Chapter 1

1.1

(a) One dimensional, multichannel, discrete time, and digital.
(b) Multi dimensional, single channel, continuous-time, analog.
(c) One dimensional, single channel, continuous-time, analog.
(d) One dimensional, single channel, continuous-time, analog.
(e) One dimensional, multichannel, discrete-time, digital.

1.2

(a) $f = \frac{0.01\pi}{2\pi} = \frac{1}{200} \Rightarrow$ periodic with $N_p = 200$.
(b) $f = \frac{30\pi}{105}(\frac{1}{2\pi}) = \frac{1}{7} \Rightarrow$ periodic with $N_p = 7$.
(c) $f = \frac{3\pi}{2\pi} = \frac{3}{2} \Rightarrow$ periodic with $N_p = 2$.
(d) $f = \frac{3}{2\pi} \Rightarrow$ non-periodic.
(e) $f = \frac{62\pi}{10}(\frac{1}{2\pi}) = \frac{31}{10} \Rightarrow$ periodic with $N_p = 10$.

1.3

(a) Periodic with period $T_p = \frac{2\pi}{5}$.
(b) $f = \frac{5}{2\pi} \Rightarrow$ non-periodic.
(c) $f = \frac{1}{12\pi} \Rightarrow$ non-periodic.
(d) $cos(\frac{n}{8})$ is non-periodic; $cos(\frac{\pi n}{8})$ is periodic; Their product is non-periodic.
(e) $cos(\frac{\pi n}{2})$ is periodic with period N_p=4
$sin(\frac{\pi n}{8})$ is periodic with period N_p=16
$cos(\frac{\pi n}{4} + \frac{\pi}{3})$ is periodic with period N_p=8
Therefore, x(n) is periodic with period N_p=16. (16 is the least common multiple of 4,8,16).

1.4

(a) $w = \frac{2\pi k}{N}$ implies that $f = \frac{k}{N}$. Let

$$\alpha = \text{ GCD of } (k, N), \text{ i.e.,}$$

$$k = k'\alpha, N = N'\alpha.$$

Then,

$$f = \frac{k'}{N'}, \text{ which implies that}$$

$$N' = \frac{N}{\alpha}.$$

(b)

$$\begin{aligned} N &= 7 \\ k &= 0\ 1\ 2\ 3\ 4\ 5\ 6\ 7 \\ \text{GCD}(k,N) &= 7\ 1\ 1\ 1\ 1\ 1\ 1\ 7 \\ N_p &= 1\ 7\ 7\ 7\ 7\ 7\ 7\ 1 \end{aligned}$$

(c)

$$\begin{aligned} N &= 16 \\ k &= 0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ \ldots\ 16 \\ \text{GCD}(k,N) &= 16\ 1\ 2\ 1\ 4\ 1\ 2\ 1\ 8\ 1\ 2\ 1\ 4\ \ldots\ 16 \\ N_p &= 1\ 6\ 8\ 16\ 4\ 16\ 8\ 16\ 2\ 16\ 8\ 16\ 4\ \ldots\ 1 \end{aligned}$$

1.5

(a) Refer to fig 1.5-1
(b)

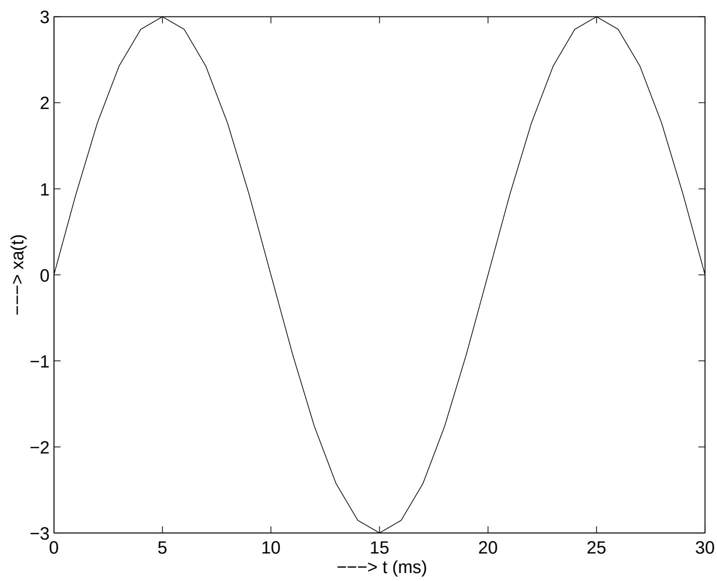

Figure 1.5-1:

$$\begin{aligned} x(n) &= x_a(nT) \\ &= x_a(n/F_s) \\ &= 3sin(\pi n/3) \Rightarrow \\ f &= \frac{1}{2\pi}(\frac{\pi}{3}) \\ &= \frac{1}{6}, N_p = 6 \end{aligned}$$

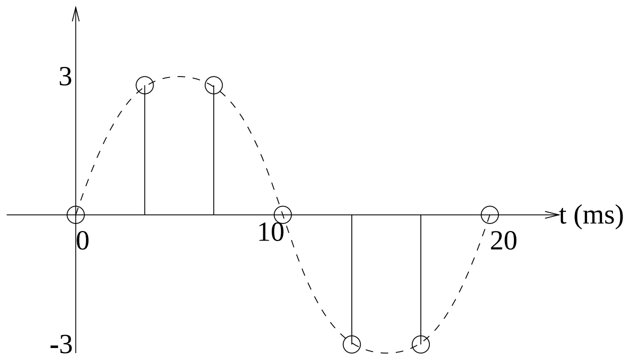

Figure 1.5-2:

(c)Refer to fig 1.5-2
$x(n) = \left\{0, \frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}, 0, -\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}\right\}, N_p = 6.$
(d) Yes.

$$x(1) = 3 = 3sin(\frac{100\pi}{F_s}) \Rightarrow F_s = 200 \text{ samples/sec.}$$

1.6

(a)

$$\begin{aligned} x(n) &= Acos(2\pi F_0 n/F_s + \theta) \\ &= Acos(2\pi(T/T_p)n + \theta) \end{aligned}$$

But $T/T_p = f \Rightarrow x(n)$ is periodic if f is rational.
(b) If x(n) is periodic, then f=k/N where N is the period. Then,

$$T_d = (\frac{k}{f}T) = k(\frac{T_p}{T})T = kT_p.$$

Thus, it takes k periods (kT_p) of the analog signal to make 1 period (T_d) of the discrete signal.
(c) $T_d = kT_p \Rightarrow NT = kT_p \Rightarrow f = k/N = T/T_p \Rightarrow$ f is rational $\Rightarrow$ x(n) is periodic.

1.7

(a) $F_{\max} = 10kHz \Rightarrow F_s \geq 2F_{\max} = 20kHz.$
(b) For $F_s = 8kHz, F_{\text{fold}} = F_s/2 = 4kHz \Rightarrow 5kHz$ will alias to 3kHz.
(c) F=9kHz will alias to 1kHz.

1.8

(a) $F_{\max} = 100kHz, F_s \geq 2F_{\max} = 200Hz.$
(b) $F_{\text{fold}} = \frac{F_s}{2} = 125Hz.$

1.9

(a) $F_{\max} = 360Hz, F_N = 2F_{\max} = 720Hz.$
(b) $F_{\text{fold}} = \frac{F_s}{2} = 300Hz.$
(c)

$$\begin{aligned} x(n) &= x_a(nT) \\ &= x_a(n/F_s) \\ &= sin(480\pi n/600) + 3sin(720\pi n/600) \\ x(n) &= sin(4\pi n/5) - 3sin(4\pi n/5) \\ &= -2sin(4\pi n/5). \end{aligned}$$

Therefore, $w = 4\pi/5$.
(d) $y_a(t) = x(F_s t) = -2sin(480\pi t)$.

1.10

(a)

$$\begin{aligned} \text{Number of bits/sample} &= log_2 1024 = 10. \\ F_s &= \frac{[10,000 \text{ bits/sec}]}{[10 \text{ bits/sample}]} \\ &= 1000 \text{ samples/sec.} \\ F_{\text{fold}} &= 500Hz. \end{aligned}$$

(b)

$$\begin{aligned} F_{\max} &= \frac{1800\pi}{2\pi} \\ &= 900Hz \\ F_N &= 2F_{\max} = 1800Hz. \end{aligned}$$

(c)

$$\begin{aligned} f_1 &= \frac{600\pi}{2\pi}(\frac{1}{F_s}) \\ &= 0.3; \\ f_2 &= \frac{1800\pi}{2\pi}(\frac{1}{F_s}) \\ &= 0.9; \\ \text{But } f_2 &= 0.9 > 0.5 \Rightarrow f_2 = 0.1. \\ \text{Hence, } x(n) &= 3cos[(2\pi)(0.3)n] + 2cos[(2\pi)(0.1)n] \end{aligned}$$

(d) $\triangle = \frac{x_{\max} - x_{\min}}{m-1} = \frac{5-(-5)}{1023} = \frac{10}{1023}$.

1.11

$$\begin{aligned} x(n) &= x_a(nT) \\ &= 3cos\left(\frac{100\pi n}{200}\right) + 2sin\left(\frac{250\pi n}{200}\right) \end{aligned}$$

$$
\begin{aligned}
&= 3cos\left(\frac{\pi n}{2}\right) - 2sin\left(\frac{3\pi n}{4}\right) \\
T' &= \frac{1}{1000} \Rightarrow y_a(t) = x(t/T') \\
&= 3cos\left(\frac{\pi 1000t}{2}\right) - 2sin\left(\frac{3\pi 1000t}{4}\right) \\
y_a(t) &= 3cos(500\pi t) - 2sin(750\pi t)
\end{aligned}
$$

1.12

(a) For $F_s = 300Hz$,

$$
\begin{aligned}
x(n) &= 3cos\left(\frac{\pi n}{6}\right) + 10sin(\pi n) - cos\left(\frac{\pi n}{3}\right) \\
&= 3cos\left(\frac{\pi n}{6}\right) - 3cos\left(\frac{\pi n}{3}\right)
\end{aligned}
$$

(b) $x_r(t) = 3cos(10000\pi t/6) - cos(10000\pi t/3)$

1.13

(a)

$$
\begin{aligned}
\text{Range} &= x_{\text{max}} - x_{\text{min}} = 12.7. \\
m &= 1 + \frac{\text{range}}{\triangle} \\
&= 127 + 1 = 128 \Rightarrow log_2(128) \\
&= 7 \text{ bits.}
\end{aligned}
$$

(b) $m = 1 + \frac{127}{0.02} = 636 \Rightarrow log_2(636) \Rightarrow 10$ bit A/D.

1.14

$$
\begin{aligned}
R &= (20\frac{\text{samples}}{\text{sec}}) \times (8\frac{\text{bits}}{\text{sample}}) \\
&= 160\frac{\text{bits}}{\text{sec}} \\
F_{\text{fold}} &= \frac{F_s}{2} = 10Hz. \\
\text{Resolution} &= \frac{1\text{volt}}{2^8 - 1} \\
&= 0.004.
\end{aligned}
$$

1.15

(a) Refer to fig 1.15-1. With a sampling frequency of 5kHz, the maximum frequency that can be represented is 2.5kHz. Therefore, a frequency of 4.5kHz is aliased to 500Hz and the frequency of 3kHz is aliased to 2kHz.

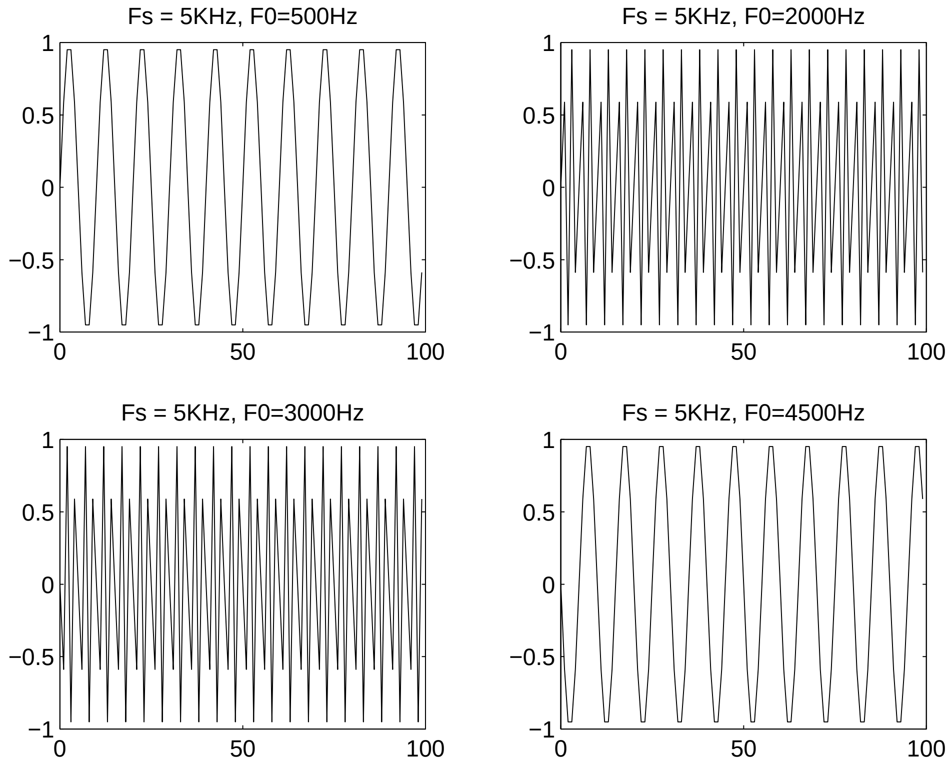

Figure 1.15-1:

(b) Refer to fig 1.15-2. y(n) is a sinusoidal signal. By taking the even numbered samples, the sampling frequency is reduced to half i.e., 25kHz which is still greater than the nyquist rate. The frequency of the downsampled signal is 2kHz.

1.16

(a) for levels = 64, using truncation refer to fig 1.16-1.
for levels = 128, using truncation refer to fig 1.16-2.
for levels = 256, using truncation refer to fig 1.16-3.

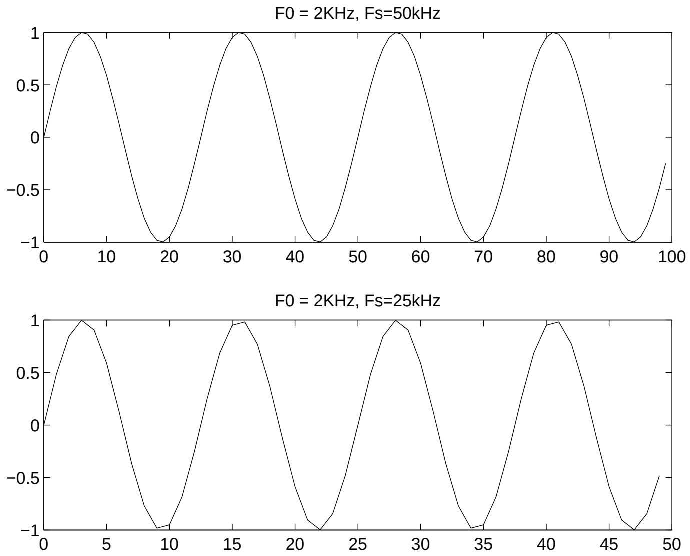

Figure 1.15-2:

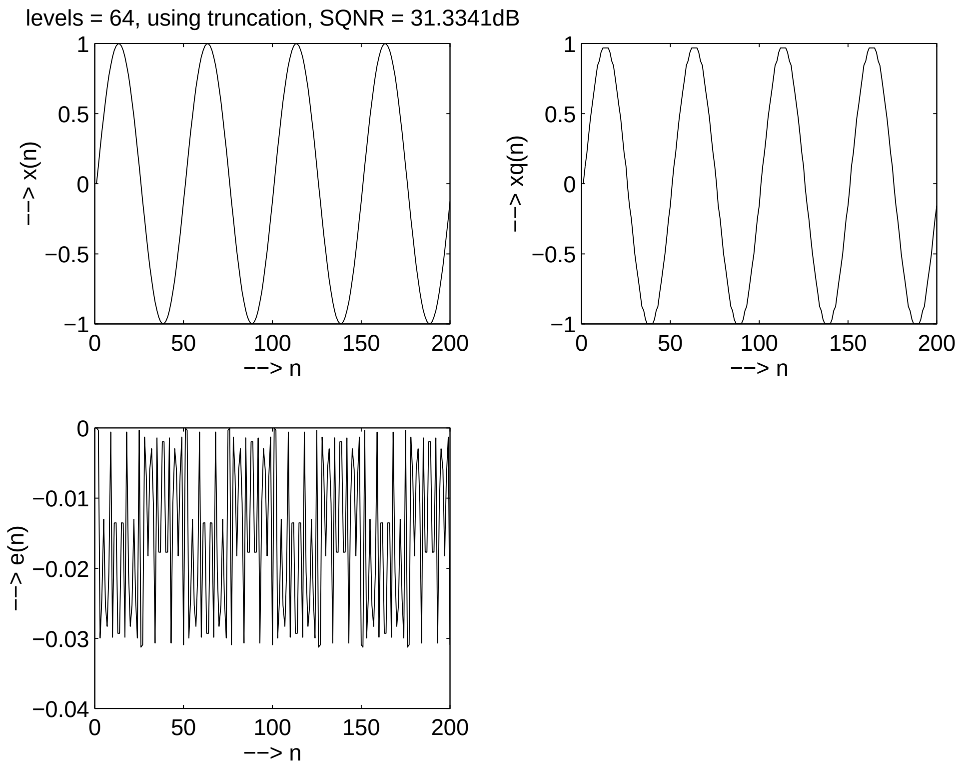

Figure 1.16-1:

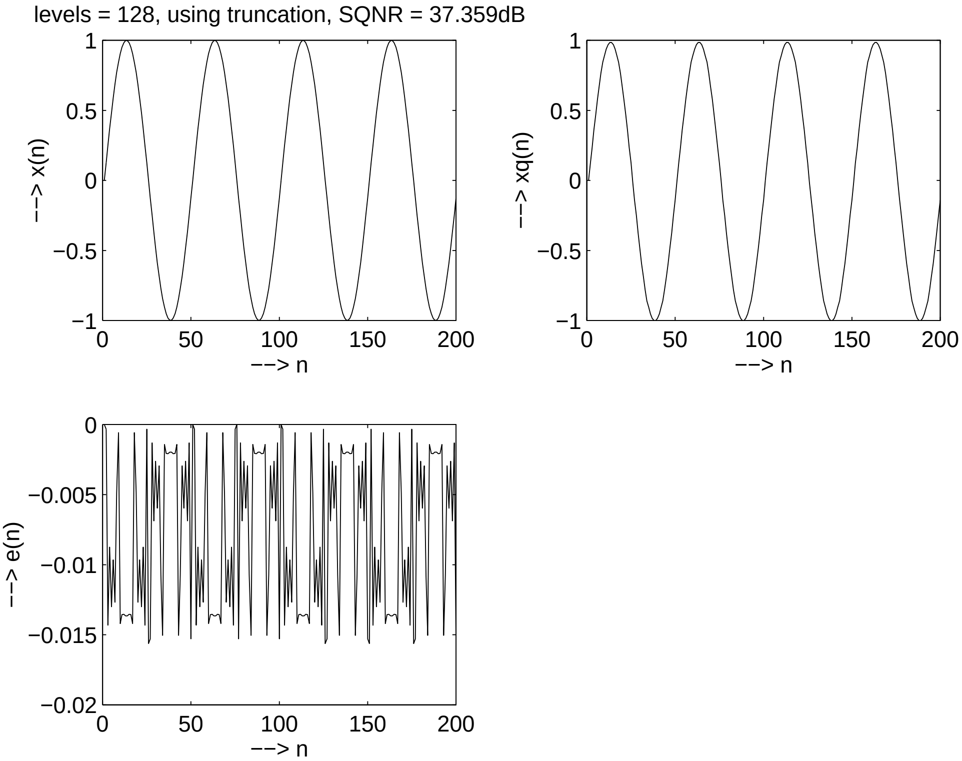

Figure 1.16-2:

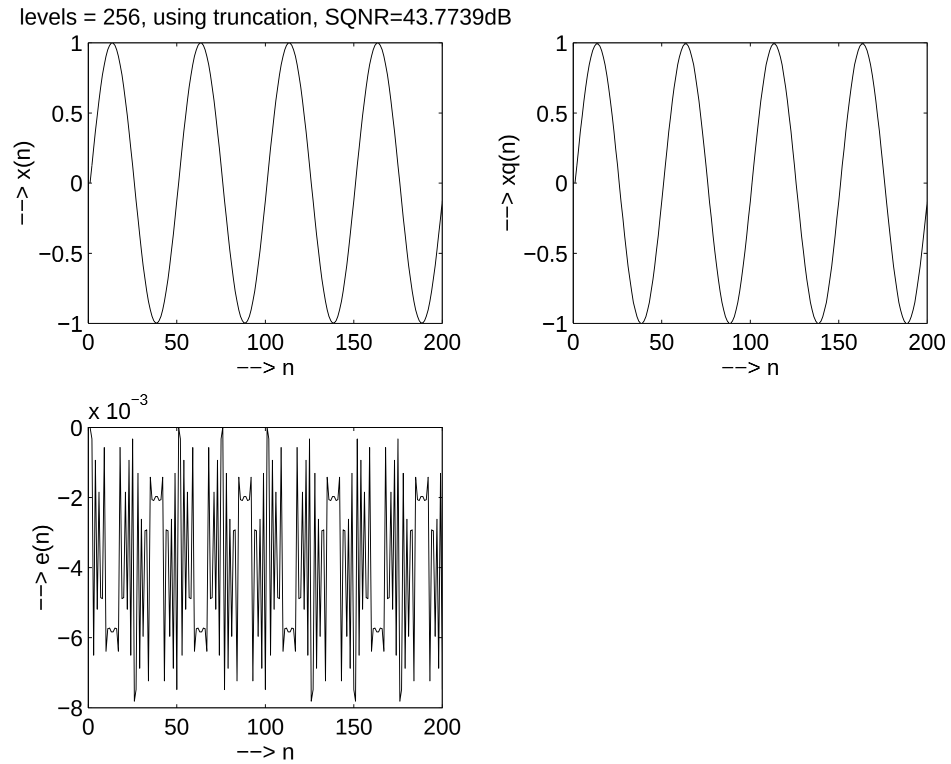

Figure 1.16-3:

(b) for levels = 64, using rounding refer to fig 1.16-4.
for levels = 128, using rounding refer to fig 1.16-5.
for levels = 256, using rounding refer to fig 1.16-6.

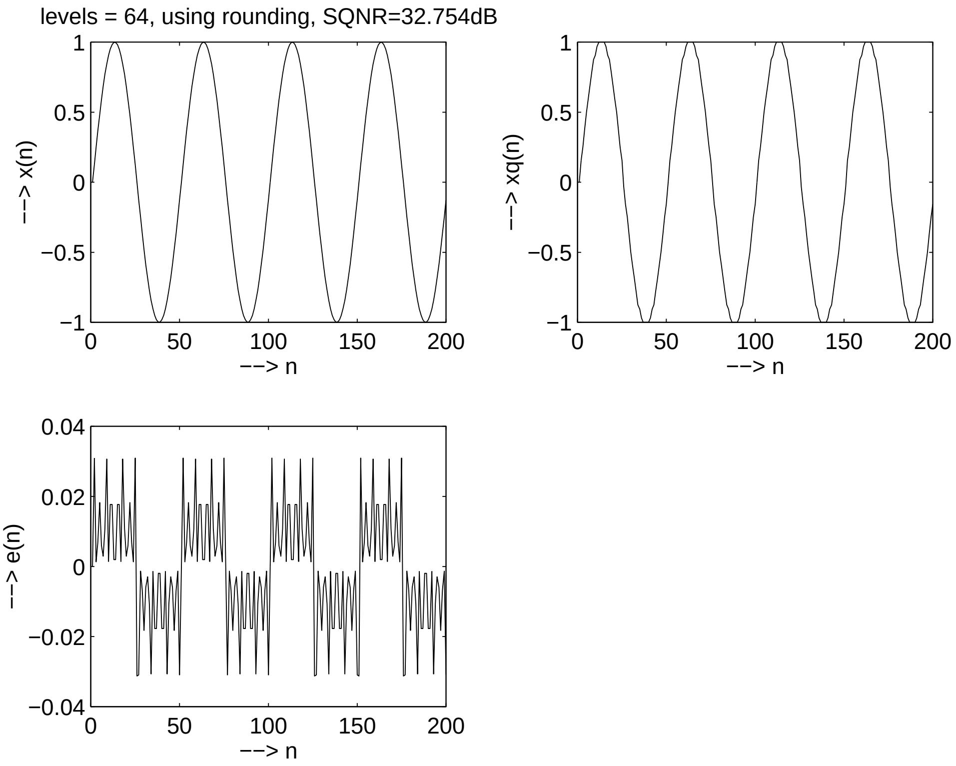

Figure 1.16-4:

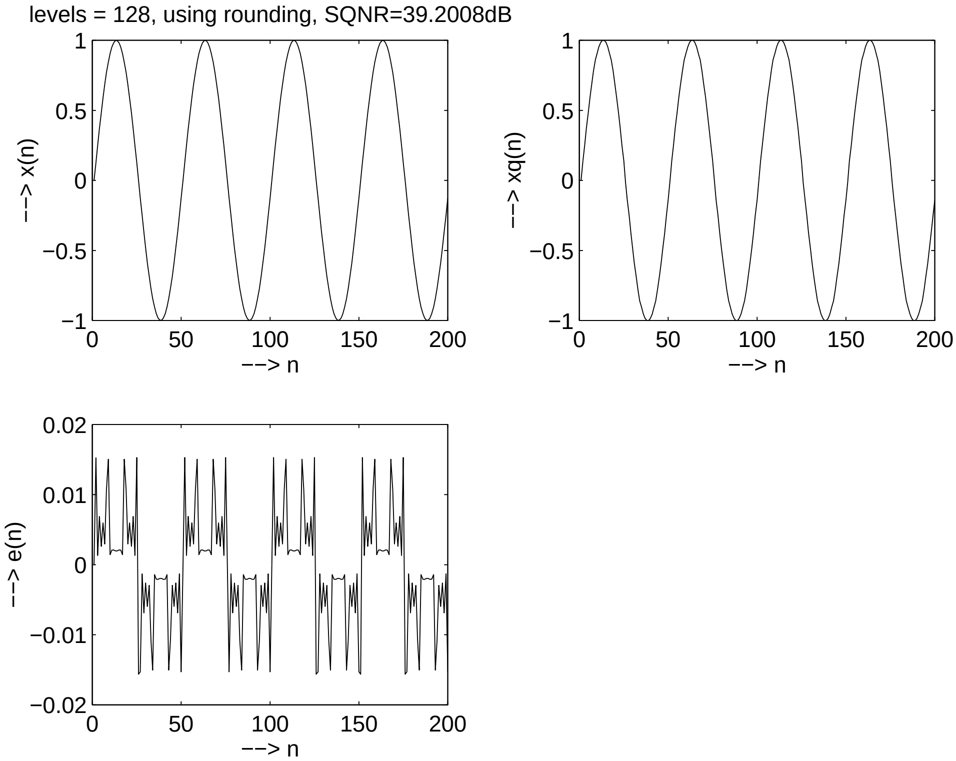

Figure 1.16-5:

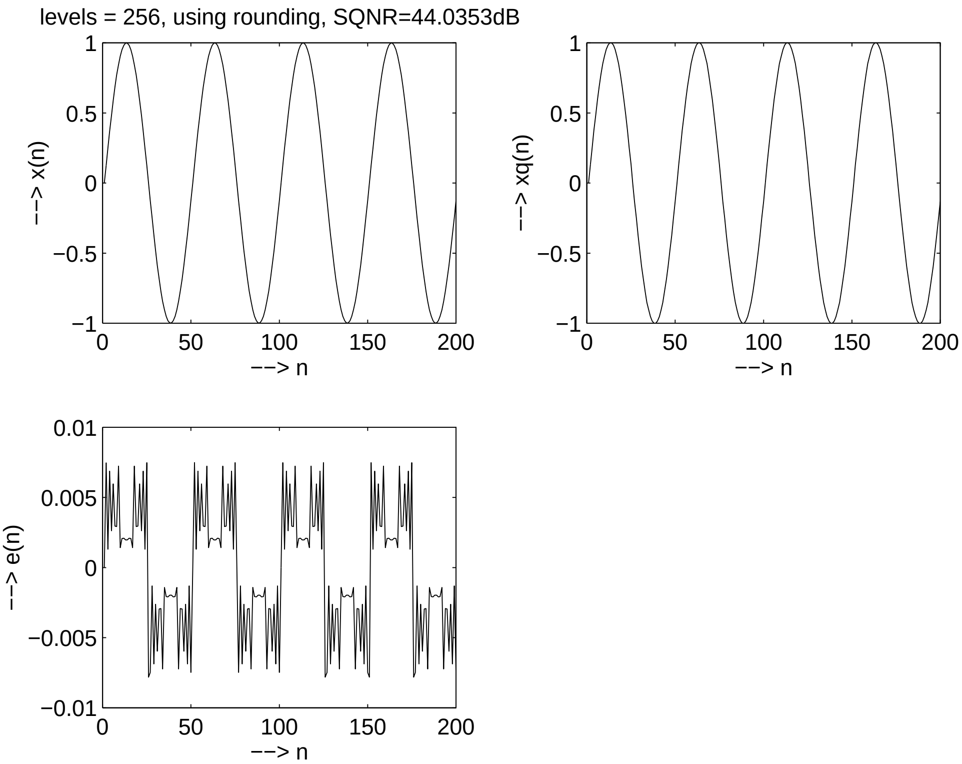

Figure 1.16-6:

(c) The sqnr with rounding is greater than with truncation. But the sqnr improves as the number of quantization levels are increased.

(d)

levels	64	128	256
theoretical sqnr	43.9000	49.9200	55.9400
sqnr with truncation	31.3341	37.359	43.7739
sqnr with rounding	32.754	39.2008	44.0353

The theoretical sqnr is given in the table above. It can be seen that theoretical sqnr is much higher than those obtained by simulations. The decrease in the sqnr is because of the truncation and rounding.