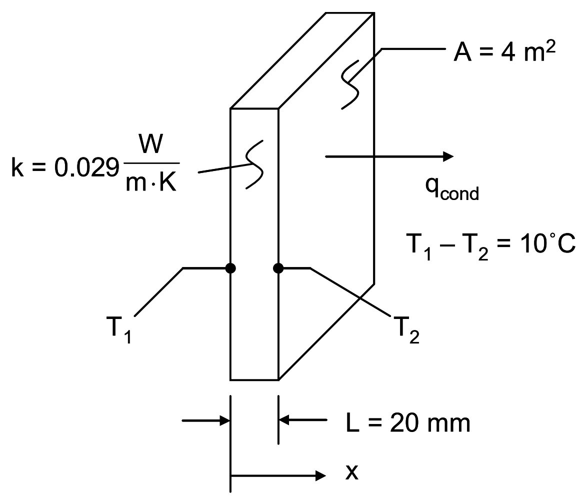
PROBLEM 1.1

KNOWN: Thermal conductivity, thickness and temperature difference across a sheet of rigid extruded insulation.

FIND: (a) The heat flux through a 2 m × 2 m sheet of the insulation, and (b) The heat rate through the sheet.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: From Equation 1.2 the heat flux is

$$q''_x = -k\frac{dT}{dx} = k\frac{T_1 - T_2}{L}$$

Solving,

$$q''_x = 0.029\frac{W}{m \cdot K} \times \frac{10\ K}{0.02\ m}$$

$$q''_x = 14.5\frac{W}{m^2}$$ <

The heat rate is

$$q_x = q''_x \cdot A = 14.5\frac{W}{m^2} \times 4\ m^2 = 58\ W$$ <

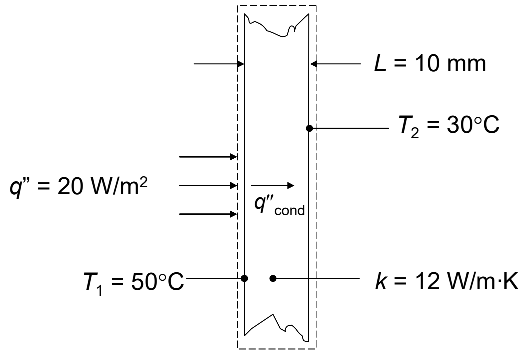
COMMENTS: (1) Be sure to keep in mind the important distinction between the heat flux (W/m^2) and the heat rate (W). (2) The direction of heat flow is from hot to cold. (3) Note that a temperature *difference* may be expressed in kelvins or degrees Celsius.

PROBLEM 1.2

KNOWN: Thickness and thermal conductivity of a wall. Heat flux applied to one face and temperatures of both surfaces.

FIND: Whether steady-state conditions exist.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal energy generation.

ANALYSIS: Under steady-state conditions an energy balance on the control volume shown is

$$q''_{in} = q''_{out} = q''_{cond} = k(T_1 - T_2)/L = 12 \text{ W/m} \cdot \text{K}(50°\text{C} - 30°\text{C})/0.01 \text{ m} = 24{,}000 \text{ W/m}^2$$

Since the heat flux in at the left face is only 20 W/m^2, the conditions are not steady state. <

COMMENTS: If the same heat flux is maintained until steady-state conditions are reached, the steady-state temperature difference across the wall will be

$$\Delta T = q''L/k = 20 \text{ W/m}^2 \times 0.01 \text{ m}/12 \text{ W/m} \cdot \text{K} = 0.0167 \text{ K}$$

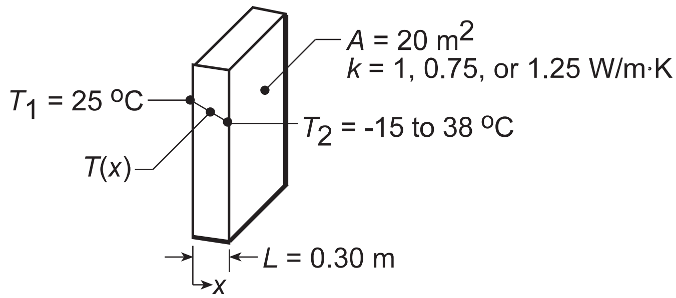
which is much smaller than the specified temperature difference of 20°C.

PROBLEM 1.3

KNOWN: Inner surface temperature and thermal conductivity of a concrete wall.

FIND: Heat loss by conduction through the wall as a function of outer surface temperatures ranging from -15 to 38°C.

SCHEMATIC:



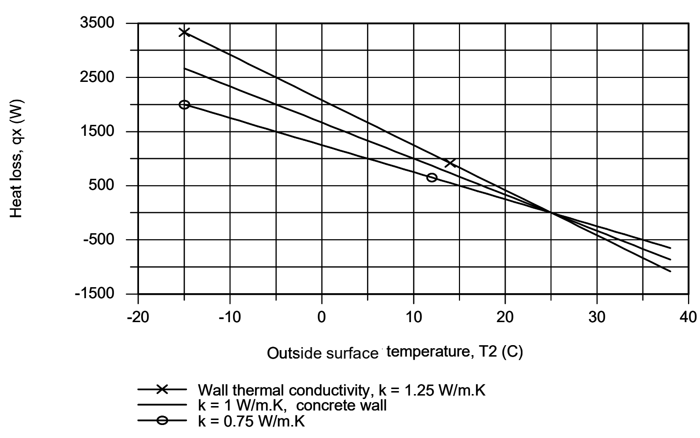
ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: From Fourier's law, if q''_x and k are each constant it is evident that the gradient, $dT/dx = -q''_x/k$, is a constant, and hence the temperature distribution is linear. The heat flux must be constant under one-dimensional, steady-state conditions; and k is approximately constant if it depends only weakly on temperature. The heat flux and heat rate when the outside wall temperature is T_2 = -15°C are

$$q''_x = -k\frac{dT}{dx} = k\frac{T_1 - T_2}{L} = 1\,W/m\cdot K\frac{25^\circ C - (-15^\circ C)}{0.30\,m} = 133.3\,W/m^2 . \qquad (1)$$

$$q_x = q''_x \times A = 133.3\,W/m^2 \times 20\,m^2 = 2667\,W . \qquad (2) <$$

Combining Eqs. (1) and (2), the heat rate q_x can be determined for the range of outer surface temperature, $-15 \le T_2 \le 38$°C, with different wall thermal conductivities, k.



For the concrete wall, k = 1 W/m·K, the heat loss varies linearly from +2667 W to -867 W and is zero when the inside and outer surface temperatures are the same. The magnitude of the heat rate increases with increasing thermal conductivity.

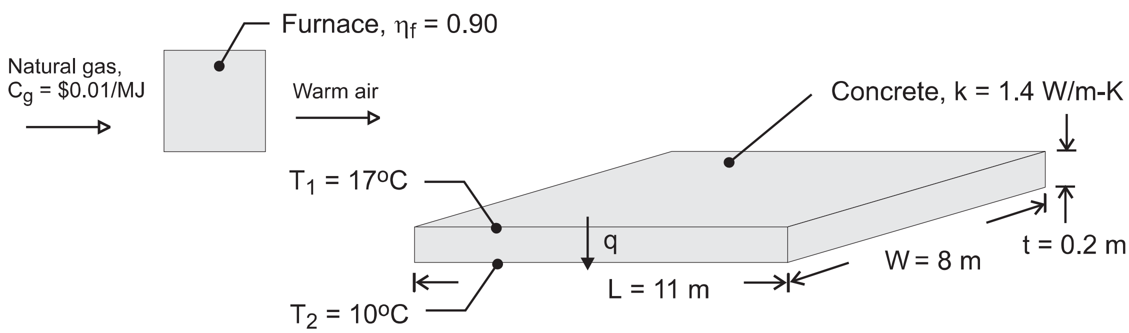
COMMENTS: Without steady-state conditions and constant k, the temperature distribution in a plane wall would not be linear.

PROBLEM 1.4

KNOWN: Dimensions, thermal conductivity and surface temperatures of a concrete slab. Efficiency of gas furnace and cost of natural gas.

FIND: Daily cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: The rate of heat loss by conduction through the slab is

$$q = k\,(LW)\frac{T_1 - T_2}{t} = 1.4\,W/m\cdot K\,(11\,m \times 8\,m)\frac{7°C}{0.20\,m} = 4312\ W$$
<

The daily cost of natural gas that must be combusted to compensate for the heat loss is

$$C_d = \frac{q\,C_g}{\eta_f}(\Delta t) = \frac{4312\,W \times \$0.02/MJ}{0.9 \times 10^6 J/MJ}(24\,h/d \times 3600\,s/h) = \$8.28/d$$
<

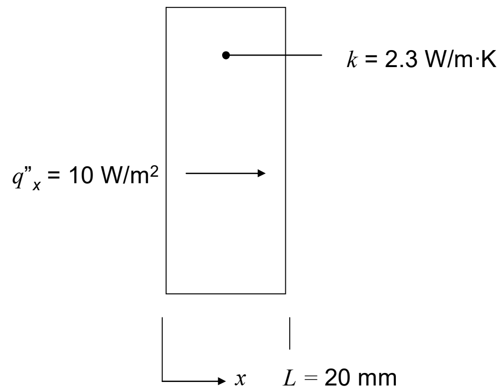
COMMENTS: The loss could be reduced by installing a floor covering with a layer of insulation between it and the concrete.

PROBLEM 1.5

KNOWN: Thermal conductivity and thickness of a wall. Heat flux through wall. Steady-state conditions.

FIND: Value of temperature gradient in K/m and in °C/m.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties.

ANALYSIS: Under steady-state conditions,

$$\frac{dT}{dx} = -\frac{q_x''}{k} = -\frac{10 \text{ W/m}^2}{2.3 \text{ W/m} \cdot \text{K}} = -4.35 \text{ K/m} = -4.35 \text{ °C/m}$$ $<$

Since the K units here represent a temperature *difference*, and since the temperature difference is the same in K and °C units, the temperature gradient value is the same in either units.

COMMENTS: A negative value of temperature gradient means that temperature is decreasing with increasing *x*, corresponding to a positive heat flux in the *x*-direction.