

## CHAPTER 2. NATURE OF MATERIALS

- 2.1. See Section 2.2.1.
- 2.2. See Section 2.1.
- 2.3. See Section 2.1.1.
- 2.4. See Section 2.1.1.
- 2.5. See Section 2.1.2.
- 2.6. See Section 2.2.1.
- 2.7. See Section 2.1.2.
- 2.8. See Section 2.2.1.
- 2.9. See Section 2.2.1.
- 2.10. If the atomic masses and radii are the same, then the material that crystalizes into a lattice with a higher APF will have a larger density. The FCC structure has a higher APF than the BCC structure.

- 2.11. For the face-center cubic crystal structure, number of equivalent whole atoms in each unit cell = 4

By inspection the diagonal of the face of a FCC unit cell =  $4r$

Using Pythagorean theory:

$$(4r)^2 = a^2 + a^2$$

$$16r^2 = 2a^2$$

$$8r^2 = a^2$$

$$a = 2\sqrt{2}r$$

- 2.12. a. Number of equivalent whole atoms in each unit cell in the BCC lattice structure = 2

b. Volume of the sphere =  $(4/3) \pi r^3$

$$\text{Volume of atoms in the unit cell} = 2 \times (4/3) \pi r^3 = (8/3) \pi r^3$$

By inspection, the diagonal of the cube of a BCC unit cell

$$= 4r = \sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$$

$$a = \text{Length of each side of the unit cell} = \frac{4r}{\sqrt{3}}$$

c. Volume of the unit cell =  $\left[ \frac{4r}{\sqrt{3}} \right]^3$

$$APF = \frac{\text{volume of atoms in the unit cell}}{\text{total unit volume of the cell}} = \frac{(8/3)\pi.r^3}{(4r/\sqrt{3})^3} = \mathbf{0.68}$$

**2.13.** For the BCC lattice structure:  $a = \frac{4r}{\sqrt{3}}$

$$\text{Volume of the unit cell of iron} = \left[ \frac{4r}{\sqrt{3}} \right]^3 = \left[ \frac{4 \times 0.124 \times 10^{-9}}{\sqrt{3}} \right]^3 = \mathbf{2.348 \times 10^{-29} \text{ m}^3}$$

**2.14.** For the FCC lattice structure:  $a = 2\sqrt{2}r$

$$\text{Vol. of unit cell of aluminum} = (2\sqrt{2}r)^3 = (2\sqrt{2} \times 0.143)^3 = 0.06616725 \text{ nm}^3 = \mathbf{6.6167 \times 10^{-29} \text{ m}^3}$$

**2.15.** From Table 2.3, copper has an FCC lattice structure and  $r$  of 0.1278 nm

$$\text{Volume of the unit cell of copper} = (2\sqrt{2}r)^3 = (2\sqrt{2} \times 0.1278)^3 = \mathbf{0.04723 \text{ nm}^3} = \mathbf{4.723 \times 10^{-29} \text{ m}^3}$$

**2.16.** For the BCC lattice structure:  $a = \frac{4r}{\sqrt{3}}$

$$\text{Volume of the unit cell of iron} = \left[ \frac{4r}{\sqrt{3}} \right]^3 = \left[ \frac{4 \times 0.124 \times 10^{-9}}{\sqrt{3}} \right]^3 = 2.348 \times 10^{-29} \text{ m}^3$$

$$\text{Density} = \rho = \frac{nA}{V_C N_A}$$

$$n = \text{Number of equivalent atoms in the unit cell} = 2$$

$$A = \text{Atomic mass of the element} = 55.9 \text{ g/mole}$$

$$N_A = \text{Avogadro's number} = 6.023 \times 10^{23}$$

$$\rho = \frac{2 \times 55.9}{2.348 \times 10^{-29} \times 6.023 \times 10^{23}} = 7.904 \times 10^6 \text{ g/m}^3 = \mathbf{7.904 \text{ Mg/m}^3}$$

**2.17.** For the BCC lattice structure:  $a = \frac{4r}{\sqrt{3}}$

$$\text{Vol. of the unit cell of molybdenum} = \left[ \frac{4r}{\sqrt{3}} \right]^3 = \left[ \frac{4 \times 0.1363 \times 10^{-9}}{\sqrt{3}} \right]^3 = 3.119 \times 10^{-29} \text{ m}^3$$

$$\rho = \frac{nA}{V_C N_A} = \frac{2 \times 95.94}{3.119 \times 10^{-29} \times 6.023 \times 10^{23}} = 10.215 \times 10^6 \text{ g/m}^3 = \mathbf{10.215 \text{ Mg/m}^3}$$

**2.18.** For the BCC lattice structure:  $a = \frac{4r}{\sqrt{3}}$

$$\text{Volume of the unit cell of the metal} = \left[ \frac{4r}{\sqrt{3}} \right]^3 = \left[ \frac{4 \times 0.128 \times 10^{-9}}{\sqrt{3}} \right]^3 = 2.583 \times 10^{-29} \text{ m}^3$$

$$\rho = \frac{nA}{V_C N_A} = \frac{2 \times 63.5}{2.583 \times 10^{-29} \times 6.023 \times 10^{23}} = 8.163 \times 10^6 \text{ g/m}^3 = \mathbf{8.163 \text{ Mg/m}^3}$$

**2.19.** For the FCC lattice structure:  $a = 2\sqrt{2}r$

$$\text{Volume of unit cell of the metal} = (2\sqrt{2}r)^3 = (2\sqrt{2} \times 0.132)^3 = 0.05204 \text{ nm}^3 = 5.204 \times 10^{-29} \text{ m}^3$$

$$\rho = \frac{nA}{V_C N_A} = \frac{4 \times 42.9}{5.204 \times 10^{-29} \times 6.023 \times 10^{23}} = 5.475 \times 10^6 \text{ g/m}^3 = \mathbf{5.475 \text{ Mg/m}^3}$$

**2.20.** For the FCC lattice structure:  $a = 2\sqrt{2}r$

$$\text{Volume of unit cell of aluminum} = (2\sqrt{2}r)^3 = (2\sqrt{2} \times 0.143)^3 = 0.06616725 \text{ nm}^3 = 6.6167 \times 10^{-29} \text{ m}^3$$

$$\text{Density} = \rho = \frac{nA}{V_C N_A}$$

For FCC lattice structure,  $n = 4$

$A =$  Atomic mass of the element  $= 26.98 \text{ g/mole}$

$N_A =$  Avogadro's number  $= 6.023 \times 10^{23}$

$$\rho = \frac{4 \times 26.98}{6.6167 \times 10^{-29} \times 6.023 \times 10^{23}} = 2.708 \times 10^6 \text{ g/m}^3 = \mathbf{2.708 \text{ Mg/m}^3}$$

**2.21.**  $\rho = \frac{nA}{V_C N_A}$

For FCC lattice structure,  $n = 4$

$$V_c = \frac{4 \times 63.55}{8.89 \times 10^6 \times 6.023 \times 10^{23}} = 4.747 \times 10^{-29} \text{ m}^3$$

$$\text{APF} = 0.74 = \frac{4 \times (4/3) \pi \cdot r^3}{4.747 \times 10^{-29}}$$

$$r^3 = 0.2097 \times 10^{-29} \text{ m}^3$$

$$r = 0.128 \times 10^{-9} \text{ m} = \mathbf{0.128 \text{ nm}}$$

**2.22. a.**  $\rho = \frac{nA}{V_C N_A}$

For FCC lattice structure,  $n = 4$

$$V_c = \frac{4 \times 40.08}{1.55 \times 10^6 \times 6.023 \times 10^{23}} = 1.717 \times 10^{-28} \text{ m}^3$$

$$\text{b. APF} = 0.74 = \frac{4x(4/3)\pi \cdot r^3}{1.717 \times 10^{-28}}$$

$$r^3 = 0.7587 \times 10^{-29} \text{ m}^3$$

$$r = 0.196 \times 10^{-9} \text{ m} = \mathbf{0.196 \text{ nm}}$$

$$\mathbf{2.23. \quad \frac{\rho_1}{\rho_2} = \frac{n_1 A_1 V_{c2} N_A}{V_{c1} N_A n_2 A_2} = \frac{n_1 V_{c2}}{n_2 V_{c1}}}$$

$$\frac{8.87}{\rho_2} = \frac{2 \times \left(\frac{4r}{\sqrt{3}}\right)^3}{4x(2r\sqrt{2})^3}$$

$$\mathbf{\rho_2 = 32.573 \text{ g/cm}^3}$$

**2.24.** See Section 2.2.2.

**2.25.** See Section 2.2.2.

**2.26.** See Section 2.2.2.

**2.27.** See Figure 2.14.

**2.28.** See Section 2.2.5.

**2.29.**  $m_t = 100 \text{ g}$   
 $P_B = 65 \%$   
 $P_{lB} = 30 \%$   
 $P_{sB} = 80 \%$   
 From Equations 2.4 and 2.5,  
 $m_l + m_s = 100$   
 $30 m_l + 80 m_s = 65 \times 100$   
 Solving the two equations simultaneously, we get:  
 $m_l = \text{mass of the alloy which is in the liquid phase} = \mathbf{30 \text{ g}}$   
 $m_s = \text{mass of the alloy which is in the solid phase} = \mathbf{70 \text{ g}}$

**2.30.**  $m_t = 100 \text{ g}$   
 $P_B = 45 \%$   
 $P_{lB} = 17 \%$   
 $P_{sB} = 65 \%$   
 From Equations 2.4 and 2.5,  
 $m_l + m_s = 100$   
 $17 m_l + 65 m_s = 45 \times 100$   
 Solving the two equations simultaneously, we get:  
 $m_l = \text{mass of the alloy which is in the liquid phase} = \mathbf{41.67 \text{ g}}$   
 $m_s = \text{mass of the alloy which is in the solid phase} = \mathbf{58.39 \text{ g}}$

**2.31.**  $m_t = 100 \text{ g}$

$P_B = 60 \%$

$P_{lB} = 25 \%$

$P_{sB} = 70 \%$

From Equations 2.4 and 2.5,

$m_l + m_s = 100$

$25 m_l + 70 m_s = 60 \times 100$

Solving the two equations simultaneously, we get:

$m_l = \text{mass of the alloy which is in the liquid phase} = \mathbf{22.22 \text{ g}}$

$m_s = \text{mass of the alloy which is in the solid phase} = \mathbf{77.78 \text{ g}}$

**2.32.**  $m_t = 100 \text{ g}$

$P_B = 40 \%$

$P_{lB} = 20 \%$

$P_{sB} = 50 \%$

From Equations 2.4 and 2.5,

$m_l + m_s = 100$

$40 m_l + 50 m_s = 40 \times 100$

Solving the two equations simultaneously, we get:

$m_l = \text{mass of the alloy which is in the liquid phase} = \mathbf{33.33 \text{ g}}$

$m_s = \text{mass of the alloy which is in the solid phase} = \mathbf{66.67 \text{ g}}$

**2.33. a.** Spreading salt reduces the melting temperature of ice. For example, at a salt composition of 5%, ice starts to melt at  $-21^\circ\text{C}$ . When temperature increases more ice will melt. At a temperature of  $-5^\circ\text{C}$ , all ice will melt.

**b.**  $-21^\circ\text{C}$

**c.**  $-21^\circ\text{C}$

**2.34.** See Section 2.3.

**2.35.** See Section 2.3.

**2.36.** See Section 2.4.