Instructor's Resource Guide
Project Solutions

Calculus

ELEVENTH EDITION

Ron Larson
The Pennsylvania University,
The Behrend College

Bruce Edwards
University of Florida

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Cengage Learning
20 Channel Center Street
Boston, MA 02210
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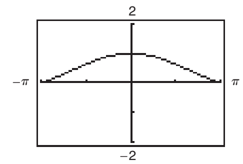
SECTION PROJECTS

Chapter 1, Section 5, page 94 Graphs and Limits of Trigonometric Functions

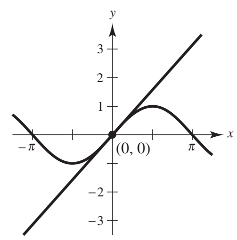
(a) On the graph of f, it appears that the y-coordinates of points lie as close to 1 as desired as long as you consider only those points with an x-coordinate near to but not equal to 0.



(b) Use a table of values of x and $f(x)$ that includes several values of x near 0. Check to see if the corresponding values of $f(x)$ are close to 1. In this case, because f is an even function, only positive values of x are needed.

x	0.5	0.1	0.01	0.001
$f(x)$	0.9589	0.9983	1.0000	1.0000

(c) The slope of the sine function at the origin appears to be 1. (It is necessary to use radian measure and have the same unit of length on both axes.)



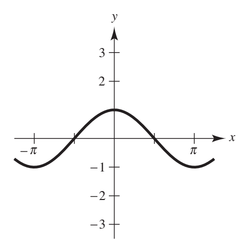
(d) In the notation of Section 1.1, $c = 0$ and $c + \Delta x = x$. Thus, $m_{\text{sec}} = \dfrac{\sin x - 0}{x - 0}$.

This formula has a value of 0.998334 if $x = 0.1$; $m_{\text{sec}} = 0.999983$ if $x = 0.01$.

The exact slope of the tangent line to g at $(0, 0)$ is $\lim\limits_{x \to 0} m_{\text{sec}} = \lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$.

(e) The slope of the tangent line to the cosine function at the point $(0, 1)$ is 0. The analytical proof is as follows:

$$\lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} \frac{\cos(0 + \Delta x) - 1}{\Delta x} = -\lim_{\Delta x \to 0} \frac{1 - \cos(\Delta x)}{\Delta x} = 0.$$



(f) The slope of the tangent line to the graph of the tangent function at $(0, 0)$ is:

$$\lim_{\Delta x \to \infty} m_{\text{sec}} = \lim_{\Delta x \to 0} \frac{\tan(0 + \Delta x) - 0}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sin(\Delta x)}{\Delta x} \cdot \frac{1}{\cos \Delta x} = 1 \cdot \frac{1}{1} = 1.$$

Chapter 2, Section 5, page 151 Optical Illusions

(a)
$$x^2 + y^2 = C^2$$
$$2x + 2yy' = 0$$
$$y' = -\frac{x}{y}$$

at the point $(3, 4)$, $y' = -\dfrac{3}{4}$

(b)
$$xy = C$$
$$xy' + y = 0$$
$$y' = -\frac{y}{x}$$

at the point $(1, 4)$, $y' = -4$

(c) $ax = by$

$a = by'$

$y' = \frac{a}{b}$

at $a = \sqrt{3}$ and $b = 1$, $y' = \sqrt{3}$

(d) $y = C \cos x$

$y' = -C \sin x$

at $x = \frac{\pi}{3}$ and

$C = \frac{2}{3}, y' = -\frac{2}{3} \sin \frac{\pi}{3} = -\frac{2}{3}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{3}}{3}$

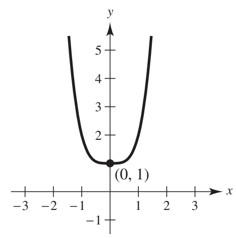
Chapter 3, Section 3, page 190 Even Fourth-Degree Polynomials

(a) (i) $f(x) = x^4 + 1, f'(x) = 4x^3$

$x = 0$, critical number

Relative minimum at $(0, 1)$

Increasing on $(0, \infty)$, decreasing on $(-\infty, 0)$



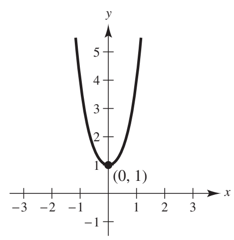
(ii) $f(x) = x^4 + 2x^2 + 1$

$f'(x) = 4x^3 + 4x = 4x(x^2 + 1)$

$x = 0$, critical number

Relative minimum at $(0, 1)$

Increasing on $(0, \infty)$, decreasing on $(-\infty, 0)$



(iii) $f(x) = x^4 - 2x^2 + 1$

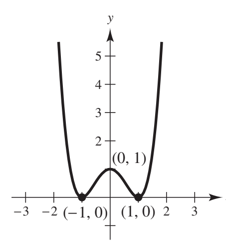
$f'(x) = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x + 1)(x - 1)$

$x = 0, \pm 1$, critical numbers

Relative minimum at $(-1, 0), (1, 0)$

Relative maximum at $(0, 1)$

Increasing on $(-1, 0)$ and $(1, \infty)$, decreasing on $(-\infty, -1)$ and $(0, 1)$



(b) $f(x) = x^4 + ax^2 + b$

$f(-x) = (-x^4) + a(-x)^2 + b = x^4 + ax^2 + b = f(x)$

Therefore, f is even.

(i) If $a = 0$, $f(x) = x^4 + b$ and $f'(x) = 4x^3$.

So, $x = 0$ is the only critical number.

f is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$.

(ii) If $a > 0$, $f(x) = x^4 + ax^2 + b$ and $f'(x) = 4x^3 + 2ax = 2x(2x^2 + a)$.

Since $a > 0$, $2x^2 + a > 0$ and there is only one critical number, $x = 0$.

f is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$.

(iii) If $a < 0$, $f(x) = x^4 + ax^2 + b$ and $f'(x) = 4x^3 + 2ax = 2x(2x^2 + a)$.

Since $a < 0$, there are three critical numbers $x = 0$, $x = \pm\sqrt{\dfrac{a}{2}}$.

f is increasing on $\left(-\sqrt{\frac{a}{2}}, 0\right)$ and $\left(\sqrt{\frac{a}{2}}, \infty\right)$ and decreasing on $\left(-\infty, -\sqrt{\frac{a}{2}}\right)$ and $\left(0, \sqrt{\frac{a}{2}}\right)$.

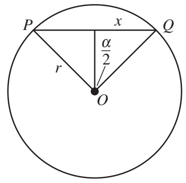
(iv) $f(x) = x^4 + ax^2 + b \Rightarrow x^2 = \dfrac{-a \pm \sqrt{a^2 - 4b}}{2}$ by the Quadratic Formula. If $a^2 < 4b$, then the term inside the radical is negative and there are no real zeros.

(v) If $a^2 \geq 4b$, then the term inside the radical is either 0 or positive. So, there would be at most four real zeros.

Chapter 3, Section 7, page 228 Minimum Time

(a) The length of arc $\widehat{AB}$ is $r\theta = 2\theta$.

(b) Consider the chord $\overline{PQ}$ determined by an angle α.



Then $\sin \dfrac{\alpha}{2} = \dfrac{x}{r}$, and $PQ = 2r \sin \dfrac{\alpha}{2}$.

So, the distance swam from point B to point C is given by

$$2r \sin\left(\frac{\pi - \theta}{2}\right) = 4 \sin\left(\frac{\pi - \theta}{2}\right) = 4 \cos \frac{\theta}{2}.$$

(c) Since time $= \dfrac{\text{distance}}{\text{rate}}$, the total time to move from point A to point C is

$$f(\theta) = \frac{2\theta}{v_1} + \frac{4 \cos \frac{\theta}{2}}{v_2}$$

The domain of f is $0 \leq \theta \leq \pi$.

(d) $f'(\theta) = \dfrac{2}{v_1} - \dfrac{2 \sin \frac{\theta}{2}}{v_2}$

(e) $f'(\theta) = \frac{2}{5} - \sin\frac{\theta}{2} = 0$

Using a graphing utility, $\theta \approx 0.823$ and $f(\theta) = \frac{2}{5}\theta + \frac{4}{2}\cos\frac{\theta}{2} \approx 2.16.$

Checking the endpoints, $f(0) = 2$ and $f(\pi) \approx 1.26.$

So, the critical number corresponds to a maximum. The point *B* should be at point *C*, and the woman should walk the entire way.

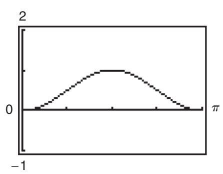
(f) $f'(\theta) = \frac{2}{3} - \sin\frac{\theta}{2} = 0.$

Using a graphing utility, $\theta \approx 1.459$ and $f(\theta) = \frac{2}{3}\theta + \frac{4}{2}\cos\frac{\theta}{2} \approx 2.46.$

Checking the endpoints, $f(0) = 2$ and $f(\pi) \approx 2.09.$

So, the critical number corresponds to a maximum. The point *B* should be at point *A*, and the woman should swim the entire way.

Chapter 4, Section 4, page 295 Demonstrating the Fundamental Theorem

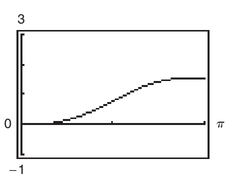


(a)

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$F(x)$	0	0.0453	0.3071	0.7854	1.2637	1.5255	1.5708

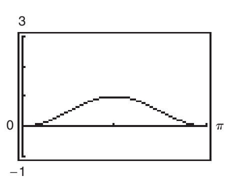
According to the Second Fundamental Theorem of Calculus, $F'(x) = \sin^2 x$, which is positive for all x in the interval $(0, \pi)$. By Theorem 3.5, F is increasing.

(b)



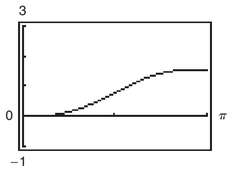
$F(x) = \int_0^x \sin^2 t\,dt$

(c)



The graph in part (b) is increasing because this graph of F' is always positive (except at the endpoints).

$F'(x) = \frac{d}{dx}\int_0^x \sin^2 t\,dt$

(d) 

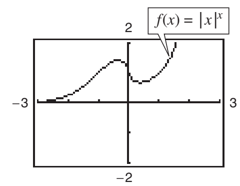
This graph is identical to the graph in part (b) and therefore, its derivative, $\sin^2 t$, has the same graph that appears in part (c).

$$y = \frac{1}{2}t - \frac{\sin 2t}{4}$$

$$\frac{dy}{dt} = \frac{1}{2} - \frac{2\cos 2t}{4} = \frac{1 - \cos 2t}{2} = \sin^2 t$$

Chapter 5, Section 5, page 361 Using Graphing Utilities to Estimate Slope

(a) The domain of f is the set of all real numbers. (If your graphing utility does not display points on the graph of $y = |x|^x$ with $x < 0$, try graphing an equivalent expression such as $y = e^{x\ln|x|}$.)



(b) Indications are that $\lim_{x\to 0} f(x) = 1 = f(0)$.

(c) If $x \neq 0$, then $f(x) = e^{x\ln|x|}$ and $f'(x) = e^{x\ln|x|}\left(x \cdot \frac{1}{x} + \ln|x|\right) = |x|^x(1 + \ln|x|)$. Thus, by Theorem 2.1, f is continuous at each nonzero real number. Furthermore, the conclusion of part (b) means that f is also continuous at zero.

(d) The y-axis appears to be a tangent line. To be more confident about this, change the viewing window to $-1 \leq x \leq 1$ and $0.5 \leq y \leq 1.5$. Apparently, f has no slope at the point $(0, 1)$.

(e) By definition,

$$f'(x) = \lim_{\Delta x\to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

if the limit exists. Because $-\Delta x$ is near zero if Δx is, you also have

$$f'(x) = \lim_{\Delta x\to 0} \frac{f(x - \Delta x) - f(x)}{-\Delta x}.$$

The formula of interest is the average:

$$\frac{1}{2}\left[\frac{f(x + \Delta x) - f(x)}{\Delta x} + \frac{f(x - \Delta x) - f(x)}{-\Delta x}\right] = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}$$

If $f'(x)$ exists, then taking limits on both sides of the above equation gives

$$f'(x) = \frac{1}{2}\left[f'(x) + f'(x)\right] = \lim_{\Delta x\to 0} \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x}.$$

Trying to estimate $f'(0)$ by calculating

$$\frac{f(\Delta x) - f(-\Delta x)}{2\Delta x}$$

produces various negative values. For instance, if $\Delta x = 0.0001$, then the estimate is about -9.21. It appears that

$$\lim_{\Delta x\to 0} \frac{f(\Delta x) - f(-\Delta x)}{2\Delta x} = -\infty.$$

The graph has no slope at $(0, 1)$.

(f) As was shown in the answer to part (b), $f'(x) = |x|^x(1 + \ln|x|)$ if $x \neq 0$. There is no formula for $f'(0)$ because it does not exist. If a graphing utility does not have the same unit of length on both axes, you might incorrectly approximate slopes. You also might approximate slopes incorrectly if the detail of the graph is insufficient, perhaps because the viewing window is too large.

(g) $f'(x) = 0$ if $\ln|x| = -1$. It follows that besides 0, the critical numbers of *f* are $\pm 1/e$. Evidently, the relative extrema of *f* are a relative minimum,

$$f\left(\frac{1}{e}\right) = 0.69220,$$

and a relative maximum,

$$f\left(-\frac{1}{e}\right) = 1.44467.$$

Chapter 5, Section 9, page 399   Mercator Map

(a) Let a positive latitude ϕ be given. If *n* is a positive integer, let $\Delta\phi = \phi n$. Then the total vertical distance from the equator to the *n*th latitude line is a Reimann sum that approximates the integral $\int_0^\phi R \sec \lambda \, d\lambda = R \ln(\sec \phi + \tan \phi)$.

(b) $6 \ln(\sec 30° + \tan 30°) \approx 3.2958$

$6 \ln(\sec 45° + \tan 45°) \approx 5.2882$

$6 \ln(\sec 60° + \tan 60°) \approx 7.9017$

(c) You fail to find a finite value. Both $\sec \phi$ and $\tan \phi$ become infinitely large as ϕ increases to $90°$.

(d) $\displaystyle\int \frac{dt}{\cosh t} = \int \frac{\cosh t}{\cosh^2 t}\,dt = \int \frac{\cosh t}{1 + \sinh^2 t}\,dt$

Let $u = \sinh t$, $du = \cosh t \, dt$.

Then you have $\displaystyle\int \frac{du}{1 + u^2} = \arctan u + C \Rightarrow \arctan(\sinh t) + C.$

So, $\displaystyle\int_0^y \frac{dt}{\cosh t} = \arctan(\sinh y) - \arctan(\sinh 0) = \arctan(\sinh y).$

Chapter 6, Section 4, page 438   Weight Loss

(a) Separate variables and integrate, using a convenient form of the constant term.

$$\frac{dw}{dt} = \frac{C}{3500} - \frac{17.5w}{3500} = \frac{C - 17.5w}{3500}$$

$$\frac{dw}{17.5w - C} = \frac{dt}{-3500}$$

$$\int \frac{dw}{17.5w - C} = \int \frac{dt}{-3500}$$

$$\frac{\ln\left|17.5w - C\right|}{17.5} = -\frac{t}{3500} + k_1$$

$$\ln\left|17.5w - C\right| = -\frac{17.5t}{3500} + k_2$$

$$e^{(-17.5t/3500)+k_2} = 17.5w - C$$

$$17.5w - C = ke^{(-17.5t/3500)}$$

(Assume that k and $17.5w - C$ have the same sign.)

If a person weighs w_0 pounds at a time $t = 0$, then $k = 17.5w_0 - C$. Therefore,

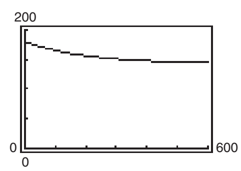
$$w = \frac{C}{17.5} + \left(w_0 - \frac{C}{17.5}\right)e^{-t/200}$$

(b) Solving for t in the answer to part (a) gives $t = 200 \ln \dfrac{17.5w_0 - C}{17.5w - C}$.

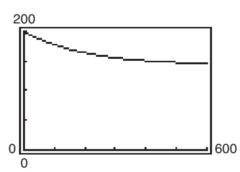
To lose 10 pounds will take $t = 200 \ln \dfrac{17.5(180) - 2500}{17.5(170) - 2500} \approx 63$ days.

Similarly, to lose 35 pounds will take approximately 571 days.

(c) The "limiting" weight is $\lim_{t\to\infty} w = \dfrac{2500}{17.5} \approx 143$ pounds.



(d) To lose 10 pounds will take approximately 38 days.
To lose 35 pounds will take about 190 days.
The "limiting" weight is still 143 pounds.



Chapter 7, Section 3, page 473 Saturn

(a) Volume of sphere:

$$V = \frac{4}{3}\pi(60{,}268)^3 \approx 9.16957 \times 10^{14}$$

Volume of oblate ellipsoid:

$$V = 2\pi \int_0^{60{,}268} 2(54{,}364)x\sqrt{1 - \frac{x^2}{60{,}268^2}}\,dx$$

$$= 217{,}456\pi \int_0^{60{,}268} \left(1 - \frac{x^2}{60{,}268^2}\right)^{1/2} x\,dx$$

$$= \left[(217{,}456\pi)\left(-\frac{60{,}268^2}{2}\right)\left(\frac{2}{3}\right)\left(1 - \frac{x^2}{60{,}268^2}\right)^{3/2}\right]_0^{60{,}268} \approx 8.27130 \times 10^{14}$$

Ratio: $\dfrac{8.27130 \times 10^{14}}{9.16957 \times 10^{14}} \approx 0.902$

(b)

$$V = \frac{4}{3}\pi r^3$$

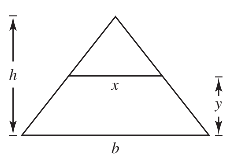
$$8.2713 \times 10^{14} = \frac{4}{3}\pi r^3$$

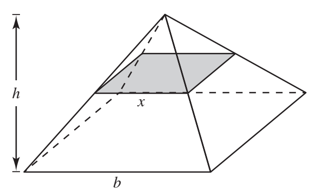
$$8.2713 \times 10^{14} \cdot \frac{3}{4\pi} = r^3$$

$$r \approx 58{,}232 \text{ km}$$

Chapter 7, Section 5, page 493 Pyramid of Khufa

(a) First calculate the work needed to build a general pyramid of square base b, height h, and material density ρ.





By similar triangles, $\dfrac{x}{h - y} = \dfrac{b}{h} \Rightarrow x = \dfrac{b}{h}(h - y)$.

The volume of a slice is $x^2\,dy = \dfrac{b^2}{h^2}(h - y)^2\,dy$.

The work is

$$\int_0^h \rho\frac{b^2}{h^2}(h - y)^2 y\,dy = \frac{1}{12}\rho b^2 h^2$$

For the Great Pyramid of Giza, $\rho = 150$, $h = 481$, and $b = 756$. So, the work is

$$w = \frac{1}{12}\rho b^2 h^2 \approx 1.653 \times 10^{12} \text{ foot-pounds.}$$

(b) Over 20 years, each worker did

$$\left(200\frac{\text{ft-lb}}{\text{hr}}\right)\left(12\frac{\text{hr}}{\text{day}}\right)\left(330\frac{\text{day}}{\text{yr}}\right)(20 \text{ yr}) \approx 1.584 \times 10^7 \text{ foot-pounds.}$$

So, the number of workers is approximately

$$\frac{1.653 \times 10^{12}}{1.584 \times 10^7} \approx 104{,}000.$$

Chapter 8, Section 3, page 540 The Wallis Product

(a) $I(2) = \left(\frac{1}{2}\right)\left(\frac{\pi}{2}\right) = \frac{\pi}{4} \approx 0.7854$

$I(3) = \frac{2}{3} \approx 0.6667$

$I(4) = \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{\pi}{2}\right) = \frac{3\pi}{16} \approx 0.5890$

$I(5) = \left(\frac{2}{3}\right)\left(\frac{4}{5}\right) = \frac{8}{15} \approx 0.5333$

It appears that $I(n + 1) \le I(n)$.

(b) $\sin^{n+1} x = \sin x \cdot \sin^n x \le \sin^n x$ on $\left[0, \frac{\pi}{2}\right]$

$I(n + 1) = \int_0^{\pi/2} \sin^{n+1} x\, dx \le \int_0^{\pi/2} \sin^n x\, dx = I(n)$

(c) First, note that $\frac{I(2n + 2)}{I(2n)} = \frac{2n + 1}{2n + 2}$.

So, $\frac{2n + 1}{2n + 2} = \frac{I(2n + 2)}{I(2n)} \le \frac{I(2n + 1)}{I(2n)} \le 1$.

By the Squeeze Theorem, $\lim_{n\to\infty} \frac{I(2n + 1)}{I(2n)} = 1$.

(d) $\lim_{n\to\infty} \frac{I_{2n+1}}{I_{2n}} = \left(\frac{2}{3} \cdot \frac{4}{5} \cdots \frac{2n}{2n + 1}\right) \Big/ \left(\frac{1}{2} \cdot \frac{3}{4} \cdots \frac{2n - 1}{2n}\right)\frac{\pi}{2} = 1$

$$\Rightarrow \lim_{n\to\infty} \frac{2 \cdot 4 \cdot 6 \cdots 2n}{3 \cdot 5 \cdot 7 \cdots (2n + 1)} \cdot \frac{2 \cdot 4 \cdots 2n}{1 \cdot 3 \cdot 5 \cdots 2n - 1} = \frac{\pi}{2}$$

$$\Rightarrow \lim_{n\to\infty} \frac{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdots (2n)(2n)}{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdots (2n - 1)(2n + 1)} = \frac{\pi}{2}$$

Chapter 9, Section 2, page 608 Cantor's Disappearing Table

$$L - \frac{1}{4}L - 2\left(\frac{1}{16}\right)L - 4\left(\frac{1}{64}\right)L - 8\left(\frac{1}{256}\right)L - \cdots = L - L\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots\right)$$

$$= L - L\sum_{n=0}^{\infty} \frac{1}{4}\left(\frac{1}{2}\right)^n = L\left(1 - \frac{1/4}{1 - (1/2)}\right) = \frac{1}{2}L$$

The remaining pieces are getting smaller and smaller, thus making the table appear to disappear.

Chapter 9, Section 3, page 615 The Harmonic Series

(a) Grouping shows that the sum of the first 2^r terms of the harmonic series exceeds $1 + \frac{r}{2}$. It follows that no number L can be the limit of the partial sums. In fact, if you pick any positive integer $r > 2L$, then $S_n > 1 + L$ for all $n > 2^r$.

(b) Since $f(x) = 1/x$, the inequality

$$\sum_{i=2}^{n} f(i) \le \int_1^n f(x)\,dx$$

appearing in the proof of Theorem 9.10 means

$$\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \le \ln n.$$

Also, the inequality

$$\int_1^n f(x)\,dx \le \sum_{i-1}^{n-1} f(i),$$

with n replaced by $n + 1$, means

$$\ln(n+1) = \int_1^{n+1} f(x)\,dx \le \sum_{i=1}^{n} f(i) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}.$$

The combination means $\ln(n+1) \le 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \le 1 + \ln n.$

(c) According to part (b), $\sum_{n=1}^{M} \frac{1}{n} > 50$ if $\ln(M+1) > 50$. Because logarithmic and exponential functions are increasing, you want

$$M + 1 = e^{\ln(M+1)} > e^{50} \approx 5.1847 \times 10^{21}.$$

Therefore, $M = 5.185 \times 10^{21}$, for example, will do.

(d) Part (b) implies that $1 + \frac{1}{2} + \cdots + \frac{1}{1{,}000{,}000} \le 1 + \ln(1{,}000{,}000) < 14.816 < 15.$

(e) Suppose that $1 < r \le s + 1$. The exact area of the plane region that lies above the interval $[r - 1, s]$ and below the graph of $y = 1/x$ is $\ln[s/(r-1)]$. The total area of $s - r + 1$ rectangles of width 1 inscribed in the region is

$$\frac{1}{r} + \cdots + \frac{1}{s}.$$

The exact area of the plane region over the interval $[r, s + 1]$ and under the curve $y = 1/x$ is $\ln[(s+1)/r]$. The total area of $s - r + 1$ rectangles of width 1 circumscribing the region is

$$\frac{1}{r} + \cdots + \frac{1}{s}.$$

Therefore,

$$\ln\frac{s+1}{r} < \frac{1}{r} + \cdots + \frac{1}{s} < \ln\frac{s}{r-1}.$$

Take $r = 10$ and $s = 20$ to get the first inequality. For the second inequality, use $r = 100$ and $s = 200$.

(f) From the solution to part (e),

$$\ln\frac{2m+1}{m} \le \sum_{n=m}^{2m} \frac{1}{n} \le \ln\frac{2m}{m-1}.$$

By the Squeeze Theorem for Sequences, Theorem 9.3, $\lim_{m\to\infty} \sum_{n=m}^{2m} \frac{1}{n} = \ln 2.$

Chapter 10, Section 2, page 709 Cycloids

I. $H(8, 3)$ matches (d).

II. $E(8, 3)$ matches (c).

III. $H(8, 7)$ matches (a).

IV. $E(24, 3)$ matches (e).

V. $H(24, 7)$ matches (f).

VI. $E(24, 7)$ matches (b).

Chapter 10, Section 4, page 728 Cassini Oval

(a)
$$\left[d\big((x, y), (-c, 0)\big)\, d\big((x, y), (c, 0)\big)\right]^2 = b^4$$
$$\left[(x + c)^2 + y^2\right]\left[(x - c)^2 + y^2\right] = b^4$$
$$\left[(x + c)(x - c)\right]^2 + y^2\left[(x + c)^2 + (x - c)^2\right] + y^4 = b^4$$
$$\left(x^2 - c^2\right)^2 + y^2\left(2x^2 + 2c^2\right) + y^4 = b^4$$
$$x^4 - 2x^2c^2 + c^4 + 2x^2y^2 + 2y^2c^2 + y^4 = b^4$$
$$\left(x^2 + y^2\right)^2 - 2c^2\left(x^2 - y^2\right) + c^4 = b^4$$

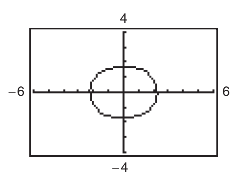
(b) Let $x = r\cos\theta$ and $y = r\sin\theta$. Then $x^2 + y^2 = r^2$ and $x^2 - y^2 = r^2\cos 2\theta$.

$$\left(x^2 + y^2\right)^2 - 2c^2\left(x^2 - y^2\right) + c^4 = b^4$$
$$r^4 - 2c^2r^2\cos 2\theta = b^4 - c^4$$

(c) Letting $b = c$, you have

$$r^4 - 2c^2r^2\cos 2\theta = 0$$
$$r^2 = 2c^2\cos 2\theta, \text{ a lemniscate.}$$

(d)



Chapter 11, Section 5, page 797 Distances in Space

(a) (i) The lines are not parallel because the direction vector $\langle 5, 5, -4\rangle$ for the line L_1 is not a scalar multiple of the direction vector $\langle 1, 8, -3\rangle$ for L_2.

(ii) At a point (x, y, z) of intersection, the parameters *s* and *t* would satisfy the system of equations $4 + 5t = x = 4 + s, 5 + 5t = y = -6 + 8s,$ and $1 - 4t = z = 7 - 3s.$ The lines do not intersect because, as the following computation shows, the system is inconsistent.

$$\begin{cases} s - 5t = 0 \\ 8s - 5t = 11 \\ -3s + 4t = -6 \end{cases}$$

$$\begin{cases} s - 5t = 0 & \\ 35t = 11 & (\text{equation } 2) - 8(\text{equation } 1) \\ -11t = -6 & (\text{equation } 3) + 3(\text{equation } 1) \end{cases}$$

$$\begin{cases} s - 5t = 0 & \\ 35t = 11 & \\ 0 = -\frac{89}{35} & (\text{equation } 3) + \frac{11}{35}(\text{equation } 2) \end{cases}$$

(iii) The vector $\langle 5, 5, -4\rangle \times \langle 1, 8, -3\rangle = \langle 17, 11, 35\rangle$ is orthogonal to the direction vectors for L_1 and L_2. Clearly $(4, 5, 1)$ is a point on L_1 and $(4, -6, 7)$ is on L_2. Therefore, L_1 lies in the plane

$$17(x - 4) + 11(y - 5) + 35(z - 1) = 0$$

and L_2 lies in the parallel plane

$$17(x - 4) + 11(y + 6) + 35(z - 7) = 0.$$

(iv) The distance between L_1 and L_2 is the same as the distance *D* from a point such as $(4, 5, 1)$ in one of the parallel planes to the other plane, L_2.

$$D = \frac{\left|17 \cdot (4 - 4) + 11 \cdot (-6 - 5) + 35 \cdot (7 - 1)\right|}{\sqrt{17^2 + 11^2 + 35^2}} = \frac{89}{\sqrt{1635}} \approx 2.2.$$

(b) The vector $\langle 2, 4, 6\rangle \times \langle -1, 1, 1\rangle = \langle -2, -8, 6\rangle$ is normal to the parallel planes containing L_1 and L_2. The origin is a point on L_1. Therefore, L_1 lies in the plane $-2x - 8y + 6z = 0$. Because the point $(1, 4, -1)$ is on L_2, the distance between L_1 and L_2 is

$$\frac{\left|-2 \cdot 1 - 8 \cdot 4 + 6 \cdot (-1)\right|}{\sqrt{4 + 64 + 36}} = \frac{20}{\sqrt{26}} \approx 3.9.$$

(c) The vector $\langle 3, -1, 1\rangle \times \langle 4, 1, -3\rangle = \langle 2, 13, 7\rangle$ is normal to the parallel planes containing L_1 and L_2. The point $(0, 2, -1)$ is on L_1. Therefore, L_1 lies in the plane $2x + 13(y - 2) + 7(z + 1) = 0$. Because the point $(1, -2, -3)$ is on L_2, the distance between the lines is

$$\frac{\left|2 + 13(-4) + 7(-2)\right|}{\sqrt{4 + 169 + 49}} = \frac{64}{\sqrt{222}} \approx 4.3.$$

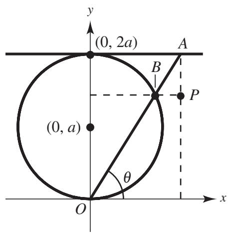
(d) Let $\mathbf{n} = \langle a_1, b_1, c_1\rangle \times \langle a_2, b_2, c_2\rangle$. The vector **n** is normal to the parallel planes containing L_1 and L_2. The point (x_1, y_1, z_1) is on L_1. Therefore, L_1 lies in the plane $\mathbf{n} \cdot \langle x - x_1, y - y_1, z - z_1\rangle = 0$. Because the point (x_2, y_2, z_2) is on L_2, the distance *D* between L_1 and L_2 is

$$D = \frac{\left|\mathbf{n} \cdot \langle x_2 - x_1, y_2 - y_1, z_2 - z_1\rangle\right|}{\|\mathbf{n}\|}.$$

Chapter 12, Section 1, page 827 Witch of Agnesi

(a) The figure shows that if $\mathbf{r}_A(\theta) = x\mathbf{i} + y\mathbf{j}$, then $y = 2a$ and, by right triangle trigonometry, $\cot\theta = \dfrac{x}{2a}$.

Therefore, $\mathbf{r}_A(\theta) = 2a\cot\theta\,\mathbf{i} + 2a\,\mathbf{j}$.



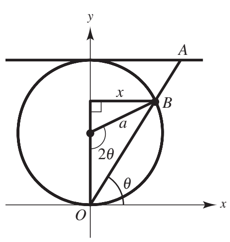
(b) The triangle with vertices at O, B, and the center of the circle $(0, a)$, is isosceles. The interior angles at O and B both equal $\pi/2 - \theta$. Because the angles of a triangle sum to π radians, the interior angle at the vertex $(0, a)$ equals 2θ. Consequently, the angle between the positive y-axis and the radius through B, equals $\pi - 2\theta$. If $\mathbf{r}_B(\theta) = x\mathbf{i} + y\mathbf{j}$, then by circular function trigonometry,

$$x = a\sin(\pi - 2\theta) = a(\sin\pi\cos 2\theta - \cos\pi\sin 2\theta) = a\sin 2\theta$$

and

$$y = a + a\cos(\pi - 2\theta) = a + a(\cos\pi\cos 2\theta + \sin\pi\sin 2\theta) = a - a\cos 2\theta.$$

That is, $\mathbf{r}_B(\theta) = a\sin 2\theta\mathbf{i} + a(1 - \cos 2\theta)\mathbf{j}$.

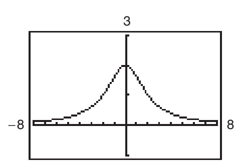


(c) Because P and A have the same x-coordinate while P and B have the same y-coordinate,

$$\mathbf{r}(\theta) = 2a\cot\theta\mathbf{i} + a(1 - \cos 2\theta)\mathbf{j}.$$

To simplify, use a double angle formula.

$$\mathbf{r}(\theta) = 2a\cot\theta\mathbf{i} + 2a\sin^2\theta\mathbf{j}.$$



$\mathbf{r}(\theta) = 2\cot\theta\mathbf{i} + 2\sin^2\theta\mathbf{j}$

(d) Both limits would seem to be indefinitely long vectors; therefore, neither limit exists.

(e) Using the answer to part (c), write the coordinates of P as $x = 2a \cot \theta$ and $y = 2a \sin^2 \theta$. Because

$$1 + \cot^2 \theta = \csc^2 \theta$$

and the cosecant is the reciprocal of the sine, you have

$$1 + \left(\frac{x}{2a}\right)^2 = \frac{2a}{y}.$$

Solve for y to get $y = \dfrac{8a^3}{x^2 + 4a^2}$. If $a = 1$, the graph of this curve is the same as the graph obtained in part (c).

Chapter 13, Section 7, page 939 Wildflowers

(a) $H = -\sum_{i=1}^{n} \rho_i \log_2 \rho_i$

May: $H = 3\left(-\dfrac{5}{16}\right) \log_2 \dfrac{5}{16} - \dfrac{1}{16} \log_2 \dfrac{1}{16} \approx 1.8232$

June: $H = 4\left(-\dfrac{1}{4}\right) \log_2 \dfrac{1}{4} = 2.0000$

August: $H = 2\left(-\dfrac{1}{4}\right) \log_2 \dfrac{1}{4} - \dfrac{1}{2} \log_2 \dfrac{1}{2} = 1.5000$

September: $H = 0 + 1 \log_2 1 = 0$

September had no diversity of wildflowers. The greatest diversity occurred in June.

(b) $H = 10\left(-\dfrac{1}{10}\right) \log_2 \dfrac{1}{10} \approx 3.3219 > 2.0000$

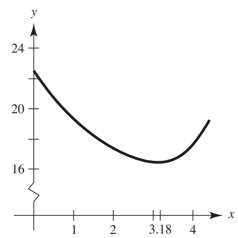
The diversity is greater with 10 types of wildflowers. An equal proportion of each type of wildflower would produce a maximum diversity.

(c) $H_n = n\left(-\dfrac{1}{n}\right) \log_2 \dfrac{1}{n} = (-1) \log_2\left(n^{-1}\right) = \log_2 n$

$\lim_{n\to\infty} H_n \to \infty$

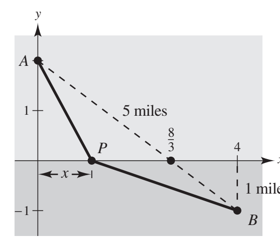
Chapter 13, Section 9, page 955 Building a Pipeline

Summary. It will cost at least \$16.4 million to build a pipeline to supply refinery *B* with oil from the offshore facility *A*. The actual cost will depend on where the pipeline crosses the shoreline. The most expensive plausible route, costing almost \$22.5 million, goes from *A* to the nearest point on the shore, then to *B*. An alternative route through the point on the shoreline closest to *B* would cost about \$17.4 million. A straight line direct route from *A* to *B* would cost approximately \$16.7 million.



At the left is a graph of the cost *C* in millions of dollars as a function of *x*, which is the number of miles between the point on the shore that is nearest to *A* and the point where the pipeline crosses the shore. The least cost occurs at $x \approx 3.18$ miles.

Analysis. It is convenient to introduce a rectangular coordinate system as shown at the right with a unit of length representing one mile. The point *P* where the pipeline crosses the shore is at $(x, 0)$ and *A* is located at $(0, 2)$. The point $(0, -1)$ is one vertex of a right triangle with other vertices at *A* and *B*. By the Pythagorean Theorem, the coordinates of *B* are $(4, -1)$. The line from *A* to *B* meets the shore at $\left(\frac{8}{3}, 0\right)$.



The cost per mile to build the pipeline is \$3 million in water and \$4 million on land. Thus, in millions of dollars, building the pipeline from *A* to *P* costs $3\sqrt{x^2 + 4}$ and building the pipeline from *P* to *B* costs $4\sqrt{(4 - x)^2 + 1}$. The total cost is

$$C(x) = 3\sqrt{x^2 + 4} + 4\sqrt{(4 - x)^2 + 1}.$$

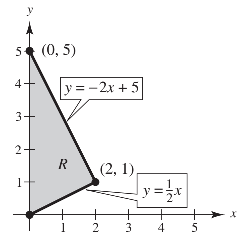
The above summary cites the costs $C(0) = 6 + 4\sqrt{17} \approx 22.49242250$, $C(4) = 6\sqrt{5} + 4 \approx 17.41640787$, and $C\left(\frac{8}{3}\right) = 16.\overline{6}$. The minimum cost occurs at a zero of the derivative

$$C'(x) = \frac{3x}{\sqrt{x^2 + 4}} - \frac{4(4 - x)}{\sqrt{(4 - x)^2 + 1}}.$$

A graphing utility can be used to find that $x = 3.1785$ is an approximate solution of $C'(x) = 0$ on the interval $[0, 4]$. (Newton's method or other methods of numeric approximation can also be used.) Consequently, the minimum cost is $C(3.1785) = 16.44$ million dollars.

Chapter 14, Section 4, page 1005 Center of Pressure on a Sail

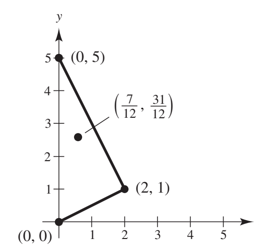
(a)
$$\int_R\int y\,dA = \int_0^2\int_{x/2}^{-2x+5} y\,dy\,dx = \int_0^2\left[\frac{y^2}{2}\right]_{x/2}^{-2x+5} dx = \frac{1}{2}\int_0^2\left[4x^2 - 20x + 25 - \frac{x^2}{4}\right]dx = \frac{1}{2}\int_0^2\left[\frac{15}{4}x^2 - 20x + 25\right]dx = \frac{1}{2}\left[\frac{5}{4}x^3 - 10x^2 + 25x\right]_0^2 = \frac{1}{2}\left[\left(\frac{5}{4}\right)(8) - 40 + 50\right] = 10$$



(b)
$$\int_R\int xy\,dA = \int_0^2\int_{x/2}^{-2x+5} xy\,dy\,dx = \int_0^2\left[\frac{xy^2}{2}\right]_{x/2}^{-2x+5} dx = \frac{1}{2}\int_0^2 x\left[\frac{15}{4}x^2 - 20x + 25\right]dx = \frac{1}{2}\int_0^2\left[\frac{15}{4}x^3 - 20x^2 + 25x\right]dx = \frac{1}{2}\left[\frac{15}{16}x^4 - \frac{20}{3}x^3 + \frac{25}{2}x^2\right]_0^2 = \frac{1}{2}\left[\frac{15}{16}(16) - \frac{20}{3}(8) + \frac{25}{2}(4)\right] = \frac{35}{6}$$

(c)
$$\int_R\int y^2\,dA = \int_0^2\int_{x/2}^{-2x+5} y^2\,dy\,dx = \int_0^2\left[\frac{y^3}{3}\right]_{x/2}^{-2x+5} dx = \frac{1}{3}\int_0^2\left[(-2x+5)^3 - \left(\frac{1}{2}x\right)^3\right]dx = \frac{1}{3}\int_0^2\left[-\frac{65}{8}x^3 + 60x^2 - 150x + 125\right]dx = \frac{1}{3}\left[-\frac{65}{8}\left(\frac{x^4}{4}\right) + 20x^3 - 75x^2 + 125x\right]_0^2 = \frac{1}{3}\left[-\frac{65}{32}(16) + 20(8) - 75(4) + 125(2)\right] = \frac{155}{6}$$

$$x_p = \frac{\int_R\int xy\,dA}{\int_R\int y\,dA} = \frac{35/6}{10} = \frac{7}{12} \qquad y_p = \frac{\int_R\int y^2\,dA}{\int_R\int y\,dA} = \frac{155/6}{10} = \frac{31}{12}$$



Chapter 14, Section 5, page 1012 Surface Area in Polar Coordinates

(a) $x = r\cos\theta,\ y = r\sin\theta,\ z = f(x, y)$

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} = \frac{\partial z}{\partial x}\cos\theta + \frac{\partial z}{\partial y}\sin\theta$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta} = \frac{\partial z}{\partial x}(-r\sin\theta) + \frac{\partial z}{\partial y}(r\cos\theta)$$

Now solve for $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

$$\frac{\partial z}{\partial x} = f_x = \frac{\partial z}{\partial r}\cos\theta - \frac{\partial z}{\partial \theta}\frac{\sin\theta}{r}$$
$$\frac{\partial z}{\partial y} = f_y = \frac{\partial z}{\partial r}\sin\theta + \frac{\partial z}{\partial \theta}\frac{\cos\theta}{r}$$

Then you have
$$\begin{aligned} 1 + f_x{}^2 + f_y{}^2 &= 1 + \left(\frac{\partial z}{\partial r}\cos\theta - \frac{\partial z}{\partial \theta}\frac{\sin\theta}{r}\right)^2 + \left(\frac{\partial z}{\partial r}\sin\theta + \frac{\partial z}{\partial \theta}\frac{\cos\theta}{r}\right)^2 \\ &= 1 + \left(\frac{\partial z}{\partial r}\right)^2\left(\cos^2\theta + \sin^2\theta\right) + \left(\frac{\partial z}{\partial \theta}\right)^2\frac{1}{r^2}\left(\sin^2\theta + \cos^2\theta\right) \\ &= 1 + \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right) \end{aligned}$$

Finally,

$$S = \int_R\int \sqrt{1 + f_r^2 + \frac{1}{r^2}f_\theta^2}\ r\,dr\,d\theta.$$

(b) $z = x^2 + y^2 = r^2,\ f_r = 2r,\ f_\theta = 0$

$$\begin{aligned} S &= \int_R\int\sqrt{1 + f_r^2 + \frac{1}{r^2}f_\theta^2}\ dA \\ &= \int_0^{2\pi}\int_0^2 \sqrt{1 + 4r^2}\ r\,dr\,d\theta \\ &= \int_0^{2\pi}\left[\frac{\left(4r^2 + 1\right)^{3/2}}{12}\right]_0^2 d\theta = \int_0^{2\pi}\frac{17\sqrt{17} - 1}{12}\,d\theta = \left[\frac{17\sqrt{17} - 1}{12}\,\theta\right]_0^{2\pi} = \frac{17\sqrt{17} - 1}{6}\pi \end{aligned}$$

(c) $z = xy = r^2\cos\theta\sin\theta = \dfrac{1}{2}r^2\sin^2\theta$

$f_r = r\sin 2\theta,\ f_\theta = r^2\cos 2\theta$

$$\begin{aligned} S &= \int_0^{2\pi}\int_0^4 \sqrt{1 + r^2\sin^2 2\theta + \frac{1}{r^2}\left(r^4\cos^2 2\theta\right)}\ r\,dr\,d\theta \\ &= \int_0^{2\pi}\int_0^4 \sqrt{1 + r^2}\ r\,dr\,d\theta \\ &= \int_0^{2\pi}\left[\frac{\left(r^2 + 1\right)^{3/2}}{3}\right]_0^4 d\theta \\ &= \int_0^{2\pi}\frac{17\sqrt{17} - 1}{3}\,d\theta \\ &= \left[\frac{17\sqrt{17} - 1}{3}\theta\right]_0^{2\pi} \\ &= \frac{2\pi}{3}\left(17\sqrt{17} - 1\right) \end{aligned}$$

Chapter 14, Section 7, page 1030   Wrinkled and Bumpy Spheres

(a) $\int_0^{2}\int_0^{\pi}\int_0^{1+0.2\sin 8\theta\sin\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$$= \frac{1}{3}\int_0^{2\pi}\int_0^{\pi} (1 + 0.2\sin 8\theta \sin\phi)^3 \sin\phi \, d\phi \, d\theta$$

$$= \frac{1}{3}\int_0^{2\pi}\int_0^{\pi} \left[\sin\phi + 0.6\sin 8\theta\sin^2\phi + 0.12\sin^2 8\theta\sin^3\phi + 0.008\sin^3 8\theta\sin^4\phi\right] d\phi \, d\theta$$

$$= \frac{1}{3}\int_0^{2\pi} \Bigg[-\cos\phi + 0.6\sin 8\theta\left(\frac{1}{2}\right)(\phi - \sin\phi\cos\phi) + 0.12\sin^2 8\theta\left(-\frac{\sin^2\phi\cos\phi}{3} - \frac{2}{3}\cos\phi\right)$$

$$+ 0.008\sin^3 8\theta\left(-\frac{\sin^3\phi\cos\phi}{4} + \frac{3}{8}(\phi - \sin\phi\cos\phi)\right)\Bigg]_0^{\pi} d\theta$$

$$= \frac{1}{3}\int_0^{2\pi} \left(2 + 0.3\pi\sin 8\theta + 0.16\sin^2 8\theta + 0.003\pi\sin^3 8\theta\right) d\theta$$

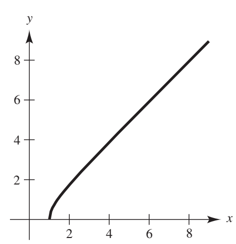
$$= \frac{1}{3}\left[2\theta - \frac{0.3\pi}{8}\cos 8\theta + \frac{0.16}{8}\left(\frac{1}{2}\right)(8\theta - \sin 8\theta\cos 8\theta) + \frac{0.003\pi}{8}\left(-\frac{\sin^2 8\theta\cos 8\theta}{3} - \frac{2}{3}\cos 8\theta\right)\right]_0^{2\pi}$$

$$= \frac{104\pi}{75}$$

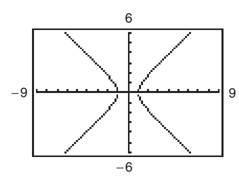
(b) $\int_0^{2\pi}\int_0^{\pi}\int_0^{1+0.2\sin 8\theta\sin 4\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = \frac{1}{3}\int_0^{2\pi}\int_0^{\pi} (1 + 0.2\sin 8\theta\sin 4\phi)^3 \sin\phi \, d\phi \, d\theta \approx 4.316$

Chapter 15, Section 4, page 1087   Hyperbolic and Trigonometric Functions

(a) If $x = \cosh t$ and $y = \sinh t$, then $x^2 - y^2 = \cosh^2 t - \sinh^2 t = 1$.



$\mathbf{r}(t) = \cosh t\,\mathbf{i} + \sinh t\,\mathbf{j}, 0 \le t \le 5$

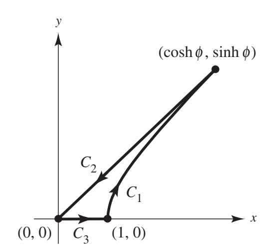


$x^2 - y^2 = 1$

(b) Divide the boundary C into three curves as shown at the right and introduce the parameterization

C_1: $x = \cosh t$, $y = \sinh t$, $0 \le t \le \phi$.

Because $x\,dy - y\,dx = 0\,dt$ for any parameterization of straight lines through the origin,



$$A = \frac{1}{2}\int_C x\,dy - y\,dx = \frac{1}{2}\int_{C_1} x\,dy - y\,dx + \frac{1}{2}\int_{C_2} 0\,dt + \frac{1}{2}\int_{C_3} 0\,dt$$

$$= \frac{1}{2}\int_{C_1} \cosh^2 t\,dt - \sinh^2 t\,dt + 0 + 0 = \frac{1}{2}\int_0^{\phi} dt = \frac{1}{2}\phi.$$

(c) The area in the first quadrant to the right of the line

$$x = \frac{\cosh \phi}{\sinh \phi} y$$

and to the left of the curve $x = \sqrt{y^2 + 1}$ is

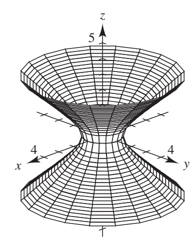
$$A = \int_0^{\sinh \phi} \left[\sqrt{y^2 + 1} - (\coth \phi) y \right] dy.$$

Using a tolerance of 1×10^{-5}, the numeric approximations $A = 0.5, 1, 2,$ and 5 were obtained for $\phi = 1, 2, 4,$ and 10, respectively. That is, $A = \frac{1}{2}\phi$.

(d) The hyperbolic functions $f(\phi) = \cosh \phi$ and $g(\phi) = \sinh \phi$ could be defined to be the coordinates $(\cosh \phi, \sinh \phi)$ of the point of intersection of the right branch of the hyperbola $x^2 - y^2 = 1$ with the straight line through the origin that (along with the x-axis) bounds a region of area $\frac{1}{2}|\phi|$.

Chapter 15, Section 6, page 1109 Hyperboloid of One Sheet

(a) The ratio b/a determine the shape of the surface. For a fixed value of a, larger values of b suggest a (bottomless) stem vase and smaller values of b suggest a pulley for a rope.



(b) $x^2 + y^2 = a^2 \cosh^2 u \cos^2 v + a^2 \cosh^2 u \sin^2 v = a^2 \cosh^2 u$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{z^2}{b^2} = \cosh^2 u - \sinh^2 u = 1$$

(c) For each value of u_0, the curve is a circle of radius $a \cosh u_0$ centered about the z-axis in the plane $z = b \sinh u_0$.

(d) For each value of v_0, the curve is hyperbola. In terms of cylindrical coordinates, it lies in the plane $\theta = v_0$ and satisfies

$$\frac{r^2}{a^2} - \frac{z^2}{b^2} = 1.$$

(e) The hyperboloid of one sheet is a level surface of

$$f(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{z^2}{b^2}.$$

The gradient of f is

$\nabla f(x, y, z) = \frac{2x}{a^2}\mathbf{i} + \frac{2y}{a^2}\mathbf{j} - \frac{2z}{b^2}\mathbf{k}$ and $(x, y, z) = (a, 0, 0)$ if $(u, v) = (0, 0)$.

Therefore, a normal vector is $\nabla f(x, y, z) = \frac{2}{a}\mathbf{i}$.

Chapter 16, Section 3, page 1152 Parachute Jump

(a) $-5y'' - 8y' = 160$

$-5m^2 - 8m = 0$ when $m = 0, -\frac{8}{5}.$

$$y_h = C_1 + C_2e^{-1.6t}$$
$$y_p = At + B$$
$$y_p' = A$$
$$y_p'' = 0$$
$$-5y'' - 8y' = -8A = 160 \Rightarrow A = -20$$
$$y = C_1 + C_2e^{-1.6t} - 20t$$

Initial conditions: $y(0) = 2000, y'(0) = -100$

$$2000 = C_1 + C_2 \quad -100 = -1.6C_2 - 20$$
$$C_2 = 50 \Rightarrow C_1 = 1950$$

Particular solution: $y = 1950 + 50e^{-1.6t} - 20t$

(b) $-6y'' - 9y' = 192$

$-6m^2 - 9m = 0$ when $m = 0, -1.5.$

$$y_h = C_1 + C_2e^{-1.5t}$$
$$y_p = At + B$$
$$-6y'' - 9y' = -9A = 192 \Rightarrow A = -\frac{64}{3}$$
$$y = C_1 + C_2e^{-1.5t} - \frac{64}{3}t$$

Initial conditions: $y(0) = 2000, y'(0) = -100$

$$2000 = C_1 + C_2$$
$$-100 = -1.5C_2 - \frac{64}{3}$$
$$C_2 = \frac{472}{9} \Rightarrow C_1 = \frac{17{,}528}{9}$$

$$y = \frac{17{,}528}{9} + \frac{472}{9}e^{-1.5t} - \frac{64}{3}t$$