Solutions to Chapter 2 Problems

2.1 In your own words, define the following terms:

a. Time domain analysis

b. Frequency domain analysis

Answers should be consistent with the definitions given in Section 2.1 of the book: In time domain analysis, we examine the amplitude vs. time characteristics of the waveform. In frequency domain analysis, we replace the waveform with a summation of sinusoids which produce an equivalent waveform and we then examine the relative amplitudes, phases, and the frequencies of the sinusoids.

2.2 Give an example of a communication system application where bandwidth is very important.

Consider a cellular telephone system in a large city. The entire system has a certain amount of bandwith allocated to it by the Federal Communications Commission (say 600 kHz within each cell). Each cellular telephone call uses a certain amount of the 600 kHz bandwidth to communicate. If the cellular phones use an inefficient modulation technique which requires, say, 30 kHz of bandwidth for each phone call, then only 20 callers in each cell can use the system at any given time. If, on the other hand, the modulation technique used by the cellular telephone needs only 5 kHz of bandwidth, then 120 callers in each cell can simultaneously use the system. This six-fold increase in capacity will allow the cellular telephone service provider to generate six times as much revenue.

2.3 Why is it important to analyze the effects of the channel in a communication system?

As discussed in Chapter 1, all channels have physical limitations that will distort and attenuate the transmitted signal and that will add noise to the transmitted signal. Thus, the received signal will not be an exact duplicate of the transmitted signal. If we understand how a particular channel distorts, attenuates, and adds noise to a signal, then we can design our transmiter (and receiver) to use a modulation technique (and demodulation technique) which minimizes the channel's effects. By minimizing the channel's effects, we guarantee that the signal will be received more accurately.

2.4 Show that the set of harmonically related sines and cosines is orthogonal. In other words, show that for any positive integer values of m and n

a. $\int_{t_o}^{t_o+T} \cos(2\pi n f_o t)\cos(2\pi m f_o t)dt = 0$ if $m \neq n$ and $\int_{t_o}^{t_o+T} \cos(2\pi n f_o t)\cos(2\pi m f_o t)dt \neq 0$ if $m = n$

b. $\int_{t_o}^{t_o+T} \sin(2\pi n f_o t)\sin(2\pi m f_o t)dt = 0$ if $m \neq n$ and $\int_{t_o}^{t_o+T} \sin(2\pi n f_o t)\sin(2\pi m f_o t)dt \neq 0$ if $m = n$

c. $\int_{t_o}^{t_o+T} \cos(2\pi n f_o t)\sin(2\pi m f_o t)dt = 0$ for all values of m and n

Solution

a. *Using trig identities (see the page opposite the inside front cover of the textbook),*

$$\int_{t_o}^{t_o+T} \cos(2\pi n f_o t)\cos(2\pi m f_o t)dt = \int_{t_o}^{t_o+T} 0.5\{\cos[2\pi(n+m)f_o t] + \cos[2\pi(n-m)f_o t]\}dt$$

$$= 0.5\int_{t_o}^{t_o+T} \cos[2\pi(n+m)f_o t]dt + 0.5\int_{t_o}^{t_o+T} \cos[2\pi(n-m)f_o t]dt$$

Let's evaluate the terms one at a time.
Evaluating the first term:

$0.5\int_{t_o}^{t_o+T} \cos[2\pi(n+m)f_o t]dt = 0$ *for any positive values of m and n, since we are integrating a sinusoid over a whole number of periods.*

Evaluating the second term:

If $m \neq n$, $0.5\int_{t_o}^{t_o+T} \cos[2\pi(n-m)f_o t]dt = 0$ *because again we are integrating a sinusoid over a whole number of periods.*

If $m = n$, $0.5\int_{t_o}^{t_o+T} \cos[2\pi(n-m)f_o t]dt = 0.5\int_{t_o}^{t_o+T} 1dt = 0.5T$

Combining the results of the first and second terms,

$$\int_{t_o}^{t_o+T} \cos(2\pi n f_o t)\cos(2\pi m f_o t)dt = \begin{cases} 0 & m \neq n \\ 0.5T & m = n \end{cases}$$

b. *Using trig identities,*

$$\int_{t_o}^{t_o+T} \sin(2\pi n f_o t)\sin(2\pi m f_o t)dt = \int_{t_o}^{t_o+T} 0.5\{\cos[2\pi(n-m)f_o t] - \cos[2\pi(n+m)f_o t]\}dt$$

$$= 0.5\int_{t_o}^{t_o+T} \cos[2\pi(n-m)f_o t]dt - 0.5\int_{t_o}^{t_o+T} \cos[2\pi(n+m)f_o t]dt$$

As shown in Part a,

$0.5\int_{t_o}^{t_o+T} \cos[2\pi(n+m)f_o t]dt = 0$ *for any positive values of m and n*

and $0.5\int_{t_o}^{t_o+T} \cos[2\pi(n-m)f_o t]dt = \begin{cases} 0 & m \neq n \\ 0.5T & m = n \end{cases}$

Thus, $\int_{t_o}^{t_o+T} \sin(2\pi n f_o t)\sin(2\pi m f_o t)dt = \begin{cases} 0 & m \neq n \\ 0.5T & m = n \end{cases}$

c. *Using trig identities,*

$$\int_{t_o}^{t_o+T}\cos(2\pi nf_ot)\sin(2\pi mf_ot)dt = \int_{t_o}^{t_o+T}0.5\{\sin[2\pi(n+m)f_ot]+\sin[2\pi(n-m)f_ot]\}dt$$

$$= 0.5\int_{t_o}^{t_o+T}\sin[2\pi(n+m)f_ot]dt + 0.5\int_{t_o}^{t_o+T}\cos[2\pi(n-m)f_ot]dt$$

Evaluating the first term,

$0.5\int_{t_o}^{t_o+T}\sin[2\pi(n+m)f_ot]dt = 0$ *for any positive values of m and n, since we are integrating a sinusoid over a whole number of periods.*

Evaluating the second term,

if $m \neq n$, $0.5\int_{t_o}^{t_o+T}\sin[2\pi(n-m)f_ot]dt = 0$ *because again we are integrating a sinusoid over a whole number of periods.*

If $m = n$, $0.5\int_{t_o}^{t_o+T}\sin[2\pi(n-m)f_ot]dt = 0.5\int_{t_o}^{t_o+T}0dt = 0$

Thus, $\int_{t_o}^{t_o+T}\cos(2\pi nf_ot)\sin(2\pi mf_ot)dt = 0$

2.5 Using the concept of orthogonality, derive the equations for the coefficients of the trigonometric form of the Fourier series (i.e., derive the expressions for a_0, a_n, and b_n). (Hint: to find the expression for a_n, start with the expression for the trigonometric form of the Fourier series, multiply both the left-hand and right-hand sides by $\cos 2\pi mf_ot$, and integrate over one period of the fundamental frequency. The a_0 and b_n expressions can be found in a similar manner.)

Starting with the trigonometric form of the Fourier series,

$$s(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos 2\pi nf_ot + b_n\sin 2\pi nf_ot\right)$$

Multiplying both sides by $a_m\cos 2\pi mf_ot$ $(m \neq 0)$ *and then integrating over one period of* $s(t)$,

$$\int_{t_0}^{t_o+T}s(t)\cos 2\pi mf_otdt = \int_{t_0}^{t_o+T}\left\{a_0 + \sum_{n=1}^{\infty}\left(a_n\cos 2\pi nf_ot + b_n\sin 2\pi nf_ot\right)\right\}\cos 2\pi mf_otdt$$

$$= \int_{t_0}^{t_o+T}a_0\cos 2\pi mf_otdt + \int_{t_0}^{t_o+T}\sum_{n=1}^{\infty}a_n\cos 2\pi nf_ot\cos 2\pi mf_otdt + \int_{t_0}^{t_o+T}\sum_{n=1}^{\infty}b_n\sin 2\pi nf_ot\cos 2\pi mf_otdt$$

Examining the first term,

$\int_{t_0}^{t_o+T} a_0 \cos 2\pi m f_o t dt = 0$ *because we are integrating a sinusoid over a whole number of periods.*

Examining the second term,

$$\int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} a_n \cos 2\pi n f_o t \cos 2\pi m f_o t dt = \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_o+T} \cos 2\pi n f_o t \cos 2\pi m f_o t dt$$

As we proved in Problem 2.4, due to orthogonality

$$\int_{t_o}^{t_o+T} \cos(2\pi n f_o t)\cos(2\pi m f_o t) dt = \begin{cases} 0 & n \neq m \\ 0.5T & n = m \end{cases}$$

and so our second term reduces to

$$\int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} a_n \cos 2\pi n f_o t \cos 2\pi m f_o t dt = \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_o+T} \cos 2\pi n f_o t \cos 2\pi m f_o t dt = a_m (0.5T)$$

Examining the third term,

$$\int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} b_n \sin 2\pi n f_o t \cos 2\pi m f_o t dt = \sum_{n=1}^{\infty} b_n \int_{t_0}^{t_o+T} \sin 2\pi n f_o t \cos 2\pi m f_o t dt$$

As we proved in Problem 2.4, due to orthogonality

$\int_{t_o}^{t_o+T} \sin(2\pi n f_o t)\cos(2\pi m f_o t) dt = 0$ *for all values of m and n, so*

$$\int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} b_n \sin 2\pi n f_o t \cos 2\pi m f_o t dt = \sum_{n=1}^{\infty} b_n \int_{t_0}^{t_o+T} \sin 2\pi n f_o t \cos 2\pi m f_o t dt = 0$$

Combining the three terms,

$$\int_{t_0}^{t_o+T} s(t) \cos 2\pi m f_o t dt = 0 + a_m (0.5T) + 0 = \frac{a_m T}{2}, \text{ or } a_m = \frac{2}{T} \int_{t_o}^{t_o+T} s(t) \cos(2\pi m f_o t) dt$$

Similarly, starting with the trigonometric form of the Fourier series, multiplying both sides by $\sin 2\pi m f_o t$*, and exploiting the orthogonality which we proved in Problem 2.4,*

$$\int_{t_0}^{t_o+T} s(t) \sin 2\pi m f_o t dt = \int_{t_0}^{t_o+T} \left\{ a_0 + \sum_{n=1}^{\infty} \left(a_n \cos 2\pi n f_o t + b_n \sin 2\pi n f_o t \right) \right\} \sin 2\pi m f_o t dt$$

$$= \int_{t_0}^{t_o+T} a_0 \sin 2\pi m f_o t dt + \int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} a_n \cos 2\pi n f_o t \sin 2\pi m f_o t dt + \int_{t_0}^{t_o+T} \sum_{n=1}^{\infty} b_n \sin 2\pi n f_o t \sin 2\pi m f_o t dt$$

$$= 0 + 0 + b_m (0.5T) = \frac{b_m T}{2}$$