-2) + 4 = -32 + 4 = -28$

102. $3 \cdot (-12) - 8 = -36 - 8 = -44$

103. $5 + 6 \div (-3) = 5 + (-2) = 3$

104. $-6 + (-10) \div (-2) = -6 + 5 = -1$

105. $16 + (-3) \cdot 12 \div 4 = 16 + (-36) \div 4 = 16 + (-9) = 7$

106. $-12 + 25 \cdot 1 \div (-5) = -12 + 25 \div (-5)$
 $= -12 + (-5)$
 $= -17$

107. $4^3 - (8 - 3)^2 = 4^3 - (5)^2 = 64 - 25 = 39$

108. $4^3 - 90 = 64 - 90 = -26$

109. $-(-4) \cdot |-3| - 5 = 4 \cdot |-3| - 5 = 4 \cdot 3 - 5 = 12 - 5 = 7$

110. $|5 - 1|^2 \cdot (-5) = |4|^2 \cdot (-5)$
 $= 4^2 \cdot (-5)$
 $= 16 \cdot (-5)$
 $= -80$

111. $\frac{(-4)(-3) - (-2)(-1)}{-10 + 5} = \frac{12 - 2}{-5} = \frac{10}{-5} = -2$

112. $\frac{4(12 - 18)}{-10 \div (-2 - 3)} = \frac{4(-6)}{-10 \div (-5)} = \frac{-24}{2} = -12$

113. $\frac{-18 + 25 + (-30) + 7 + 0 + (-2)}{6} = \frac{-18}{6} = -3$

114. $\frac{-45 + (-40) + (-30) + (-25)}{4} = \frac{-140}{4} = -35$

115. $2x - y = 2(-2) - 1 = -4 - 1 = -5$

116. $y^2 + x^2 = (1)^2 + (-2)^2 = 1 + 4 = 5$

117. $\frac{3x}{6} = \frac{3(-2)}{6} = \frac{-6}{6} = -1$

118. $\frac{5y - x}{-y} = \frac{5(1) - (-2)}{-(-1)} = \frac{5 + 2}{-1} = \frac{7}{-1} = -7$

119. $x^2 = (-2)^2 = 4$

120. $-x^2 = -(-2)^2 = -4$

$$121. 7 - x^2 = 7 - (-2)^2 = 7 - 4 = 3$$

$$122. 100 - x^3 = 100 - (-2)^3 \\ = 100 - (-8) \\ = 100 + 8 \\ = 108$$

$$123. (-4)^2 = (-4)(-4) = 16$$

$$124. -4^2 = -(4 \cdot 4) = -16$$

$$125. (-6) + (-9) = -15$$

$$126. -16 - 3 = -16 + (-3) = -19$$

$$127. -4(-12) = 48$$

$$128. \frac{84}{-4} = -21$$

$$129. -76 - (-97) = -76 + 97 = 21$$

$$130. -9 + 4 = -5$$

131. The bar that extends the farthest in the negative direction corresponds to Elevator D, so Elevator D extends the farthest below ground.

132. The bar that extends the farthest in the positive direction corresponds to Elevator B, so Elevator B extends the highest above the ground.

133. $12,923 - (-195) = 12,923 + 195 = 13,118$
There is a difference of 13,118 feet between the top of the mountain and the bottom of the valley.

134. $-18 - 9 = -27$
The temperature on Friday was -27°C .

$$135. (3 - 7)^2 \div (6 - 4)^3 = (-4)^2 \div (2)^3 = 16 \div 8 = 2$$

$$136. (4 + 6)^2 \div (2 - 7)^2 = (10)^2 \div (-5)^2 = 100 \div 25 = 4$$

$$137. 3(4 + 2) + (-6) - 3^2 = 3(6) + (-6) - 9 \\ = 18 + (-6) - 9 \\ = 12 - 9 \\ = 3$$

$$138. 4(5 - 3) - (-2) + 3^3 = 4(2) - (-2) + 27 \\ = 8 + 2 + 27 \\ = 10 + 27 \\ = 37$$

$$139. 2 - 4 \cdot 3 + \sqrt{25} = 2 - 12 + 5 = -10 + 5 = -5$$

$$140. 4 - 6 \cdot 5 + \sqrt{1} = 4 - 30 + 1 = -26 + 1 = -25$$

$$141. \frac{-|-14| - 6}{7 + 2(-3)} = \frac{-14 - 6}{7 + (-6)} = \frac{-20}{1} = -20$$

$$142. 5(7 - 6)^3 - 4(2 - 3)^2 + 2^4 = 5(1)^3 - 4(-1)^2 + 16 \\ = 5 \cdot 1 - 4 \cdot 1 + 16 \\ = 5 - 4 + 16 \\ = 1 + 16 \\ = 17$$

Chapter 2 Test

$$1. -5 + 8 = 3$$

$$2. 18 - 24 = 18 + (-24) = -6$$

$$3. 5 \cdot (-20) = -100$$

$$4. (-16) \div (-4) = 4$$

$$5. (-18) + (-12) = -30$$

$$6. -7 - (-19) = -7 + 19 = 12$$

$$7. (-5) \cdot (-13) = 65$$

$$8. \frac{-25}{-5} = 5$$

$$9. |-25| + (-13) = 25 + (-13) = 12$$

$$10. 14 - |-20| = 14 - 20 = 14 + (-20) = -6$$

$$11. |5| \cdot |-10| = 5 \cdot 10 = 50$$

$$12. \frac{|-10|}{-|-5|} = \frac{10}{-5} = -2$$

$$13. (-8) + 9 \div (-3) = (-8) + 9 \div (-3) = -8 + (-3) = -11$$

$$14. -7 + (-32) - 12 + 5 = -39 + (-12) + 5 \\ = -51 + 5 \\ = -46$$

$$15. (-5)^3 - 24 \div (-3) = -125 - 24 \div (-3) \\ = -125 - (-8) \\ = -125 + 8 \\ = -117$$

$$16. \quad (5-9)^2 \cdot (8-2)^3 = (-4)^2 \cdot (6)^3 = 16 \cdot 216 = 3456$$

$$\begin{array}{r} 216 \\ \times 16 \\ \hline 1296 \\ 2160 \\ \hline 3456 \end{array}$$

$$17. \quad -(-7)^2 \div 7 \cdot (-4) = -49 \div 7 \cdot (-4) = -7 \cdot (-4) = 28$$

$$18. \quad \begin{aligned} 3 - (8 - 2)^3 &= 3 - (6)^3 \\ &= 3 - 216 \\ &= 3 + (-216) \\ &= -213 \end{aligned}$$

$$19. \quad -6 + (-15) \div (-3) = -6 + (5) = -1$$

$$20. \quad \frac{4}{2} - \frac{8^2}{16} = \frac{4}{2} - \frac{64}{16} = 2 - 4 = 2 + (-4) = -2$$

$$21. \quad \frac{-3(-2) + 12}{-1(-4 - 5)} = \frac{6 + 12}{-1(-9)} = \frac{18}{9} = 2$$

$$22. \quad \frac{|25 - 30|^2}{2(-6) + 7} = \frac{|-5|^2}{-12 + 7} = \frac{5^2}{-5} = \frac{25}{-5} = -5$$

$$23. \quad \begin{aligned} 5(-8) - [6 - (2 - 4)] + (12 - 16)^2 &= 5(-8) - [6 - (-2)] + (-4)^2 \\ &= 5(-8) - [6 + 2] + 16 \\ &= -40 - 8 + 16 \\ &= -48 + 16 \\ &= -32 \end{aligned}$$

$$24. \quad -2^3 - 2^2 = -8 - 4 = -8 + (-4) = -12$$

$$25. \quad 3x + y = 3(0) + (-3) = 0 + (-3) = -3$$

$$26. \quad |y| + |x| + |z| = |-3| + |0| + |2| = 3 + 0 + 2 = 5$$

$$27. \quad \frac{3z}{2y} = \frac{3(2)}{2(-3)} = \frac{6}{-6} = -1$$

$$28. \quad 2y^3 = 2(-3)^3 = 2(-27) = -54$$

$$29. \quad 10 - y^2 = 10 - (-3)^2 = 10 - 9 = 1$$

$$\begin{aligned}
 30. \quad 7x + 3y - 4z &= 7(0) + 3(-3) - 4(2) \\
 &= 0 + (-9) - 8 \\
 &= -9 - 8 \\
 &= -17
 \end{aligned}$$

$$31. \quad 4(-22) = -88$$

Mary was at -88 ft after the descents.

$$\begin{aligned}
 32. \quad 129 - 79 - 40 + 35 &= 129 + (-79) + (-40) + 35 \\
 &= 129 + 35 + (-79) + (-40) \\
 &= 164 + (-119) \\
 &= 45
 \end{aligned}$$

The new balance is $+\$45$ or $\$45$.

$$\begin{aligned}
 33. \quad \boxed{\text{Difference in elevation of}} &= \boxed{\text{Elevation of}} - \boxed{\text{Elevation of the}} \\
 \boxed{\text{Mt. Washington and the}} & \quad \boxed{\text{Mt. Washington}} \quad - \quad \boxed{\text{Romanche Gap}} \\
 \boxed{\text{Romanche Gap}} & \\
 &= 6288 - (-25,354) \\
 &= 6288 + 25,354 \\
 &= 31,642
 \end{aligned}$$

The difference in the elevations is 31,642 feet.

$$\begin{aligned}
 34. \quad \boxed{\text{Difference in elevation of}} &= \boxed{\text{Elevation of}} - \boxed{\text{Depth of}} \\
 \boxed{\text{Lake Baykal and its depth}} & \quad \boxed{\text{Lake Baykal}} \quad - \quad \boxed{\text{Lake Baykal}} \\
 &= 1495 - 5315 \\
 &= -3820
 \end{aligned}$$

The deepest point in the lake is 3820 ft below sea level.

$$\begin{aligned}
 35. \quad \frac{-12 + (-13) + 0 + 9}{4} &= \frac{-25 + 0 + 9}{4} \\
 &= \frac{-25 + 9}{4} \\
 &= \frac{-16}{4} \\
 &= -4
 \end{aligned}$$

36. a. The product of a number and 17 is written $17x$.

b. Twice a number subtracted from 20 is written $20 - 2x$.

Cumulative Review Chapters 1–2

1. The place value of 3 in 396,418 is hundred-thousands.
2. The place value of 3 in 4308 is hundreds.
3. The place value of 3 in 93,192 is thousands.
4. The place value of 3 in 693,298 is thousands.
5. The place value of 3 in 534,275,866 is ten-millions.
6. The place value of 3 in 267,301,818 is hundred-thousands.

7. a. $-7 < 7$ since -7 is to the left of 7 on a number line.
 b. $0 > -4$ since 0 is to the right of -4 on a number line.
 c. $-9 > -11$ since -9 is to the right of -11 on a number line.
8. a. $12 > 4$ since 12 is to the right of 4 on a number line.
 b. $13 < 31$ since 13 is to the left of 31 on a number line.
 c. $82 > 79$ since 82 is to the right of 79 on a number line.

$$\begin{array}{r} 2 \\ 9. \quad 13 \\ 2 \\ 7 \\ 8 \\ + 9 \\ \hline 39 \end{array}$$

$$\begin{array}{r} 1 \\ 10. \quad 11 \\ 3 \\ 9 \\ + 16 \\ \hline 39 \end{array}$$

$$\begin{array}{r} 11. \quad 7826 \\ - 505 \\ \hline 7321 \end{array} \quad \begin{array}{r} \text{Check: } 7321 \\ + 505 \\ \hline 7826 \end{array}$$

$$\begin{array}{r} 12. \quad 3285 \\ - 272 \\ \hline 3013 \end{array} \quad \begin{array}{r} \text{Check: } 3013 \\ + 272 \\ \hline 3285 \end{array}$$

$$\begin{array}{r} 3 \text{ } 13 \quad 3 \text{ } 13 \text{ } 11 \\ 13. \quad \cancel{43}, \cancel{44} \cancel{1} \\ - 7 \text{ } 25 \text{ } 7 \\ \hline 36,184 \end{array}$$

$$\begin{array}{r} 5 \text{ } 12 \\ 14. \quad \cancel{762} \\ - 237 \\ \hline 525 \end{array}$$

C.J. will have \$525 in her account after buying the camera.

15. To round 568 to the nearest ten, observe that the digit in the ones place is 8. Since 8 is at least 5, add 1 to the digit in the tens place. 568 rounded to the nearest ten is 570.

16. To round 568 to the nearest hundred, observe that the digit in the tens place is 6. Since 6 is at least 5, add 1 to the digit in the hundreds place. 568 rounded to the nearest hundred is 600.

$$\begin{array}{r} 3 \text{ } 17 \\ 17. \quad 4725 \approx \cancel{4700} \\ - 2879 \approx 2900 \\ \hline 1800 \end{array}$$

$$\begin{array}{r} 18. \quad 8394 \approx 8000 \\ - 2913 \approx -3000 \\ \hline 5000 \end{array}$$

19. a. $3(4 + 5) = 3 \cdot 4 + 3 \cdot 5$

b. $10(6 + 8) = 10 \cdot 6 + 10 \cdot 8$

c. $2(7 + 3) = 2 \cdot 7 + 2 \cdot 3$

20. a. $5(2 + 12) = 5 \cdot 2 + 5 \cdot 12$

b. $9(3 + 6) = 9 \cdot 3 + 9 \cdot 6$

c. $4(8 + 1) = 4 \cdot 8 + 4 \cdot 1$

$$\begin{array}{r} 21. \quad 631 \\ \times 125 \\ \hline 3155 \\ 12620 \\ 63100 \\ \hline 78,875 \end{array}$$

$$\begin{array}{r} 22. \quad 299 \\ \times 104 \\ \hline 1196 \\ 0000 \\ 29900 \\ \hline 31,096 \end{array}$$

23. a. $42 \div 7 = 6$

Check: $6 \times 7 = 42$

b. $\frac{64}{8} = 8$

Check: $8 \times 8 = 64$

$$\text{c. } \begin{array}{r} 7 \\ 3 \overline{)21} \\ \underline{21} \\ 00 \end{array}$$

$$\text{Check: } 3 \times 7 = 21$$

$$24. \text{ a. } \frac{35}{5} = 7$$

$$\text{Check: } 5 \times 7 = 35$$

$$\text{b. } 64 \div 8 = 8$$

$$\text{Check: } 8 \times 8 = 64$$

$$\text{c. } \begin{array}{r} 12 \\ 4 \overline{)48} \\ \underline{4} \\ 08 \\ \underline{08} \\ 00 \end{array}$$

$$\text{Check: } 4 \times 12 = 48$$

$$25. \begin{array}{r} 741 \\ 5 \overline{)3705} \\ \underline{35} \\ 20 \\ \underline{20} \\ 05 \\ \underline{5} \\ 0 \end{array}$$

$$\text{Check: } \begin{array}{r} 741 \\ \times 5 \\ \hline 3705 \end{array}$$

$$26. \begin{array}{r} 456 \\ 8 \overline{)3648} \\ \underline{32} \\ 44 \\ \underline{40} \\ 48 \\ \underline{48} \\ 0 \end{array}$$

$$\text{Check: } \begin{array}{r} 456 \\ \times 8 \\ \hline 3648 \end{array}$$

$$27. \begin{array}{r} 12 \\ 19 \overline{)238} \\ \underline{-19} \\ 48 \\ \underline{-38} \\ 10 \end{array}$$

Each will receive 12 cards and there will be 10 left over.

$$28. \begin{array}{r} 9 \\ 36 \overline{)324} \\ \underline{324} \\ 0 \end{array}$$

Mrs. Mallory paid \$9.00 for each ticket.

$$29. 9^2 = 9 \cdot 9 = 81$$

$$30. 5^3 = 5 \cdot 5 \cdot 5 = 25 \cdot 5 = 125$$

$$31. 6^1 = 6$$

$$32. 4^1 = 4$$

$$33. 5 \cdot 6^2 = 5 \cdot 36 = 180$$

$$34. 2^3 \cdot 7 = 8 \cdot 7 = 56$$

$$35. \frac{7 - 2 \cdot 3 + 3^2}{5(2-1)} = \frac{7 - 6 + 9}{5(1)} = \frac{1+9}{5} = \frac{10}{5} = 2$$

$$36. \frac{6^2 + 4 \cdot 4 + 2^3}{37 - 5^2} = \frac{36 + 16 + 8}{37 - 25} = \frac{52 + 8}{12} = \frac{60}{12} = 5$$

$$37. x + 7 = 8 + 7 = 15$$

$$38. 5 + x = 5 + 9 = 14$$

$$39. \text{ a. } |-2| = 2$$

$$\text{b. } |8| = 8$$

$$\text{c. } |0| = 0$$

$$40. \text{ a. } |4| = 4$$

$$\text{b. } |-7| = 7$$

$$41. -14 + 35 = 21$$

$$42. 8 + (-3) = 5$$

$$43. 2a - b = 2(8) - (-6) = 16 - (-6) = 16 + 6 = 22$$

$$44. x - y = -2 - (-7) = -2 + 7 = 5$$

$$45. -7 \cdot 3 = -21$$

$$46. 5(-2) = -10$$

$$47. 0 \cdot (-4) = 0$$

$$48. -6 \cdot 9 = -54$$

$$\begin{aligned} 49. 3(4-7) + (-2) - \sqrt{25} &= 3(-3) + (-2) - 5 \\ &= -9 + (-2) + (-5) \\ &= -11 + (-5) \\ &= -16 \end{aligned}$$

50. $4 - 8(7 - 3) - (-1) = 4 - 8(4) + 1$
 $= 4 - 32 + 1$
 $= 4 + (-32) + 1$
 $= -28 + 1$
 $= -27$

Mini-Lecture 1.1

Study Skill Tips for Success in Mathematics

Learning Objectives:

1. Get ready for this course.
2. Understand some general tips for success.
3. Know how to use this text.
4. Know how to use text resources.
5. Get help as soon as you need it.
6. Learn how to prepare for and take an exam.
7. Develop good time management.

Examples:

1. Get ready for this course.
 - a) Positive attitude
 - b) Allow adequate time for class arrival
 - c) Bring all required material
2. Understand some general tips for success.
 - a) Organize class materials
 - b) Form study groups; exchange names and email addresses
 - c) Attend all classes
 - d) Do your homework
 - e) Check your work; learn from mistakes
 - f) Know how to get help
 - g) Ask questions
 - h) Hand in all assignments on time
3. Know how to use the text.
 - a) Each example in every section has a Practice Problem associated with it.
 - b) Review the meaning of icons used in text.
 - c) Each chapter ends with Chapter Highlights, Reviews, Chapter Tests, and Cumulative Reviews.
 - d) Student Resources section at the back of the text book contains Study Skill Builders, Study Guide Outline, a Practice Final, and Answers to Selected Exercises.
4. Know how to use text resources.
 - a) Refer to the Lecture Videos and Chapter Test Prep Videos that accompany this text
 - b) Use the Video Organizer and Student Organizer that accompany this text.
5. Get help as soon as you need it.
 - a) Try your instructor, a tutoring center, or a math lab, or you may want to form a study group with fellow classmates.
6. Learn how to prepare for and take an exam.
 - a) Review previous homework assignments, class notes, quizzes, etc.
 - b) Read Chapter Highlights to review concepts and definitions.
 - c) Practice working out exercises in the end-of-the-chapter Review and Test.
 - d) When taking a test, read directions and problems carefully.
 - e) Pace yourself. Use all available time. Check your work and answers.
7. Develop good time management.
 - a) Make a list of all weekly commitments with estimated time needed.
 - b) Be sure to schedule study time. Don't forget eating, sleeping, and relaxing!

Teaching Notes:

- Most developmental students have a high anxiety level with mathematics.
- Many developmental students are hesitant to ask questions and seek extra help.
- Be sure to include your individual expectations. Keep your expectations clear and concise.

Mini-Lecture 1.2

Place Value, Names for Numbers, and Reading Tables

Learning Objectives:

1. Find the place value of a digit in a whole number.
2. Write a whole number in words and in standard form.
3. Write a whole number in expanded form.
4. Read tables.
5. Key Vocabulary: *whole numbers, place value, standard form, period, expanded form, tables.*

Examples:

1. Find the place value of the digit 7 in each whole number.

a) 7,352 b) 607 c) 702,433 d) 17,009,321

2. Write each whole number in words.

a) 62 b) 698 c) 17,403 d) 1,067,599

Write each number in standard form.

- e) nine hundred fifty-two
f) three hundred sixty-two thousand, five hundred eighty-six
g) three million, four hundred thousand, one hundred two

3. Write each number in expanded form.

h) 398 i) 2,907 j) 4,089,347

4. Use the following table of Number of Students Enrolled to answer the questions.

Subject	Section 1	Section 2	Section 3	Total
Basic Mathematics	23	27	19	69
Statistics	20	25	22	67

- a) How many total students are enrolled in Basic Mathematics?
b) How many students are enrolled in Section 3 of Statistics?

Teaching Notes:

- Students who do not have English as their first language will need additional assistance learning place value vocabulary.
- Students who do not have English as their first language may use periods instead of commas in writing numbers.

Answers: 1a) thousands, b) ones, c) hundred thousands, d) ten million; 2a) sixty-two, b) six hundred ninety-eight, c) seventeen thousand, four hundred three, d) one million, sixty-seven thousand, five hundred ninety-nine, e) 952, f) 362,586, g) 3,400,102; 3a) $300+90+8$, b) $2000+900+7$, c) $4,000,000+80,000+9,000+300+40+7$; 4a) 69, b) 22.

Mini-Lecture 1.3

Adding Whole Numbers and Perimeter

Learning Objectives:

1. Add whole numbers.
2. Find the perimeter of a polygon.
3. Solve problems by adding whole numbers.
4. Key Vocabulary: *sum, addend, commutative property of addition, associate property of addition, polygon, perimeter,*

Examples:

1. Add.

a) $3 + 9$

b) $40 + 70$

c) $1900 + 17$

d) $5703 + 0$

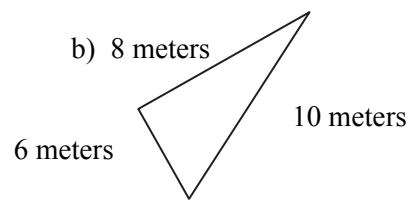
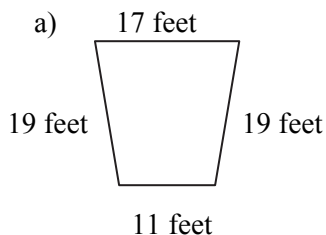
e)
$$\begin{array}{r} 51 \\ +27 \\ \hline \end{array}$$

f)
$$\begin{array}{r} 7329 \\ + 683 \\ \hline \end{array}$$

g) $93 + 145 + 69$

h)
$$\begin{array}{r} 6,403 \\ + 793 \\ \hline 17,187 \end{array}$$

2. Find the perimeter of each figure.



3. Solve the following word problems.

a) What is the sum of 8,932 and 14,799?

b) What is 830 plus 4,562 plus 88?

c) On Monday, Karen drove 57 miles; on Tuesday, she drove 39 miles; and on Wednesday, Karen drove 92 miles. How many total miles did Karen drive?

Teaching Notes:

- Some students need additional practice with basic addition facts.
- Remind students that it is acceptable to write the carry digit in order to obtain the correct answer.
- Most students will find this section easy but may need assistance with word problems.

Answers: 1a) 12, b) 110, c) 1917, d) 5703, e) 78, f) 8012, g) 307, h) 24,383; 2a) 66 ft., b) 24 m.; 3a) 23,731, c) 5,480, c) 188 miles.

Mini-Lecture 1.4

Subtracting Whole Numbers

Learning Objectives:

1. Subtract whole numbers.
2. Solve problems by subtracting whole numbers.
3. Key Vocabulary: *subtraction, minuend, subtrahend, difference, borrowing, regrouping, less than, take away, decreased by, subtracted from.*

Examples:

1. Subtract. Check by adding.

a) $11 - 7$

b) $15 - 8$

c) $22 - 22$

d) $31 - 0$

e)
$$\begin{array}{r} 27 \\ - 13 \\ \hline \end{array}$$

f)
$$\begin{array}{r} 198 \\ - 94 \\ \hline \end{array}$$

g)
$$\begin{array}{r} 5936 \\ - 803 \\ \hline \end{array}$$

h)
$$\begin{array}{r} 16,395 \\ - 5,132 \\ \hline \end{array}$$

i)
$$\begin{array}{r} 71 \\ - 32 \\ \hline \end{array}$$

j)
$$\begin{array}{r} 712 \\ - 285 \\ \hline \end{array}$$

k)
$$\begin{array}{r} 3004 \\ - 2965 \\ \hline \end{array}$$

l)
$$\begin{array}{r} 20,003 \\ - 16,867 \\ \hline \end{array}$$

m) $53,425 - 7977$

n) $71,217 - 56,839$

o) $60,000 - 26,386$

2. a) Find the difference of 93 and 27.
- b) Subtract 376 from 803.
- c) The Library Renovation Project has set a goal of \$75,000 to fundraise. To date, \$47,908 has been fundraised. How much more money does the Library Renovation Project need to fundraise?

Teaching Notes:

- Some students need additional practice with basic subtraction facts.
- Most students find subtraction without borrowing easy.
- Many students need to write the borrowing/regrouping step to maintain accuracy.
- Many students are challenged when borrowing with zeros.

Answers: 1a) 4, b) 7, c) 0, d) 31, e) 14, f) 104, g) 5,133, h) 11,263; i) 39, j) 427, k) 39, l) 3,136, m) 45,448, n) 14,378, o) 33,614; 2a) 66, b) 427, c) \$27,092.

Mini-Lecture 1.5

Rounding and Estimating

Learning Objectives:

1. Round whole numbers.
2. Use rounding to estimate sums and differences.
3. Solve problems by estimating.
4. Key Vocabulary: *rounding, estimating*

Examples:

1. Round to the nearest ten.

a) 31 b) 57 c) 346 d) 2,795

Round to the nearest hundred.

e) 312 f) 6,658 g) 8,672 h) 1,899

2. Round to the nearest thousand to find the estimated sum or difference.

a) 4892	b) 2731	c) 17,032	d) 24,803
<u>-2305</u>	<u>+ 3020</u>	<u>- 12,513</u>	<u>+ 14,587</u>

3. a) At the last 3 dances, attendance was 657 students, 403 students, and 559 students. Estimate the total attendance by rounding each to the nearest hundred.

b) Enrollment figures at the Town of Johnson's School Department increased from 6,721 students to 7,653 students. Round each number to the nearest hundred to estimate the increase.

c) The Carlisle family needs to buy a refrigerator for \$999, a stove for \$459, and a dishwasher for \$449. Round each cost to the nearest hundred to estimate the total cost.

Teaching Notes:

- Some students need to be repeatedly reminded to look at the digit to the right of the rounding position. Have students draw a line after the digit in the rounding position.
- A common error students make is to leave the digits to the right of the rounding position the same instead of changing them to zeros after rounding.
- Stress the importance of rounding and estimating with applications.

Answers: 1a) 30, b) 60, c) 350, d) 2,800, e) 300, f) 6,700, g) 8,700, h) 1,900; 2a) 3000, b) 6000, c) 4000, d) 40,000; 3a) 177, b) 1,000, c) \$1,900

Mini-Lecture 1.6

Multiplying Whole Numbers and Area

Learning Objectives:

1. Use the properties of multiplication.
2. Multiply whole numbers.
3. Multiply by whole numbers ending in zero(s).
4. Find the area of a rectangle.
5. Solve problems by multiplying whole numbers.
6. Key Vocabulary: *multiplication sign, product, multiplication property of 0, multiplication property of 1, commutative property of multiplication, associative property of multiplication, distributive property, partial products, area.*

Examples:

1. Multiply.

a) $37 \cdot 1$

b) $1 \cdot 22$

c) $0 \cdot 183$

d) $9 \cdot 5 \cdot 0$

Use the distributive property to rewrite each expression.

e) $2(5 + 4)$

f) $5(1 + 9)$

g) $10(9 + 6)$

h) $15(0 + 14)$

2. Multiply.

a)
$$\begin{array}{r} 37 \\ \times 6 \\ \hline \end{array}$$

b) 412×4

c)
$$\begin{array}{r} 1708 \\ \times 9 \\ \hline \end{array}$$

d)
$$\begin{array}{r} 337 \\ \times 25 \\ \hline \end{array}$$

e)
$$\begin{array}{r} 643 \\ \times 27 \\ \hline \end{array}$$

f)
$$\begin{array}{r} 309 \\ \times 800 \\ \hline \end{array}$$

g) $825 \times 1,000$

h)
$$\begin{array}{r} 2,477 \\ \times 963 \\ \hline \end{array}$$

3. Find the area of a rectangle with length 14 feet and width 8 feet.
4. At a recent football game, 413 adult tickets were sold at a price of \$5 each. There were 127 child tickets sold at a price of \$3 each. How much total amount of money in ticket sales for the game?

Teaching Notes:

- Some students need additional practice with basic multiplication facts.
- Some students do not know the different types of symbols used for multiplication.
- When using distributive property, many students forget to distribute over both terms.
- When multiplying, remind students to carefully line up the ones, tens, hundreds, etc.

Answers: 1a) 37, b) 22, c) 0, d) 0, e) 18, f) 50, g) 150, h) 210; 2a) 222, b) 1648, c) 15,372, d) 8,425, e) 17,361, f) 247,200, g) 825,000, h) 2,385,351; 3) 112 sq .ft.; 4) \$2,44

Mini-Lecture 1.7

Dividing Whole Numbers

Learning Objectives:

1. Divide whole numbers
2. Perform long division.
3. Solve problems that require dividing by whole numbers.
4. Find the average of a list of numbers
5. Key Vocabulary: *division, fraction bar, quotient, divisor, dividend, division by zero, undefined, long division, remainders, divide, quotient, divided by, divided or shared equally among, average.*

Examples:

1. Find each quotient. Check by multiplying.

a) $3\overline{)12}$ b) $13 \div 1$ c) $\frac{5}{5}$ d) $15 \div 15$ e) $0\overline{)5}$

2. Divide. Check by multiplying.

a) $228 \div 4$ b) $\frac{572}{7}$ c) $1570 \div 3$
d) $14\overline{)7070}$ e) $97\overline{)41,270}$ f) $603\overline{)604,911}$

3. a) Find the quotient of 94 and 5.
b) Recently, Amy earned \$1,722 selling calendars. If each calendar cost \$14, how many calendars did Amy sell?
4. a) During the semester, Kyle's test scores were: 87, 93, 62, 83 and 100. What was Kyle's average for the semester?

Teaching Notes:

- Some students need additional practice with basic division facts.
- Many students confuse division by zero (undefined) and zero divided by any non-zero number ($= 0$).
- Many students need to be cautious with placement of digits in quotient and dividend. Be sure appropriate place values are lined up. Stress organization!

Answers: 1a) 4, b) 13, c) 1, d) 1, e) undefined; 2a) 57, b) $81r5$, c) $523r1$; d) 505, e) $425r45$, f) $1003r102$;
3a) $18r4$, b) 123; 4a) 85

Mini-Lecture 1.8

An Introduction to Problem Solving

Learning Objectives:

1. Solve problems by adding, subtracting, multiplying, or dividing whole numbers.
2. Solve problems that require more than one operation.
3. Key Vocabulary: *understand, translate, solve, interpret,*

Examples:

1. Using Problem-Solving Steps, solve problems involving one type of operation.
 - a) Alicia owns a home that is 4,500 square feet. Anastasia owns a home that is 2,300 square feet. How much larger is Alicia's house than Anastasia's?
 - b) How many 16-ounce cans of tomato sauce can be made from a vat of tomato sauce containing 361 ounces? Will there be any sauce leftover?
 - c) Recently, Jason purchased a DVD for \$18, a CD for \$12, a set of headphones for \$19, and a magazine for \$6. How much money did Jason spend?
 - d) Last week, Tyler worked 38 hours at his job and is paid \$17/hour. How much money did Tyler earn last week?
2. Using Problem-Solving Steps, solve problems involving more than one operation.
 - a) Find the total cost of 5 calculators at \$69 each and 5 protective cases at \$8 each.
 - b) Andrea has \$1040 in her checking account. She spent \$130 at Matt's Sports Supplies, \$170 at The Pool Center, and \$80 at the grocery store. Finally, she stopped at her bank and deposited her paycheck for \$270. How much money is in Andrea's checking account?
 - c) In preparation for school, Kayla purchased two pairs of pants at \$59 each, four shirts at \$26 each, two pairs of shoes at \$69 each, and three pairs of socks at \$6 each. What was the total cost of these items?

Teaching Notes:

- Many students have a very difficult time with word problems.
- Refer students to **Problem-Solving Steps** in the textbook.
- Encourage students to use estimation to check whether their answers are reasonable.

Answers: 1a) 2,200, b) 22, 9 oz., c) \$55, d) \$646; 2a) \$385, b) \$930, c) \$378.

Mini-Lecture 1.9

Exponents, Square Roots, and Order of Operations

Learning Objectives:

1. Write repeated factors using exponential notation.
2. Evaluate expressions containing exponents.
3. Evaluate the square root of a perfect square.
4. Use the order of operations.
5. Find the area of a square.
6. Key Vocabulary: *exponential notation, exponent, base, order of operations, square root, perfect square, area of a square.*

Examples:

1. Write using exponential notation.

a) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ b) $(7)(7)(7)$ c) $4 \cdot 4 \cdot 3 \cdot 3 \cdot 3$ d) $5 \cdot 5 \cdot 8 \cdot 8 \cdot 5 \cdot 5$

2. Evaluate.

a) 5^2 b) 7^3 c) 3^6 d) 10^4

3. Find each square root.

a) $\sqrt{9}$ b) $\sqrt{49}$ c) $\sqrt{1}$ d) $\sqrt{225}$

4. Using order of operations, simplify.

a) $3 \cdot 4 - 10 \div 2$ b) $6^2 \div 3 \cdot 2$ c) $8 \cdot 4 + \{27 \div [8 - (3 + 2)]\}$

5. a) Find the area of a square whose side measures 6 feet.

b) A square floor measures 12 feet on each side. If 1 can of floor finish can cover 16 square feet, how many cans are needed to finish this floor?

Teaching Notes:

- Students may confuse exponent and base.
- Many students have trouble with order of operations.

Answers: 1a) 2^6 , b) 7^3 , c) $3^3 \cdot 4^2$, d) $5^4 \cdot 8^2$; 2a) 25, b) 343, c) 729, d) 10,000; 3a) 3, b) 7, c) 1, d) 15; 4a) 7, b) 24, c) 41; 5a) 36 sq. ft.; b) 9 cans.

Mini-Lecture 2.1

Introduction to Variables and Algebraic Expressions

Learning Objectives:

1. Evaluate algebraic expressions given replacement values.
2. Translate phrases into variable expressions.

Addition (+)	Subtraction (-)	Multiplication (•)	Division (÷)
Sum, plus, added to, more than, increased by, total	Difference, minus, subtract, less than, decreased by, less	Product, times, multiply, multiply by, of, double, triple	Quotient, divide, shared equally among, divided by, divided into

3. Key Vocabulary: *variable, algebra, algebraic expression, expression, evaluating the expression.*

Examples:

1. Evaluate each expression for $x = 12$, $y = 4$, and $z = 3$

a) $x - y + z$ b) $x - (y + z)$ c) $5(3x + 7)$ d) $2xy - 3z$

e) $x^2 - 4y$ f) $y^3 - 2x$ g) $\frac{3x}{4} - \frac{yz}{3}$ h) $\left(\frac{2yz - x}{2}\right)^3$

2. Write each phrase as a variable expression. Use x to represent “a number.”

- a) The sum of a number and eleven
- b) Fifteen added to a number
- c) The difference between a number and three hundred
- d) A number subtracted from forty-two
- e) The product of sixteen and a number
- f) A number times thirteen
- g) The quotient of thirty and a number
- h) Seven divided by a number
- i) The quotient of eighteen and a number, decreased by two

Teaching Notes:

- Remind students that order of operations apply with variables.
- Many students will have difficulty translating a phrase into an algebraic expression.
- Refer students to textbook for ***Translating Phrases into Variable Expressions Chart***.

Answers: 1a) 11; 1b) 5; 1c) 215; 1d) 87; 1e) 128; 1f) 40; 1g) 5; 1h) 216; 2a) $x=11$; 2b) $15+x$; 2c) $300-x$; 2d) $42-x$; 2e) $16x$; 2f) $13x$; 2g) $30/x$; 2h) $7/x$; 2i) $\frac{18}{x} - 2$

Mini-Lecture 2.2

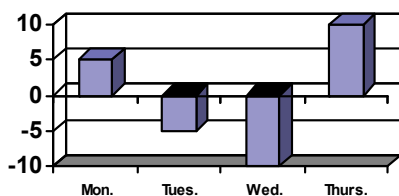
Introduction to Integers

Learning Objectives:

1. Represent real-life situations with integers.
2. Graph integers on a number line.
3. Compare integers.
4. Find the absolute value of a number.
5. Find the opposite of a number.
6. Read bar graphs containing integers.
7. Key Vocabulary: *positive numbers, negative numbers, signed numbers, integers, is less than, is greater than, opposite, absolute value.*

Examples:

1. Represent each quantity by an integer.
 - a) A scuba diver is swimming 25 feet below sea level.
 - b) The record high temperature for the town is 113°F .
 - c) The number of televisions sold reflected a 35 percent loss from the previous year.
2. Graph each integer in the list on the same number line.
 - a) 1, 3, 5, 6
 - b) 2, -2, 3, -3
 - c) 4, 0, -2, -5
 - d) 0, -1, -2, -5
3. Insert $<$ or $>$ between each pair of integers to make a true statement.
 - a) $5 \underline{\hspace{1cm}} 10$
 - b) $0 \underline{\hspace{1cm}} -3$
 - c) $-42 \underline{\hspace{1cm}} -38$
 - d) $-22 \underline{\hspace{1cm}} 22$
4. Simplify.
 - a) $|2|$
 - b) $|-12|$
 - c) $|-3|$
 - d) $-|14|$
 - e) $-|45|$
 - f) $-|-103|$
 - g) $|-x|$ if $x = -25$
 - h) $|x|$ if $x = -8$
5. Find the opposite of each integer.
 - a) 9
 - b) -15
 - c) 0
 - d) -16
6. The bar graph shows the January temperatures for four days in Boston..




a) Which day was the coldest?

b) Which day was the warmest?

Teaching Notes:

- Many students will confuse absolute value and opposite.
- Encourage students to list everyday situation where negative numbers are used.

Answers: 1a) -25; 1b) +113; 1c) -35; 2a)  2b)  2c) 

2d)  ; 3a) $<$; 3b) $>$; 3c) $<$; 3d) $<$; 4a) 2, 4b) 12; 4c) 3; 4d) -14; 4e) -45; 4f) -103;

4g) 25; 4h) 8; 5a) -9; 5b) 15; 5c) 0; 5d) 16; 6a) Wed.; 6b) Thurs.

Mini-Lecture 2.3

Adding Integers

Learning Objectives:

1. Add integers.
2. Evaluate an algebraic expression by adding.
3. Solve problems by adding integers.

Examples:

1. Add.

- | | | | |
|------------------------|----------------------|-------------------------------|------------------|
| a) $23 + 12$ | b) $-23 + (-17)$ | c) $-11 + (-2)$ | d) $-21 + (-13)$ |
| e) $6 + (-8)$ | f) $-3 + 5$ | g) $-74 + 27$ | h) $-51 + (24)$ |
| i) $-8 + (-13)$ | j) $-79 + 97$ | k) $46 + (-54)$ | l) $-4 + (-24)$ |
| m) $23 + (-19) + (-8)$ | n) $14 + 25 + (-16)$ | o) $-25 + (-4) + (-2) + (-6)$ | |

2. Evaluate $x + y$ for the given replacement values.

- | | | |
|--------------------------|----------------------------|---------------------------|
| a) $x = -5$ and $y = 14$ | b) $x = -33$ and $y = -27$ | c) $x = -43$ and $y = 38$ |
|--------------------------|----------------------------|---------------------------|

3. Translate each phrase; then simplify.

- | | |
|------------------------------------|---------------------------------------------|
| a) Find the sum of -7 and 25 . | b) Find the sum of -52 , 13 , and -82 |
|------------------------------------|---------------------------------------------|

4. During a storm in Anchorage Alaska, the temperature was 10°F at Noon. At 1 p.m., the temperature had dropped 7° . At 2 p.m., the temperature dropped another 5° ; and finally, at 3 p.m., the temperature had dropped an additional 9° . Use positive and negative numbers to represent his situation. Then find the present temperature.

Teaching Notes:

- Some students need to see adding integers done on a number line first.
- Many students have a better understanding if they think of depositing and withdrawing money from a bank account.
- Refer students to the rules for adding signed numbers in the textbook.

Answers: 1a) 35; 1b) -40 ; 1c) -13 ; 1d) -34 ; 1e) -2 ; 1f) 2; 1g) -47 ; 1h) -27 ; 1i) -21 ; 1j) 18; 1k) -8 ; 1l) -28 ; 1m) -4 ; 1n) 23; 1o) -37 ; 2a) 9; 2b) -60 ; 2c) -5 ; 3a) 18; 3b) -121 ; 4) -11° .

Mini-Lecture 2.4

Subtracting Integers

Learning Objectives:

1. Subtract integers.
2. Add and subtract integers.
3. Evaluate an algebraic expression by subtracting.
4. Solve problems by subtracting integers.
5. Key Vocabulary: *additive inverse*.

Examples:

1. Subtract.

a) $-9 - (-2)$	b) $-14 - (-2)$	c) $4 - (-3)$	d) $20 - 20$
e) $2 - 5$	f) $-2 - 12$	g) $-150 - 410$	h) $-147 - (-85)$

2. Simplify.

a) $6 + 20 - 15$	b) $-1 - 11 - 12$	c) $-1 - 20 + 10$	d) $-16 + 11 - 18 + (-4)$
------------------	-------------------	-------------------	---------------------------

3. Evaluate $x - y$ for the given replacement values.

a) $x = -2$ and $y = -8$	b) $x = 8$ and $y = -32$
c) $x = -9$ and $y = -9$	d) $x = 3$ and $y = -15$

5. Solve.

- a) Amy has \$545 in her checking account. She writes a check for \$257, makes a deposit of \$75, and then writes another check for \$409. Find the balance in her account. (Write the amount as an integer.)
- b) The city of Manchester has an elevation of 13,005 feet above sea level while the city of Catherine has an elevation of 17,532 feet below sea level. Find the difference in elevation between the two cities.
- c) The temperature on a January morning in Worcester is -5°F at 2 a.m. If the temperature drops 4° by 3 a.m., rise 6° by 4 a.m., and then drops 8° by 5 a.m., find the temperature by 8 a.m.

Teaching Notes:

- Many students find subtracting signed numbers difficult at first.
- Some students like to see subtracting signed numbers on a number line.
- Many students make errors when evaluating $x - y$ when y is a negative number. Encourage students to make a direct substitution first so they do not forget to write the subtraction symbol.

Answers: 1a) -7 ; 1b) -12 ; 1c) 7 ; 1d) 0 ; 1e) -3 ; 1f) -14 ; 1g) -560 ; 1h) -62 ; 2a) 11 ; 2b) -24 ; 2c) -11 ; 2d) -27 ; 3a) 6 ; 3b) 40 ; 3c) 0 ; 3d) 18 ; 4a) $-\$46$; 4b) 4527 ft. ; 4c) -11°F .

Mini-Lecture 2.5

Multiplying and Dividing Integers

Learning Objectives:

1. Multiply integers.
2. Divide integers.
3. Evaluate an algebraic expression by multiplying or dividing.
4. Solve problems by multiplying and dividing integers.

Examples:

1. Multiply.

- | | | | |
|------------------|----------------------|-----------------------|-------------------------|
| a) $7(-6)$ | b) $-4(10)$ | c) $-20(13)$ | d) $-10(-19)$ |
| e) $(-4)(-3)(6)$ | f) $(-50)(0)(-5)(8)$ | g) $(-4)(-5)(-4)(-3)$ | h) $(-2)(3)(-1)(-4)(2)$ |
| i) -4^2 | j) $(-3)^3$ | k) -3^3 | l) $(-8)^2$ |

2. Find each quotient.

- | | | | |
|----------------|-------------------|--------------------|--------------------|
| a) $21 \div 7$ | b) $36 \div (-6)$ | c) $\frac{-48}{6}$ | d) $\frac{-17}{0}$ |
|----------------|-------------------|--------------------|--------------------|

3. Evaluate xy and also $\frac{x}{y}$ for the given replacement values.

- | | | |
|-------------------------|----------------------------|--------------------------|
| a) $x = 8$ and $y = -4$ | b) $x = -30$ and $y = -10$ | c) $x = 0$ and $y = -16$ |
|-------------------------|----------------------------|--------------------------|

4. Solve.

- a) Find the product of -13 and -5 .
- b) Find the quotient of 63 and -9 .
- c) Better Electric Co. marked \$15 off the price of each microwave in stock. If there are 57 microwaves in stock, write the total reduction in price of all microwaves as an integer.
- d) During a cold front in Canada the temperature dropped 4°F each hour for 7 hours. Express the total drop in temperature as an integer.

Teaching Notes:

- Some students need a review of basic multiplication and division facts before they begin working with integers.
- Some students mix up the rules for addition of integers and the rules for multiplication/division of integers.
- Many students have a hard time understanding the difference between -3^2 and $(-3)^2$.

Answers: 1a) -42 ; 1b) -40 ; 1c) -260 ; 1d) -190 ; 1e) 72 ; 1f) 0 ; 1g) 240 ; 1h) -48 ; 1i) -16 ; 1j) -27 ; 1k) -27 ; 1l) 64 ; 2a) 3 ; 2b) -6 ; 2c) -8 ; 2d) undefined; 3a) -2 ; 3b) 3 ; 3c) 0 ; 4a) 65 ; 4b) -7 ; 4c) $-\$855$; 4d) -28°F .

Mini-Lecture 2.6

Order of Operations

Learning Objectives:

1. Simplify expressions by using the order of operations.
2. Evaluate an algebraic expression.
3. Find the average of a list of numbers.

Examples:

1. Simplify.

a) $-2 + 5 \cdot 6$

b) $-2 - 5(5 - 8)$

c) $2(-5)(7 - 3) - 7$

d) $80 \div (-8) - 15$

e) $3^3 - 8(2)$

f) $8 - 2(7 - 2^2) + 3$

g) $8^2 - 2(6) + 45 \div 5$

h) $3(-2) + (8 - 10)^2$

i) $21 \div [7 \cdot (-15 \div (-5))]$

j) $\frac{8(-2) - 4 + 3}{-85 \div 5}$

k) $\frac{[-36 \div (-4) - 1]}{[2 - (-2)]}$

l) $\frac{20(-1) - (-5)(-3)}{2[-12 \div (-3 - 3)]}$

2. Evaluate each expression for $x = -3$, $y = 6$, and $z = -1$.

a) $x + y + z$

b) $2y - 3z + x$

c) $x^2 - y + z$

d) $\frac{8x}{2y}$

e) $5y - x^2$

f) $x^3 + yz$

3. Find the average of each list of numbers.

a) $-8, 12, -14, 5, -15$

b) $-42, -30, -25, 12, 15, 28$

Teaching Notes:

- Many students confuse the addition/subtraction rules with the multiplication/division rules when working with many operations in one expression.
- Encourage students to perform one operation at a time.
- Refer students to **Order of Operations** in the textbook.

Answers: 1a) 28; 1b) 13; 1c) -47; 1d) -25; 1e) 11; 1f) 5; 1g) 61; 1h) -2; 1i) 1; 1j) 1; 1k) 2; 1l) -9;
2a) 2; 2b) 12; 2c) 2; 2d) -2; 2e) 21; 2f) -33; 3a) -4; 3b) -7

Mini-Lecture 3.1

Introduction to Fractions and Mixed Numbers

Learning Objectives:

1. Identify the numerator and the denominator of a fraction.
2. Write a fraction to represent parts of figures or real-life data.
3. Identify proper fractions, improper fractions, and mixed numbers.
4. Graph fractions on a number line.
5. Review division properties for 0 and 1.
6. Key Vocabulary: *fractions, numerator, denominator, fraction bar, proper fraction, improper fraction, mixed number.*

Examples:

1. Identify the numerator and the denominator of a fraction.

a) $\frac{3}{7}$

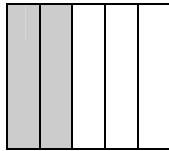
b) $\frac{12}{13}$

c) $\frac{10}{7}$

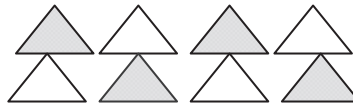
d) $\frac{13}{13}$

2. Represent the shaded part and unshaded part of each figure as a fraction..

a)



b)



Draw and shade a part of a diagram to represent each fraction.

c) $\frac{1}{6}$ of a diagram

d) $\frac{5}{9}$ of a diagram.

Write a fraction to represent the following information.

- e) Of the 207 students taking Basic Mathematics, 143 are freshman. What fraction of the class is freshman?
 - f) A bag of 50 marbles contains 27 red and 23 blue. What fraction of the bag is blue marbles.
3. Identify the following fractions as a proper fraction, improper fraction, or a mixed numbers.
 - a) $\frac{3}{10}$
 - b) $\frac{16}{7}$
 - c) $2\frac{2}{7}$
 - d) $\frac{6}{6}$
 4. Simplify.
 - a) $\frac{4}{4}$
 - b) $\frac{-7}{1}$
 - c) $\frac{0}{2}$
 - d) $\frac{-12}{0}$

Teaching Notes:

- Students need to have a firm grasp of fraction vocabulary before continuing.
- Many students confuse $\frac{0}{x}$ with $\frac{x}{0}$. Be sure to stress the difference.
- Many students can write a fraction to represent a real-life situation, but they do not truly understand the meaning.

Answers: 1a) $n=3, d=7$; 1b) $n=12, d=13$; 1c) $n=10, d=7$; 1d) $n=13, d=13$; 2a) $\frac{2}{5}, \frac{3}{5}$; 2b) $\frac{4}{8}, \frac{4}{8}$; 2c – 2d) diagrams will vary; 2e) $\frac{143}{207}$; 2f) $\frac{23}{50}$; 3a) prop.; 3b) improp.; 3c) mix; 3d) improp.; 4a) 1; 4b) -7 ; 4c) 0; 4d) undefined.

Mini-Lecture 3.2

Factors and Simplest Form

Learning Objectives:

1. Write a number as a product of prime numbers.
2. Write a fraction in simplest form.
3. Determine whether two fractions are equivalent.
4. Solve problems by writing fractions in simplest form.
5. Key Vocabulary: *factor, prime factorization, prime numbers, composite number, simplest form, lowest terms.*

Examples:

1. Write the prime factorization of each number.
a) 30 b) 75 c) 170 d) 360
2. Write each fraction in simplest form.
a) $\frac{10}{16}$ b) $\frac{36}{63}$ c) $\frac{77}{88}$ d) $-\frac{12}{42}$
e) $\frac{11}{34}$ f) $\frac{-27}{36}$ g) $\frac{30}{80}$ d) $\frac{6}{-105}$
3. Determine whether each pair of fractions is equivalent.
a) $\frac{5}{10}$ and $\frac{11}{22}$ b) $\frac{7}{21}$ and $\frac{8}{24}$ c) $\frac{2}{7}$ and $\frac{8}{15}$ d) $\frac{6}{0}$ and $\frac{0}{6}$
4. Solve. Write each fraction in simplest form.
a) Alicia was scheduled to work 6 hours at the tanning salon. What fraction of Alicia's shift is represented by 4 hours?
b) There are 36 inches in a yard. What fraction of a yard is represent by 9 inches?
c) There are 140 students in a freshman lecture class. If 16 students are absent, what fraction of the students are absent?

Teaching Notes:

- Many students will understand equivalent fractions if they are shown drawings.
- Some students will confuse cross products and simplifying. Stress that cross products is only a check to determine equality of fractions.
- Some students prefer to reduce fractions by factoring the numerator and denominator as products of prime numbers, then canceling all common factors. Others prefer to repeatedly divide the numerator and the denominator by a common factor.

Answers: 1a) $2 \cdot 3 \cdot 5$; 1b) $3 \cdot 5^2$ 1c) $2 \cdot 5 \cdot 17$; 1d) $2^3 \cdot 3^2 \cdot 5$; 2a) $5/8$; 2b) $2c) 7/8$; 2d) $-2/7$; 2e) cannot be simplified; 2f) $-3/4$; 2g) $3/8$; 2d) $-2/35$; 3a) yes; 3b) yes; 3c) no; 3d) no; 4a) $2/3$; 4b) $1/4$; 4c) $4/35$

Mini-Lecture 3.3

Multiplying and Dividing Fractions

Learning Objectives:

1. Multiply fractions.
2. Evaluate exponential expressions with fractional bases.
3. Divide fractions.
4. Multiply or divide given fractional replacement values.
5. Solve applications that require multiplication of fractions.
6. Key Vocabulary: *reciprocal*, "of".

Examples:

1. Multiply. Write the product in simplest form.

a) $\frac{1}{9} \cdot \frac{1}{7}$

b) $\frac{2}{3} \cdot \frac{1}{4}$

c) $-\frac{5}{6} \cdot -\frac{2}{3}$

d) $-\frac{7}{2} \cdot \frac{6}{3}$

e) $\frac{5}{2} \cdot \frac{18}{15}$

f) $\frac{7}{8} \cdot 0$

g) $\frac{1}{2} \cdot -\frac{3}{5} \cdot \frac{1}{5}$

h) $-\frac{12}{14} \cdot -\frac{3}{9} \cdot -\frac{2}{10}$

2. Evaluate.

a) $\left(\frac{1}{2}\right)^2$

b) $\left(-\frac{1}{3}\right)^4$

c) $\left(-\frac{2}{5}\right)^3$

d) $\left(\frac{2}{7}\right)^2 \cdot \frac{1}{4}$

3. Divide. Write all quotients in simplest form.

a) $\frac{3}{5} \div \frac{4}{7}$

b) $\frac{1}{4} \div \frac{1}{4}$

c) $-\frac{1}{5} \div \frac{9}{19}$

d) $\frac{8}{17} \div \frac{12}{15}$

e) $-\frac{2}{17} \div -\frac{3}{17}$

f) $\frac{1}{14} \div 0$

g) $\frac{27}{-7} \div \frac{4}{7}$

h) $0 \div \frac{-3}{11}$

4. Given the following replacement values, evaluate (a) xy and (b) $x \div y$.

a) $x = -\frac{1}{3}$ and $y = \frac{4}{9}$

b) $x = \frac{5}{7}$ and $y = -\frac{5}{9}$

5. Solve. Write each answer in simplest form.

a) Find $\frac{1}{3}$ of 48.

b) Find $\frac{3}{7}$ of -63

c) A bike trail is 27 miles long. Michelle bikes $\frac{2}{3}$ of the trail. How many miles did Michelle bike?

Teaching Notes:

- Encourage students to divide out common factors in the numerator and denominator before multiplying.
- When dividing, encourage students take the time and rewrite the problem by changing the division symbol to multiplication and multiply by the reciprocal. Many students begin "simplifying" and forget to multiply by the reciprocal.
-

Answers: 1a) $1/63$; 1b) $1/6$; 1c) $5/9$; 1d) -7 ; 1e) 3 ; 1f) 0 ; 1g) $-3/50$; 1h) $-2/35$; 2a) $1/4$; 2b) $1/81$; 2c) $-8/125$; 2d) $1/49$; 3a) $21/20$; 3b) 1 ; 3c) $-19/45$; 3d) $10/51$; 3e) $2/3$; 1f) undef; 1g) $-27/4$; 3h) 0 ; 4a) $-4/27$, $-3/4$; 4b) $-25/63$, $-9/7$; 5a) 16 ; 5b) -27 ; 5c) 18 .

Mini-Lecture 3.4

Adding and Subtracting Like Fractions, Least Common Denominator, and Equivalent Fractions

Learning Objectives:

1. Add or subtract like fractions.
2. Add or subtract given fractional replacement values.
3. Solve problems by adding or subtracting like fractions.
4. Find the least common denominator of a list of fractions.
5. Write equivalent fractions.
6. Key Vocabulary: *like fractions, unlike fractions, (LCD) least common denominator, (LCM) least common multiple, and equivalent fractions.*

Examples:

1. Add and simplify.

a) $\frac{1}{9} + \frac{4}{9}$

b) $\frac{1}{8} + \frac{5}{8}$

c) $-\frac{1}{10} + \frac{9}{10}$

d) $-\frac{3}{14} + \left(-\frac{5}{14}\right)$

Subtract and simplify.

e) $\frac{6}{8} - \frac{3}{8}$

f) $-\frac{6}{21} - \frac{5}{21}$

g) $\frac{25}{42} - \left(-\frac{7}{42}\right)$

h) $-\frac{28}{13} - \left(-\frac{5}{13}\right)$

2. Solve.

a) Find the perimeter of a triangle with sides: $\frac{6}{25}$ inch, $\frac{9}{25}$ inch, and $\frac{5}{25}$ inch.

b) Cori read $\frac{2}{11}$ of her book on Friday, $\frac{3}{11}$ of her book on Saturday, and $\frac{4}{11}$ of her book on Sunday.
What part of her book has she read?

3. Find the LCD of each list of numbers.

a) $\frac{3}{10}, \frac{7}{12}$

b) $\frac{11}{12}, \frac{9}{14}$

c) $\frac{13}{30}, \frac{17}{35}$

d) $\frac{7}{30}, \frac{19}{20}, \frac{29}{50}$

4. Write each fraction as an equivalent fraction with the given denominator.

a) $\frac{4}{9} = \frac{\quad}{18}$

b) $\frac{7}{11} = \frac{\quad}{55}$

c) $\frac{5}{2} = \frac{\quad}{6}$

d) $\frac{2}{3} = \frac{\quad}{24}$

Teaching Notes:

- Many students add or subtract both numerator and denominator.
- When subtracting, some students may need to take the intermediate step of writing out the operations performed on the numerators. For example: $-\frac{1}{7} - \left(-\frac{3}{7}\right) = -\frac{1}{7} + \left(+\frac{3}{7}\right) = \frac{-1+3}{7} = \frac{2}{7}$.
- Some students forget to multiply the numerator when building equivalent fractions.

Answers: 1a) $\frac{5}{9}$; 1b) $\frac{3}{4}$; 1c) $\frac{4}{5}$; 1d) $-\frac{4}{7}$; 1e) $\frac{3}{8}$; 1f) $-\frac{11}{21}$; 1g) $\frac{16}{21}$; 1h) $-\frac{23}{13}$; 2a) $\frac{4}{5}$ inch; 2b) $\frac{9}{11}$; 3a) 60; 3b) 84; 3c) 210; 3d) 300; 4a) 8; 4b) 35; 4c) 15; 4d) 16.

Mini-Lecture 3.5

Adding and Subtracting Unlike Fractions

Learning Objectives:

1. Add or subtract unlike fractions.
2. Write fractions in order.
3. Evaluate expressions given fractional replacement values.
4. Solve problems by adding or subtracting unlike fractions.
5. Key Vocabulary: *least common denominator (LCD)*.

Examples:

1. Add or subtract as indicated.

a) $\frac{1}{10} + \frac{2}{5}$

b) $-\frac{1}{5} + \left(-\frac{2}{25}\right)$

c) $\frac{1}{7} + \left(-\frac{9}{10}\right)$

d) $\frac{3}{5} + \frac{1}{20}$

e) $\frac{4}{5} - \frac{3}{20}$

f) $\frac{7}{9} - \left(-\frac{1}{12}\right)$

g) $-\frac{5}{7} - \left(-\frac{1}{2}\right)$

h) $\frac{1}{20} - \frac{8}{15}$

i) $-\frac{7}{5} + \frac{8}{16} + \frac{4}{20}$

2. Insert $<$ or $>$ to form a true sentence.

a) $\frac{2}{3}$ _____ $\frac{1}{9}$

b) $\frac{5}{12}$ _____ $\frac{1}{2}$

c) $-\frac{5}{6}$ _____ $-\frac{4}{5}$

3. Evaluate each expression if $x = \frac{1}{4}$ and $y = -\frac{3}{5}$.

a) $x + y$

b) $x \cdot y$

c) $x - y$

d) $x \div y$

4. Solve.

a) Find the perimeter of a rectangle with width $\frac{3}{4}$ feet and length $\frac{3}{14}$ feet.

b) Sharon is making matching holiday outfits for her three children. Each outfit required $\frac{7}{8}$ yards.

How many yards of material will be needed to make the three outfits?

Teaching Notes:

- Refer students back to Section 3.4: *Method 1: Finding the LCM of a List of numbers Using Multiples of the Largest Number* and *Method 2: Finding the LCM of a List of Numbers Using Prime Factorization*.
- Some students try to cross-cancel when adding or subtracting.
- Some students add and subtract both the numerator and denominator.
- Some students forget to multiply the numerator when building equivalent fractions.

Answers: 1a) $\frac{1}{2}$; 1b) $-\frac{7}{25}$; 1c) $-\frac{53}{70}$; 1d) $\frac{13}{20}$; 1e) $\frac{13}{20}$; 1f) $\frac{31}{36}$; 1g) $-\frac{3}{14}$; 1h) $-\frac{29}{60}$; 1i) $-\frac{7}{10}$; 2a) $>$; 2b) $<$; 2c) $<$; 3a) $-\frac{7}{20}$; 3b) $\frac{17}{20}$; 3d) $-\frac{5}{12}$; 4a) $\frac{27}{14}$ feet; 4b) $\frac{21}{8}$ yds.

Mini-Lecture 3.6

Complex Fractions, Order of Operations, and Mixed Numbers

Learning Objectives:

1. Simplify complex fractions.
2. Review the order of operations.
3. Evaluate expressions given replacement values.
4. Write mixed numbers as improper fractions.
5. Write improper fractions as mixed numbers or whole numbers.
6. Key Vocabulary: *complex fraction, improper fraction, mixed number, whole number.*

Examples:

1. Simplify each complex fraction.

a) $\frac{\frac{1}{6}}{\frac{2}{3}}$

b) $\frac{\frac{16}{7}}{\frac{8}{7}}$

c) $\frac{\frac{1}{6} + \frac{1}{2}}{\frac{1}{3} + \frac{3}{4}}$

2. Use the order of operations to simplify each expression.

a) $\frac{1}{4} + \frac{1}{4} \cdot \frac{1}{3}$

b) $\frac{3}{2} \div \left(\frac{7}{8} + \frac{7}{16} \right)$

c) $\left(\frac{2}{7} + \frac{3}{14} \right) \left(\frac{2}{7} - \frac{3}{14} \right)$

d) $\left(\frac{2}{5} \right)^2 \cdot \frac{1}{2}$

e) $\frac{1}{2} + \left(\frac{2}{3} \right)^2 - \frac{1}{3}$

f) $\frac{2}{3} \cdot \left(\frac{1}{4} + \frac{1}{2} \right) \cdot 6$

3. Evaluate each expression if $x = -\frac{1}{2}$, $y = \frac{3}{5}$, and $z = \frac{7}{10}$.

a) $3x - z$

b) $\frac{y}{z}$

c) $x^2 + 2y$

d) $\frac{x+y}{z}$

4. Write each mixed number as an improper fraction.

a) $3\frac{1}{2}$

b) $2\frac{8}{9}$

c) $13\frac{2}{9}$

d) $103\frac{3}{11}$

5. Write each improper fraction as a mixed number or a whole number.

a) $\frac{16}{3}$

b) $\frac{38}{5}$

c) $\frac{156}{12}$

d) $\frac{159}{143}$

Teaching Notes:

- Many students make careless errors when using Method 2 for simplifying complex fractions. If this is the case, encourage students to use Method 1 (rewrite as a division problem).
- Remind students that when dividing fractions, you must change division to multiplication and multiply by the reciprocal.
- Some students will try to apply procedures for simplifying complex fractions to adding and subtracting fractions.

Answers: $\frac{1}{4}$; 1b) 2; 1c) $\frac{8}{13}$; 2a) $\frac{1}{3}$; 2b) $\frac{8}{7}$; 2c) $\frac{3}{28}$; 2d) $\frac{2}{25}$; 2e) $\frac{11}{18}$; 2f) 3; 3a) $-\frac{11}{5}$; 3b) $\frac{6}{7}$; 1c) $\frac{29}{20}$; 1d) $\frac{1}{7}$; 4a) $\frac{7}{2}$; 4b) $\frac{26}{9}$; 4c) $\frac{119}{9}$; 4d) $\frac{1136}{11}$; 5a) $5\frac{1}{3}$; 5b) $7\frac{3}{5}$; 5c) 13; 5d) $1\frac{16}{143}$

Mini-Lecture 3.7

Operations on Mixed Numbers.

Learning Objectives:

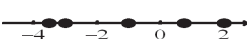

1. Graph positive and negative fractions and mixed numbers.
2. Multiply or divide mixed or whole numbers.
3. Add or subtract mixed numbers.
4. Solve problems containing mixed numbers.
5. Perform operations on negative mixed numbers.

Examples:

1. Graph each list of numbers on a number line.
a) $-3, -3\frac{1}{2}, -1, \frac{3}{4}, 2$ b) $4, -3\frac{3}{4}, 0, 1\frac{1}{5}, -\frac{1}{2}$
2. Multiply or divide. Find an exact answer and an estimated answer.
a) $2\frac{2}{3} \cdot \frac{1}{2}$ b) $3\frac{3}{4} \cdot 1\frac{3}{5}$ c) $4\frac{1}{5} \div \frac{1}{5}$ d) $3\frac{1}{3} \div 2\frac{3}{5}$
3. Add or subtract. Find an exact sum and an estimated sum.
a) $10\frac{1}{2} + 7\frac{1}{9}$ b) $6\frac{1}{3} + 13\frac{4}{9}$ c) $4\frac{2}{3} + 9\frac{7}{9}$ d) $8\frac{1}{3} + 2\frac{2}{3} + 3\frac{2}{9}$
e) $15\frac{8}{9} - 6\frac{2}{9}$ f) $19\frac{1}{25} - 7\frac{1}{5}$ g) $17\frac{1}{6} - 5\frac{13}{24}$ h) $13 - 6\frac{5}{9}$
4. Solve. Write answer in simplest form.
a) Amy rode her bicycle $9\frac{4}{15}$ miles on each of 9 days. What is the total distance Amy rode?
b) John cuts a board $13\frac{3}{7}$ feet long from one 20 feet long. How long is the remaining piece?
5. Perform the indicated operation.
a) $-5\frac{7}{8} \div 5\frac{1}{4}$ b) $-7\frac{7}{9} + \left(-4\frac{2}{9}\right)$ c) $-8\frac{1}{3} \cdot \left(-1\frac{1}{5}\right)$ d) $5\frac{5}{8} \div (-9)$
e) $19\frac{3}{5} - \left(-5\frac{18}{20}\right)$ f) $\left(-15\frac{3}{7}\right) + 14\frac{1}{5}$ g) $10\frac{1}{9} + \left(-5\frac{5}{9}\right)$ h) $-9 \cdot \left(5\frac{7}{12}\right)$

Teaching Notes:

- Most students forget that mixed numbers must be changed to improper fractions before multiplying. Some try to multiply the whole number parts together, and then multiply the fractional parts together.
- Many students confuse the rules for multiplication with adding/subtracting rules.
- Many students are challenged by word problems. Students have trouble deciding which operation to use for the word problems.

Answers: 1a)  1b)  2a) $4/3, 3$; 2b) $6, 8$; 2c) $21, 20$;

2d) $1\frac{11}{39}, 1$; 3a) $17\frac{11}{18}$; 3b) $19\frac{7}{9}$; 3c) $14\frac{4}{9}$; 3d) $14\frac{2}{9}$; 3e) $9\frac{2}{3}$; 3f) $11\frac{21}{25}$; 3g) $11\frac{5}{8}$; 3h) $6\frac{4}{9}$; 4a) $83\frac{2}{5}$; 4b) $6\frac{4}{7}$; 5a) $-47\frac{42}{25}$; 5b) -12 ; 5c) 10 ; 5d) $-5\frac{8}{9}$; 5e) $25\frac{1}{2}$; 5f) $-1\frac{8}{35}$; 5g) $4\frac{5}{9}$; 5h) $-50\frac{1}{4}$

Mini-Lecture 4.1

Introduction to Decimals

Learning Objectives:

1. Know the meaning of place value for a decimal number, and write decimals in words.
2. Write decimals in standard form.
3. Write decimals as fractions.
4. Compare decimals.
5. Round decimals to a given place value.
6. Key Vocabulary: *decimals, standard form, < (less than), > (greater than), = (equal to), round the decimal part.*

Examples:

1. Determine the place value for the digit 9 in each number.

a) 90 b) 900 c) 0.9 d) 0.09

Write each decimal number in words.

e) 8.54 f) -0.382 g) 7002.09

2. Write each decimal number in standard form.

a) two and seven tenths b) negative eleven and five hundredths
c) seven hundred three and two d) negative ninety-five ten thousandths
hundred fifty-five thousandths

3. Write each decimal as a fraction or a mixed number. Write your answer in simplest form.

a) 0.7 b) -0.35 c) 0.094 d) -2.4005

4. Insert $<$, $>$, or $=$ to form a true statement.

a) $0.2 \underline{\hspace{1cm}} 0.5$ b) $0.14 \underline{\hspace{1cm}} -0.14000$
c) $0.6401 \underline{\hspace{1cm}} 0.6410$ d) $-15.0037 \underline{\hspace{1cm}} 15.00037$

5. Round each decimal to the given place value.

a) 0.39 to the nearest tenth b) -0.174 to the nearest hundredth
c) 1.4782 to the nearest thousandth d) -22.099 to the nearest hundredth

Round each monetary amount to the nearest cent or dollar as indicated.

e) \$0.058 to the nearest cent f) \$17.88 to the nearest dollar

Teaching Notes:

- Most students find problems 1 and 2 easy.
- Some students have difficulty with example 3 when a whole number is involved.
- Some students become confused when rounding monetary values. When rounding to the nearest cent, it is important to remind them that this is the hundredths position (one-hundredths-of-a-dollar position).

Answers: 1a) tens, b) hundreds, c) tenths, d) hundredths, e) eight and fifty-four hundredths, f) negative three hundred eighty-two thousandths, g) seven thousand two and nine hundredths; 2a) 2.7, b) -11.05, c) 703.255, d) -0.0095;

3a) $\frac{7}{10}$, b) $-\frac{7}{20}$, c) $\frac{47}{500}$, d) $-2\frac{801}{2000}$; 4a) $<$, b) $>$, c) $<$, d) $<$; 5a) 0.4, b) -0.17, c) 1.478, d) -22.10, e) \$0.06, f) \$18

Mini-Lecture 4.2

Adding and Subtracting Decimals

Learning Objectives:

1. Add decimals.
2. Subtract decimals.
3. Estimate when adding or subtracting decimals.
4. Evaluate expressions with decimal replacement values.
5. Solve problems that involve adding or subtracting decimals.

Examples:

1. Add. Be sure to estimate to see if the answer is reasonable.

a) $0.5 + 0.1$

b) $-2.7 + -3.2$

c) $7.2 + 3.27$

d) $-372 + 9.302$

e) $43.097 + 289.3887$

f) $5.03 + 16.988 + 0.006$

2. Subtract. Be sure to estimate to see if the answer is reasonable.

a) $0.8 - 0.2$

b) $-7.5 - 2.3$

c) $187.5 - 8.39$

d) $8.2 - 5.006$

e) $-632.021 - (-295.9)$

f) $1000 - 3.0947$

3. Evaluate each expression for $x = 2.4$, $y = 3$, and $z = 0.51$.

a) $x + z$

b) $y - x$

c) $x + y - z$

4. Solve.

a) Recently, Allison went shopping and spent \$18.92 at the bookstore, \$68.03 at the grocery store, and \$129.76 at a department store. What is the total amount of money Allison spent?

b) Find the perimeter of a rectangular lawn that measures 40.93 feet by 27.09 feet.

Teaching Notes:

- Remind students to work in a vertical format and line-up the decimal point and corresponding place values.
- Some students need to be shown how to add extra zeros to the ends of the decimal part of the numbers and where to place the decimal point with whole numbers.
- Some students must be reminded of how to borrow across zeros when subtracting.
- Refer students to the **Adding or Subtracting** decimals chart in the textbook.

Answers: 1a) 0.6, b) -5.9, c) 10.47, d) -362.698, e) 332.4857, f) 22.024; 2a) 0.6, b) -9.8, c) 179.11, d) 3.194, e) -336.121, f) 996.9053; 3a) 2.91, b) 0.6, c) 4.89; 4a) \$216.71, b) 136.04 ft

Mini-Lecture 4.3

Multiplying Decimals and Circumference of a Circle

Learning Objectives:

1. Multiply decimals.
2. Estimate when multiplying decimals.
3. Multiply decimals by powers of 10.
4. Evaluate expressions with decimal replacement values.
5. Find the circumference of circles.
6. Solve problems by multiplying decimals.
7. Key Vocabulary: π (*pi*), *perimeter*, *circumference*, *diameter*, *radius*.

Examples:

1. Multiply. Check by estimating.

a) 0.7×0.2

b) 1.33×-0.5

c) 7.2×5.8

d)
$$\begin{array}{r} 0.856 \\ \times 3.1 \\ \hline \end{array}$$

e)
$$\begin{array}{r} -2.00033 \\ \times -6.9 \\ \hline \end{array}$$

f)
$$\begin{array}{r} 0.0896 \\ \times 0.345 \\ \hline \end{array}$$

2. Multiply.

a) 4.3×10

b) 17.693×100

c) -0.0027×1000

d) -0.07×-0.1

e) 9.07×0.01

f) 2.908×0.001

3. Evaluate each expressions for $x = 2$, $y = -0.3$, and $z = 7.3$.

a) xy

b) $xz - y$

c) $-3y + z$

4. Find the circumference of a circle with the given information. Use $\pi = 3.14$.

a) radius = 7 feet

b) diameter = 16 inches

c) radius = 10.3 meters

5. a) Write 57.6 million in standard form.

- b) A 1-ounce serving of hot cocoa contains 0.375 grams of fat . How many grams of fat are in an 8 oz. mug of hot cocoa?

Teaching Notes:

- Some students do not see the pattern that develops when multiplying by powers of 10, they must be shown.
- Many students prefer to multiply numbers by a power of ten the long way.
- Some students will attempt to line up the decimal point (like adding) when multiplying.

Answers: 1a) Answers: 1a) 0.14, b) -0.665, c) 41.76, d) 2.6536, e) 13.802277, f) 0.030912; 2a) 43, b) 1769.3, c) -2.7, d) 0.007, e) 0.0907, f) 0.002908; 3a) -0.6, b) 14.9, c) 8.2; 4a) 43.96 ft. b) 50.24 in., c) 64.684 m; 4a) 57,600,000 , b) 3g

Mini-Lecture 4.4

Dividing Decimals

Learning Objectives:

1. Divide decimals.
2. Estimate when dividing decimals.
3. Divide decimals by powers of 10.
4. Evaluate expressions with decimal replacement values.
5. Solve problems by dividing decimals.

Examples:

1. Divide and check.

a) $1.5 \div 5$

b) $26 \overline{)7.826}$

c) $-518 \overline{)0.9324}$

d) $8.9 \overline{)22.25}$

e) $-1411.51 \div -36.1$

f) $0.02 \overline{)0.8}$

2. Divide decimals by powers of 10.

a) $\frac{7.74}{10}$

b) $1000 \overline{)-887.73}$

c) $1.047 \div 100$

3. Evaluate each expression for $x = 3.02$, $y = -0.3$, and $z = 1.51$.

a) $z \div y$

b) $z \div x$

c) $x \div y$

4. Solve.

- a) Divide 0.894 by -0.041 and round the quotient to the nearest hundredth.

- b) Preparing for a picnic, Carol went to the Deli and purchased: 0.52 pounds of salami at \$3.29/pound; 0.48 pounds of sliced turkey at \$8.99/pound; 1.04 pounds of ham at \$3.99/pound; and 0.98 pounds of cheese at \$4.29/pound. What was the total amount Carol spent at the deli? (Round your answer to the nearest cent.)

Teaching Notes:

- Most students have forgotten the mechanics of long division with decimals.
- Stress the importance of neatness so that the decimal ends up in the correct position. Some students find it helpful to do the division on graph paper or on lined paper that is rotated so the lines are vertical.
- Remind students when rounding the quotient to a specific place, you need to carry your division one more place than the rounding place.

Answers: 1a) 0.3, b) 0.301, c) -0.0018, d) 2.5, e) 39.1, f) 40; 2a) 0.774, b) -0.88773, c) 0.01047; 3a) -5.0 $\overline{3}$, b) 0.5, c) -10.0 $\overline{6}$; 4a) -21.81, b) \$14.38

Mini-Lecture 4.5

Fractions, Decimals, and Order of Operations

Learning Objectives:

1. Write fractions as decimals.
2. Compare decimals and fractions.
3. Simplify expressions containing decimals and fractions using order of operations.
4. Solve areas problems containing fractions and decimals.
5. Evaluate expressions given decimal replacement values.

Examples:

1. Write each fraction or mixed number as a decimal.

a) $\frac{3}{5}$

b) $-\frac{3}{20}$

c) $\frac{1}{3}$

d) $\frac{5}{16}$

e) $-\frac{13}{11}$

f) $-1\frac{7}{8}$

2. Insert $<$, $>$, or $=$ to form a true statement.

a) -0.0832 ____ -0.0823

b) 0.501 ____ $\frac{1}{2}$

c) 0.428 ____ $\frac{3}{7}$

Write the numbers in order from smallest to largest.

d) 0.331 , $\frac{1}{3}$, 0.330

e) 2.15 , 2.142 , $\frac{15}{7}$

f) 1.5833 , $1\frac{21}{36}$, $\frac{38}{36}$

3. Simplify each expression.

a) $(0.3)^2 - 0.4$

b) $(7.3)(100) - (7.2)(10)$

c) $\frac{4+0.42}{-2}$

d) $\frac{1}{4} - 3(6.5)$

e) $\frac{3}{5} - (6.4)(-3)$

f) $\frac{1}{5}(-9.1 - 3.3)$

4. Evaluate for $x = -2$, $y = 0.5$, and $z = 3.6$: $\frac{x}{y} + 2z$

Teaching Notes:

- Most students need a review of order of operations.
- Most students, once taught how to convert from fraction to decimal, have little problems.
- Some students have difficulty ordering numbers when they are mixed. Suggest that after converting to decimal, line the decimal points up vertically and compare corresponding place value.

Answers: 1a) 0.6, b) -0.15, c) $0.\bar{3}$, d) 0.3125, e) $-1.\bar{18}$, f) -1.875; 2a) $<$, b) $>$, c) $<$, d) 0.330, 0.331, $1/3$, e) 2.142, $15/7$, 2.15, f) $38/36$, 1.5833, $1\frac{21}{36}$; 3a) -0.31, b) 658, c) -2.21, d) -19.25, e) 19.8, f) -2.48; 4a) 3.2

Mini-Lecture 4.6

Square Roots and the Pythagorean Theorem

Learning Objectives:

1. Find the square root of a number.
2. Approximate square roots.
3. Use the Pythagorean Theorem.
4. Key Vocabulary: *square root, radical sign, positive square root, Pythagorean Theorem, leg, hypotenuse.*

Examples:

1. Find each square root.

a) $\sqrt{4}$

b) $\sqrt{25}$

c) $\sqrt{49}$

d) $\sqrt{81}$

e) $\sqrt{\frac{1}{64}}$

f) $\sqrt{\frac{16}{25}}$

g) $\sqrt{\frac{25}{49}}$

2. Use a calculator or the appropriate Appendix to approximate each square root to the thousandths position.

a) $\sqrt{2}$

b) $\sqrt{6}$

c) $\sqrt{11}$

d) $\sqrt{39}$

3. Sketch each right triangle and find the length of the side not given. If necessary, round the answer to the nearest thousandth.

a) leg = 3, leg = 4

b) leg = 12, hypotenuse = 15

c) hypotenuse = 6.4, leg = 3

4. Solve. If necessary, round to the nearest thousandth.

a) A section of land is a square with each side measuring 2 miles. Find the length of the diagonal of the section of land.

b) A garden is in the shape of a rectangle. The diagonal length of the garden is 25 feet, and the length of one of the sides is 15 feet. Find the length of the other side.

Teaching Notes:

- Some students have never done square roots on a calculator and will need guidance.
- When approximating square roots, encourage students to mentally estimate the answer. That way if they use the calculator incorrectly they might be able to notice the incorrect result.
- Most students do not have trouble using the Pythagorean Theorem for finding a hypotenuse, but some have trouble using it for finding a missing leg.

Answers: 1a) 2, b) 5, c) 7, d) 9, e) 1/8, f) 4/5, g) 5/7; 2a) 1.414, b) 2.449, c) 3.317, d) 6.245; 3a) 5, b) 9, c) 6.4; 4a) 2.828 mi, b) 20 ft