Chapter 1: Fundamental Concepts

1.1 (i)

(a) $x(t) = p_2(t) + p_4(t)$

(b) $x(t) = \frac{4}{3}\left(1 - \frac{|t|}{4}\right)p_8(t) - \frac{1}{3}(1 - |t|)p_2(t)$

(c) $x(t) = 2p_{12}(t) + 2p_6(t) + 2\left(1 - \frac{|t|}{3}\right)p_6(t)$

(d) $x(t) = 4p_4(t) - 2\left(1 - \frac{|t|}{2}\right)p_4(t)$

(e) $x(t) = \sum_{k=1}^{\infty} p_1(t - 2k + 1.5)$

1.1 (ii) Simplest to define *t* and *x* as follows, then use the MATLAB command `plot(t,x)`

(a) t = [-3 -2 -2 -1 -1 1 1 2 2 3];

x = [0 0 1 1 2 2 1 1 0 0];

(b) t = [-5 -4 -1 1 4 5];

x = [0 0 1 1 0 0];

(c) t = [-7 -6 -6 -3 -3 0 3 3 6 6 7];

x = [0 0 2 2 4 6 4 2 2 0 0];

(d) t = [-3 -2 -2 0 2 2 3];

x = [0 0 4 2 4 0 0];

(e) t = [-1 0 0 1 1 2 2 3 3 4 4 5 5 6];

x = [0 0 1 1 0 0 1 1 0 0 1 1 0 0];

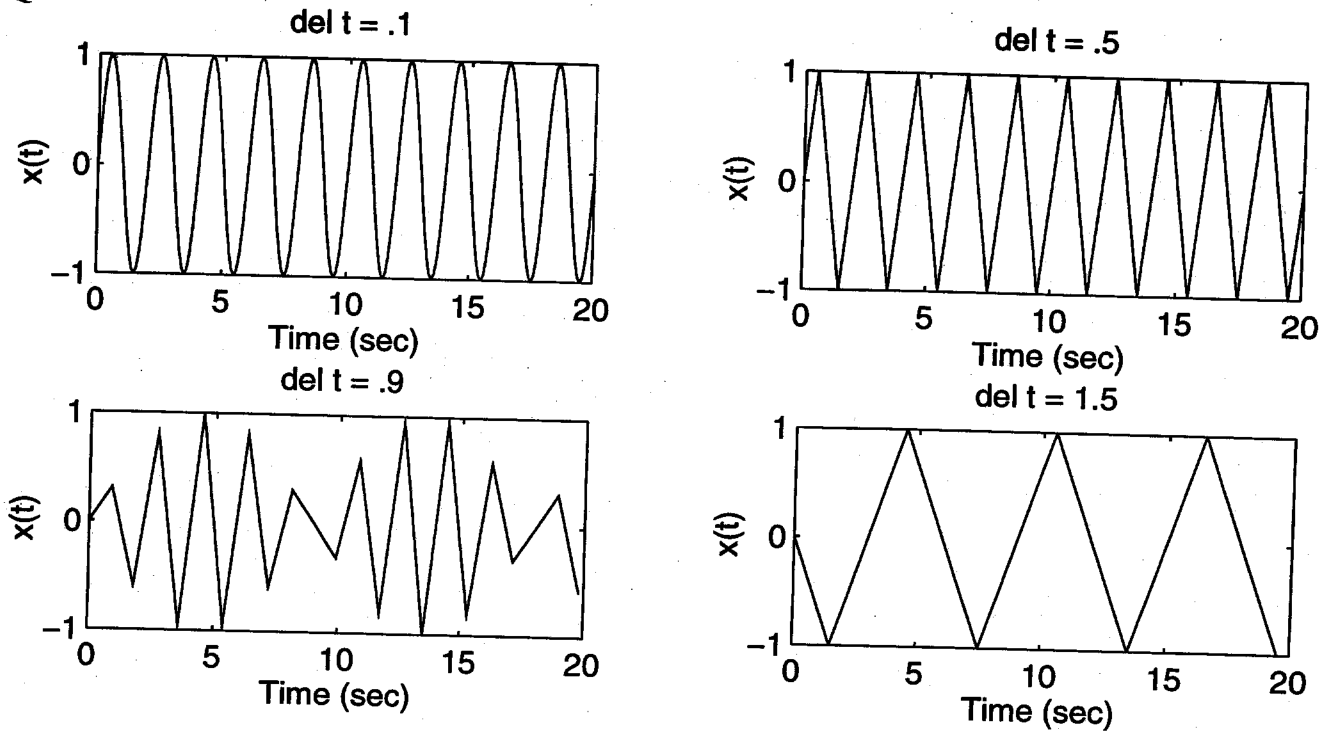
1.2 (a) $\omega = \pi$, so $\Delta t < \pi/\pi = 1$; $\sin(\pi(\Delta t n)) = 0$ for all integers n when $\Delta t = 2$

(b) $\omega = \pi$, $a = 0.1$, so $\Delta t \leq \frac{\pi}{4\sqrt{(0.1)^2 + \pi^2}} = 0.25$

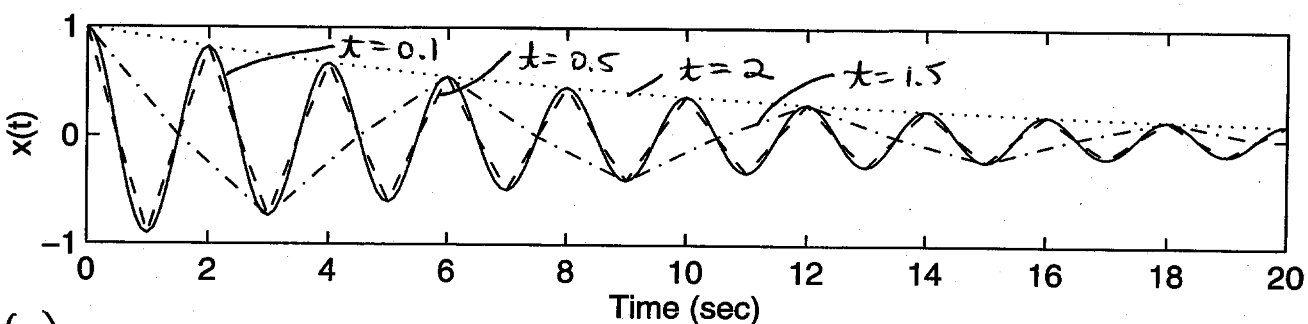
(c) $\omega = \pi/4$, $a = 1$, so $\Delta t \leq \frac{\pi}{4\sqrt{1^2 + (\pi/4)^2}} = 0.6177$

1.2 continued

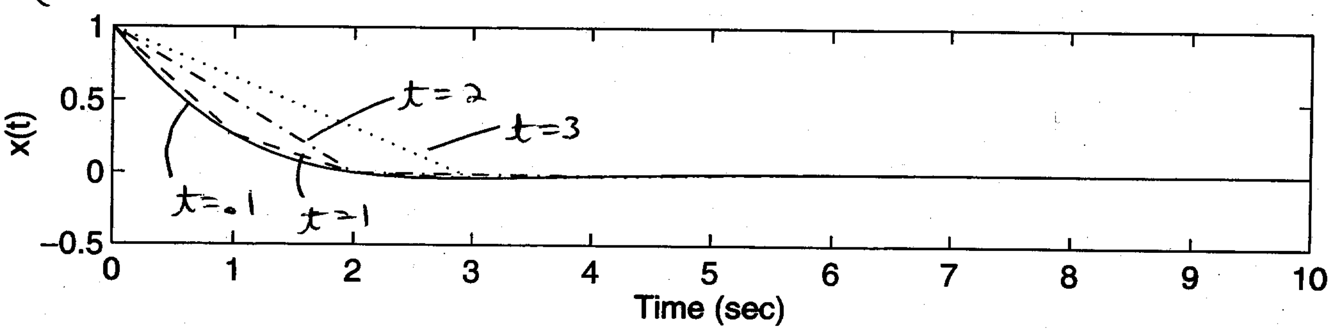
(a)



(b)



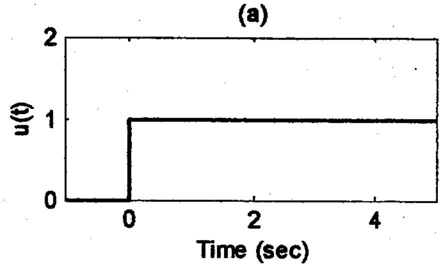
(c)



1.3

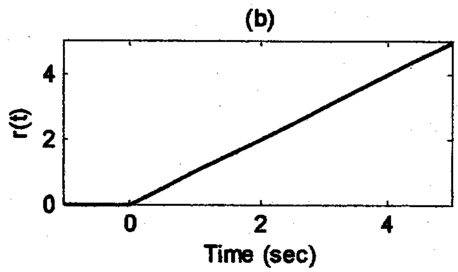
(a)

```
t=[-1 0 0 5];
x=[0 0 1 1] ;
plot(t,x,'LineWidth',1.5);
axis([-1 5 0 2])
xlabel('Time (sec)')
ylabel('u(t)')
title('(a)')
```



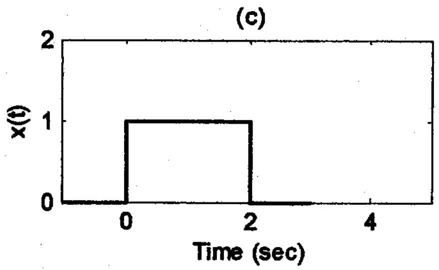
(b)

```
t=[-1 0 1 2 3 4 5];
x=[0 0 1 2 3 4 5];
plot(t,x,'LineWidth',1.5);
axis([-1 5 0 5])
xlabel('Time (sec)')
ylabel('r(t)')
title('(b)')
```



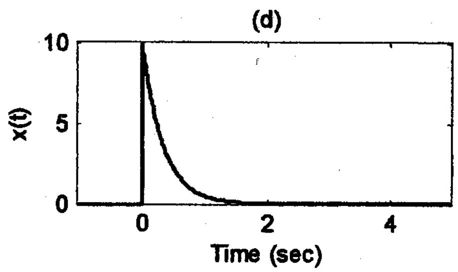
(c)

```
t=[-1 0 0 2 2 3];
x=[0 0 1 1 0 0];
plot(t,x,'LineWidth',1.5);
axis([-1 5 0 2])
xlabel('Time (sec)')
ylabel('x(t)')
title('(c)')
```



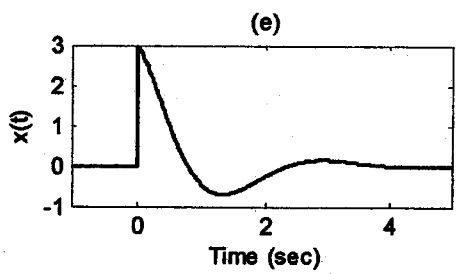
(d)

```
t1=[-1 0 0];
x1=[0 0 10];
t2=0:.01:5;
x2=10*exp(-3*t2);
t=[t1 t2];
x=[x1 x2];
plot(t,x,'LineWidth',1.5);
axis([-1 5 0 10])
xlabel('Time (sec)')
ylabel('x(t)')
title('(d)')
```



(e)

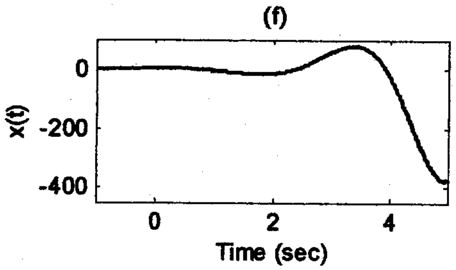
```
t1=[-1 0 0];
x1=[0 0 3];
t2=0:.01:5;
x2=3*exp(-t2).*cos(2*t2);
t=[t1 t2];
x=[x1 x2];
plot(t,x,'LineWidth',1.5);
axis([-1 5 -1 3])
xlabel('Time (sec)')
ylabel('x(t)')
title('(e)')
```



1.3 continued

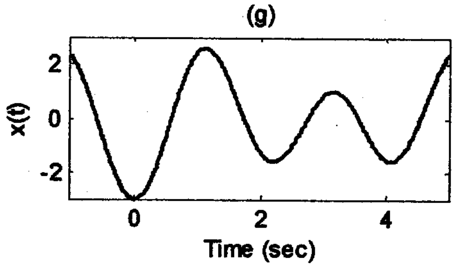
(f)

```
t1=[-1 0 0];
x1=[0 0 3];
t2=0:.01:5;
x2=3*exp(t2).*cos(2*t2);
t=[t1 t2];
x=[x1 x2];
plot(t,x,'LineWidth',1.5);
axis([-1 5 -450 100])
xlabel('Time (sec)')
ylabel('x(t)')
title('(f)')
```



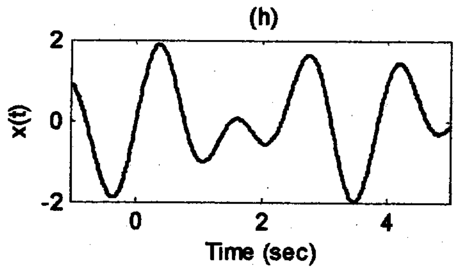
(g)

```
t=-1:.01:5;
x=2*sin(3*t-pi/2)-
cos(2*t);
plot(t,x,'LineWidth',1.5);
axis([-1 5 -3 3])
xlabel('Time (sec)')
ylabel('x(t)')
title('(g)')
```



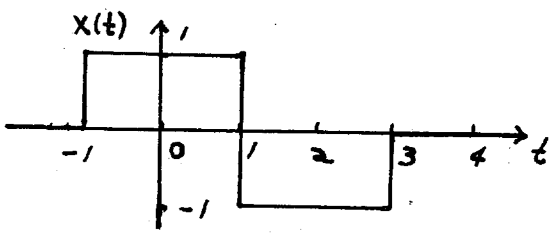
(h)

```
t=-1:.01:5;
x=sin(5*t)+sin(pi*t);
plot(t,x,'LineWidth',1.5);
axis([-1 5 -2 2])
xlabel('Time (sec)')
ylabel('x(t)')
title('(h)')
```

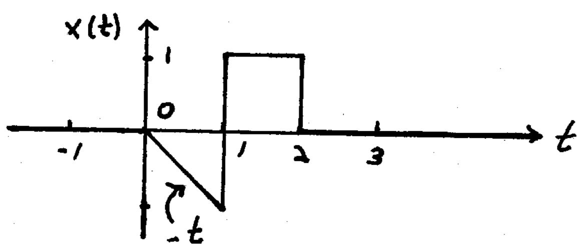


1.4

(a)

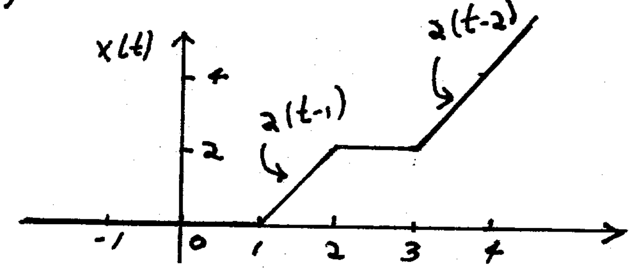


(b)



1.4 continued

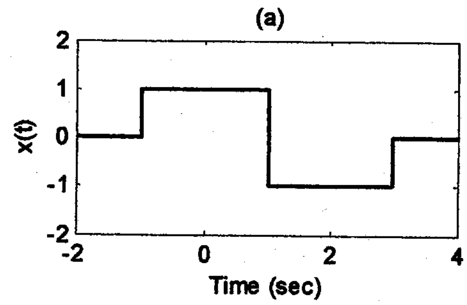
(c)



1.4 (d)

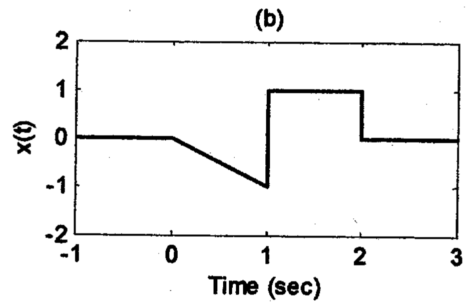
(a)

```
t=[-2 -1 -1 1 1 3 3 4];
x=[0 0 1 1 -1 -1 0 0];
plot(t,x,'LineWidth',1.5);
axis([-2 4 -2 2])
xlabel('Time (sec)')
ylabel('x(t)')
title('(a)')
```



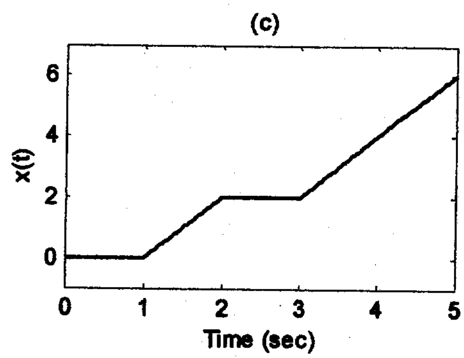
(b)

```
t=[-1 0 1 1 2 2 4];
x=[0 0 -1 1 1 0 0];
plot(t,x,'LineWidth',1.5);
axis([-1 3 -2 2])
xlabel('Time (sec)')
ylabel('x(t)')
title('(b)')
```



(c)

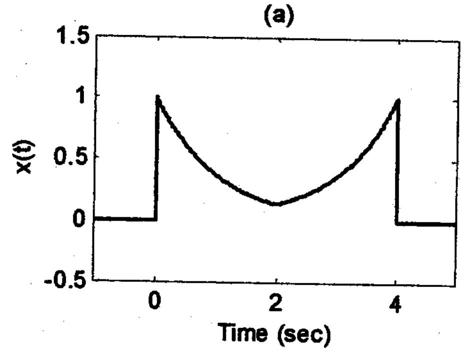
```
t1=[0 1];
x1=[0 0];
t2=1:.01:2;
x2=2*(t2-1);
t3=[2 3];
x3=[2 2];
t4=3:.01:5;
x4=2*t4-4;
t=[t1 t2 t3 t4];
x=[x1 x2 x3 x4];
plot(t,x,'LineWidth',1.5);
axis([0 5 -1 7])
xlabel('Time (sec)')
ylabel('x(t)')
title('(c)')
```



1.5

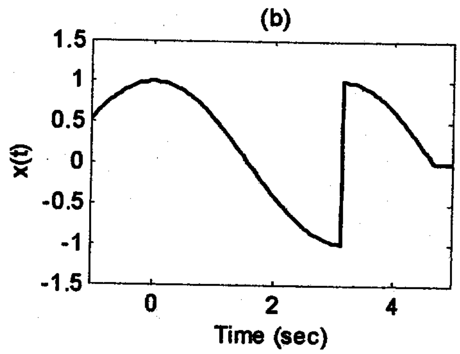
(a)

```
t1=[-1 0 0];
x1=[0 0 1];
t2=0:0.01:2;
x2=exp(-t2);
t3=2:.01:4;
x3=exp(t3-4);
t4=[4 5];
x4=[0 0];
t=[t1 t2 t3 t4];
x=[x1 x2 x3 x4];
plot(t,x,'LineWidth',1.5);
axis([-1 5 -.5 1.5])
xlabel('Time (sec)')
ylabel('x(t)')
title('(a)')
```



(b)

```
t1=-1:.05:pi;
x1=cos(t1);
t2=pi:.05:3*pi/2;
x2=-cos(t2);
t3=[3*pi/2 5];
x3=[0 0];
t=[t1 t2 t3];
x=[x1 x2 x3];
plot(t,x,'LineWidth',1.5);
axis([-1 5 -1.5 1.5])
xlabel('Time (sec)')
ylabel('x(t)')
title('(b)')
```



1.6 **(a)** Since $x(t) = v(t-c)$, replacing t by $t+c$ gives $v(t) = x(t+c)$.

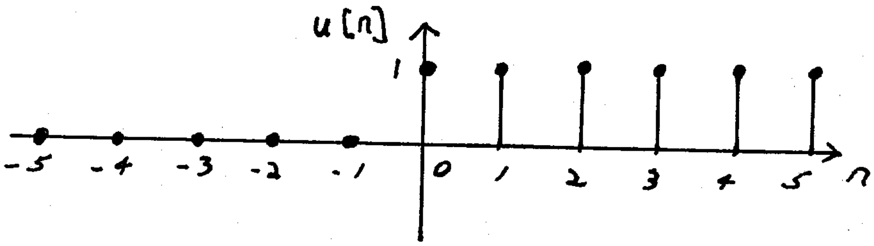
(b) (i) $v(t) = e^{-2(t+3)} = e^{-6}e^{-2t} = e^{-6}x(t)$

(ii) $v(t) = (t+2)^2 - (t+2) + 1 = t^2 + 3t + 3$

(iii) $v(t) = \sin(2(t+\pi/4)) = \sin(2t+\pi/2) = \cos(2t)$

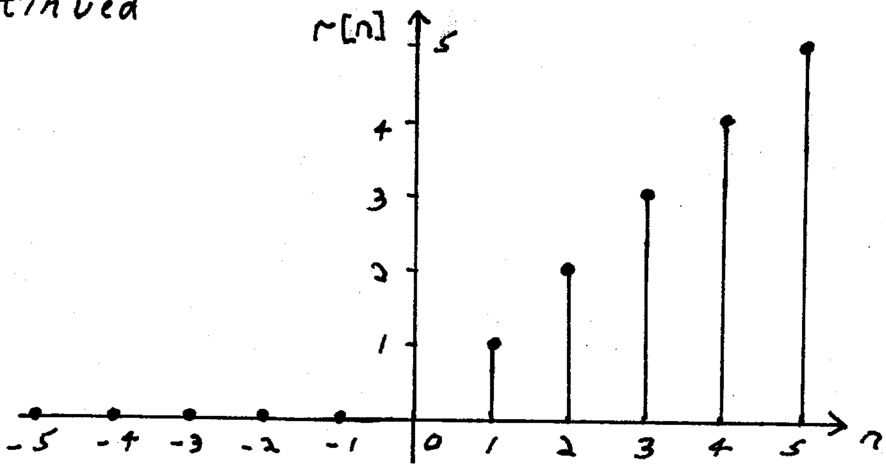
1.7

(a)

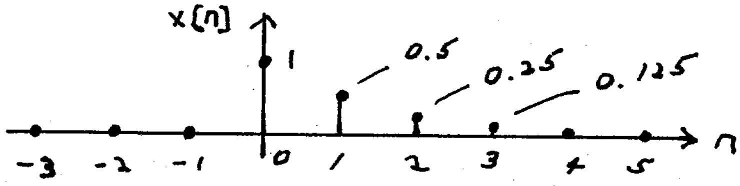


1.7 continued

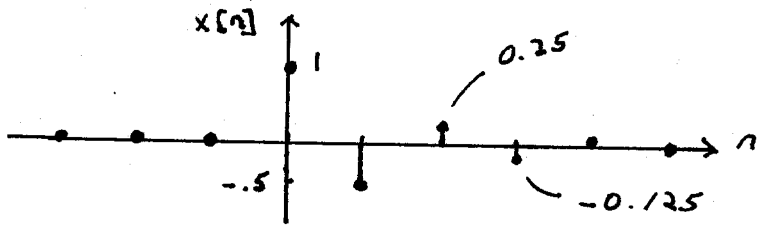
(b)



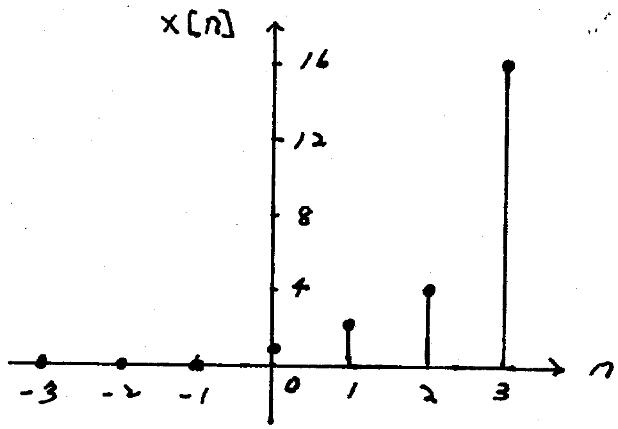
(c)



(d)

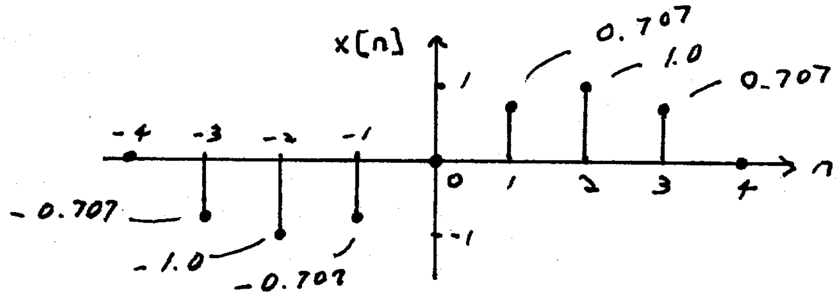


(e)

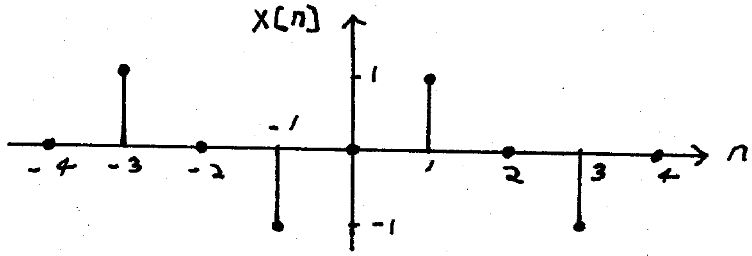


1.7 continued

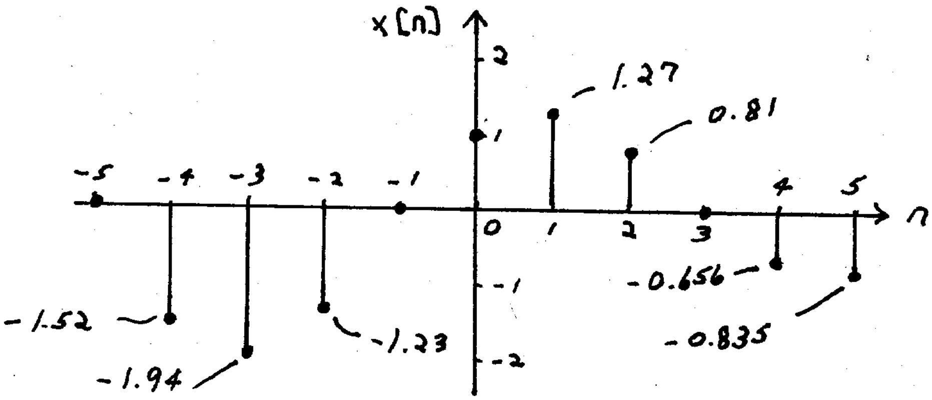
(f)



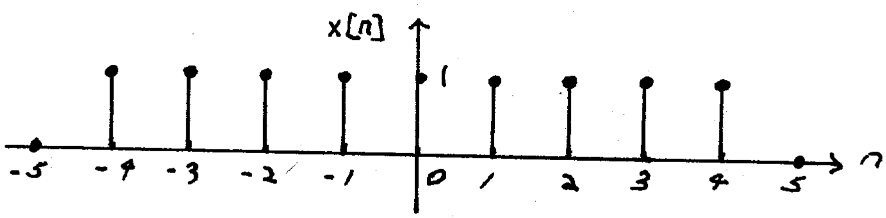
(g)



(h)

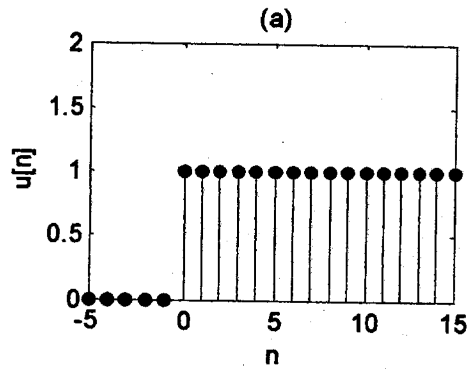


(i)

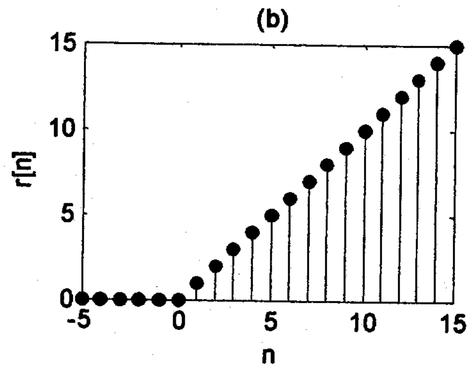


1.7 (j)

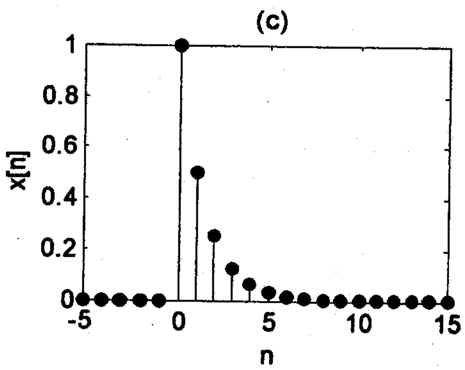
(a)
```
for n=1:5;
u(n)=0;
end;
for n=6:21;
u(n)=1;
end;
n=-5:15;
stem(n,u,'filled')
axis([-5 15 0 2])
xlabel('n')
ylabel('u[n]')
title('(a)')
```



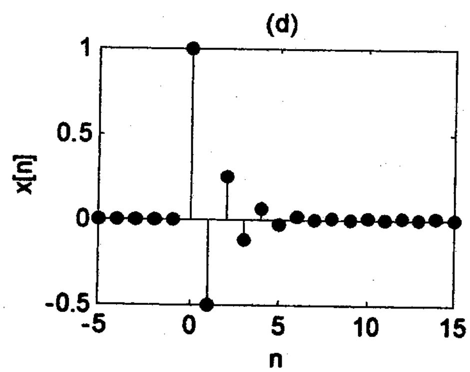
(b)
```
for n=1:5;
x(n)=0;
end;
for n=6:21;
x(n)=n-6;
end;
n=-5:15;
stem(n,x,'filled')
xlabel('n')
ylabel('r[n]')
title('(b)')
```



(c)
```
for n=1:5;
x(n)=0;
end;
for n=6:21;
x(n)=.5^(n-6);
end;
n=-5:15;
stem(n,x,'filled')
xlabel('n')
ylabel('x[n]')
title('(c)')
```

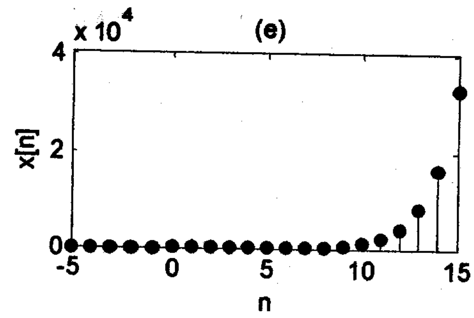


(d)
```
for n=1:5;
x(n)=0;
end;
for n=6:21;
x(n)=(-.5)^(n-6);
end;
n=-5:15;
stem(n,x,'filled')
xlabel('n')
ylabel('x[n]')
title('(d)')
```

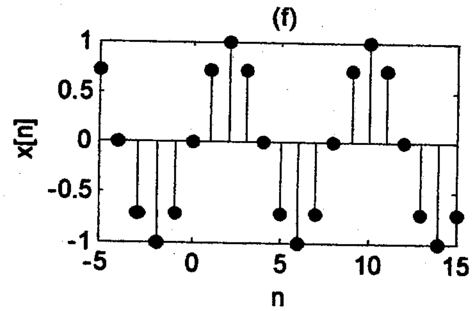


1.7 (j) continued

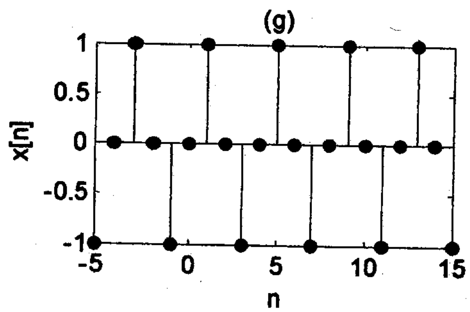
(e)
```
for n=1:5;
x(n)=0;
end;
for n=6:21;
x(n)=2^(n-6);
end;
n=-5:15;
stem(n,x,'filled')
xlabel('n')
ylabel('x[n]')
title('(e)')
```



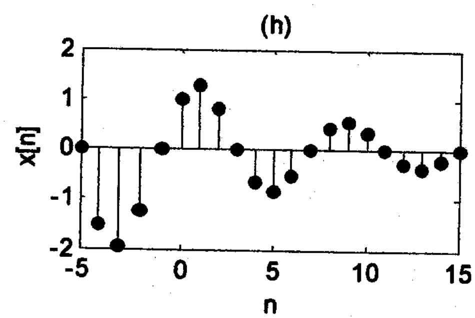
(f)
```
n=-5:15;
x=sin(pi*n/4);
stem(n,x,'filled')
xlabel('n')
ylabel('x[n]')
title('(f)')
```



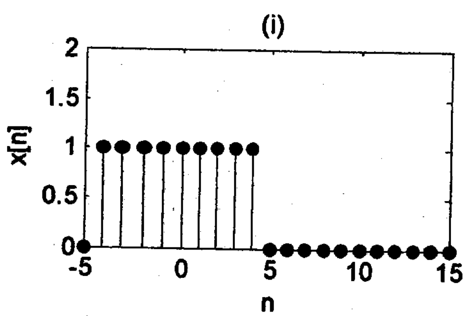
(g)
```
n=-5:15;
x=sin(pi*n/2);
stem(n,x,'filled')
xlabel('n')
ylabel('x[n]')
title('(g)')
```



(h)
```
n=-5:15;
x=(.9.^n).*(sin(pi*n/4)+cos(p
i*n/4));
stem(n,x,'filled')
xlabel('n')
ylabel('x[n]')
title('(h)')
```

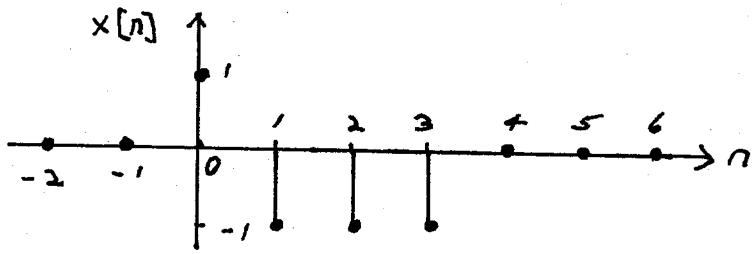


(i)
```
u(1)=0;
for n=2:10;
u(n)=1;
end;
for n=11:21;
u(n)=0;
end;
n=-5:15;
stem(n,u,'filled')
axis([-5 15 0 2])
xlabel('n')
ylabel('x[n]')
title('(i)')
```

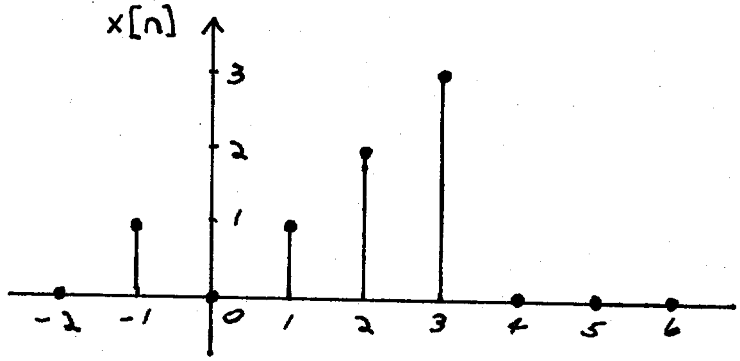


1.8

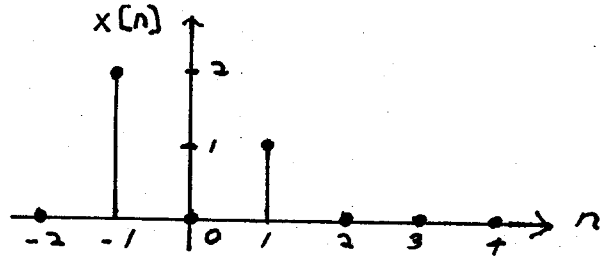
(a)


x[n]
1
-1
-2
-1
0
1
2
3
4
5
6
n

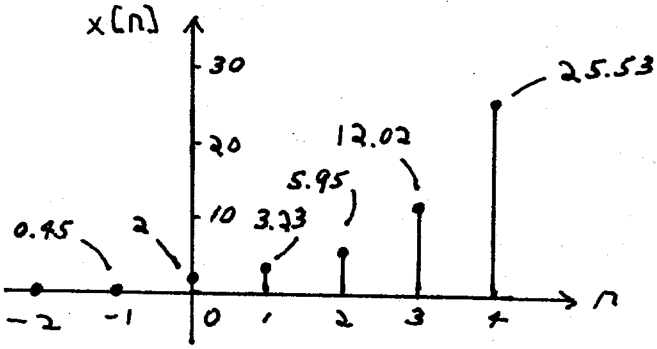
(b)


x[n]
3
2
1
-2
-1
0
1
2
3
4
5
6

(c)

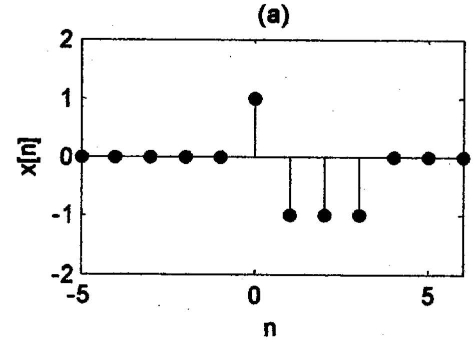

x[n]
2
1
-2
-1
0
1
2
3
4
n

(d)

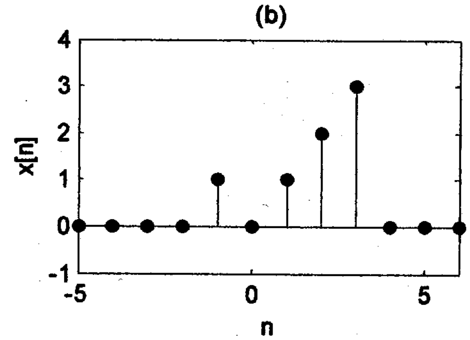

x[n]
30
20
10
25.53
12.02
5.95
3.73
0.45
2
-2
-1
0
1
2
3
4
n

1.8 (e)

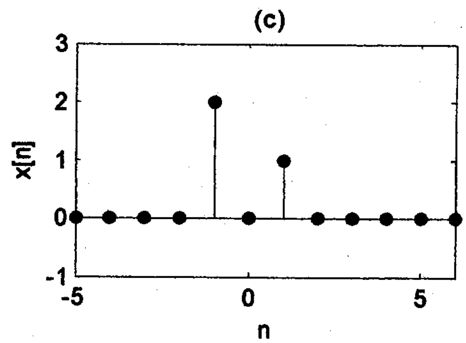
(a)
```
n=-5:6;
x=[0 0 0 0 0 1 -1 -1 -1 0 0
0];
stem(n,x,'filled')
axis([-5 6 -2 2])
xlabel('n')
ylabel('x[n]')
title('(a)')
```



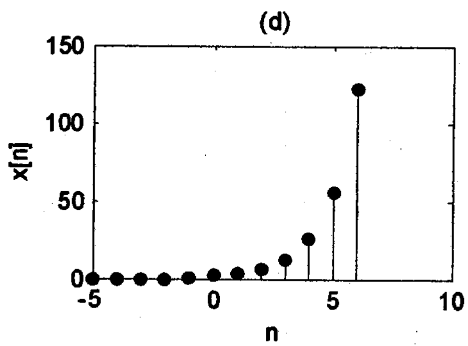
(b)
```
n=-5:6;
x=[0 0 0 0 1 0 1 2 3 0 0 0];
stem(n,x,'filled')
axis([-5 6 -1 4])
xlabel('n')
ylabel('x[n]')
title('(b)')
```



(c)
```
n=-5:6;
x=[0 0 0 0 2 0 1 0 0 0 0 0];
stem(n,x,'filled')
axis([-5 6 -1 3])
xlabel('n')
ylabel('x[n]')
title('(c)')
```

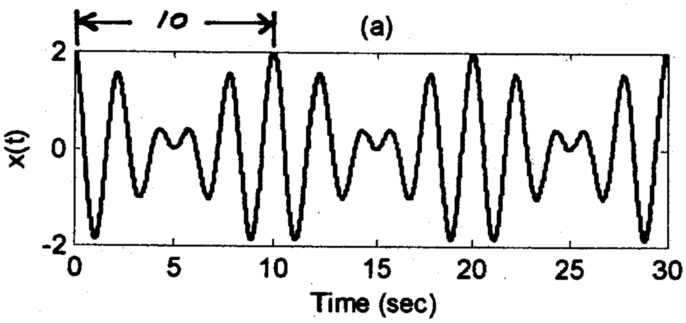


(d)
```
for n= 1:4;
x(n)=0;
end;
x(5)=exp(-.8);
for n=6:12;
x(n)=exp(.8*(n-6))+1;
end;
n=-5:6;
stem(n,x,'filled')
xlabel('n')
ylabel('x[n]')
title('(d)')
```



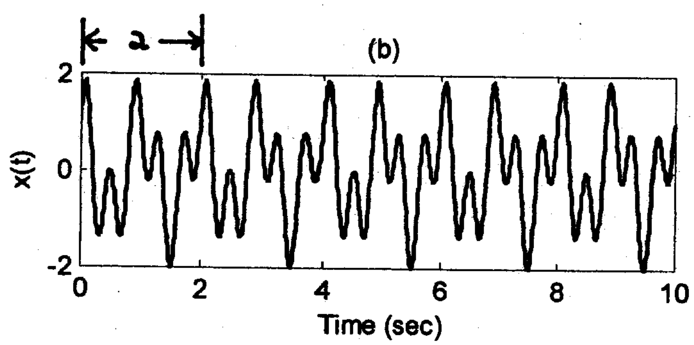
1.9 (a) $T_1 = \frac{2\pi}{\pi} = 2,\ T_2 = \frac{2\pi}{4\pi/5} = 2.5,\ \frac{T_1}{T_2} = \frac{2}{2.5} = \frac{4}{5}$, so the signal is periodic with fundamental period equal to $5T_1 = 10$.

```
t=0:.01:30;
x=cos(pi*t)+cos(4*pi*t/5);
plot(t,x,'LineWidth',1.5)
xlabel('Time (sec)')
ylabel('x(t)')
title('(a)')
```



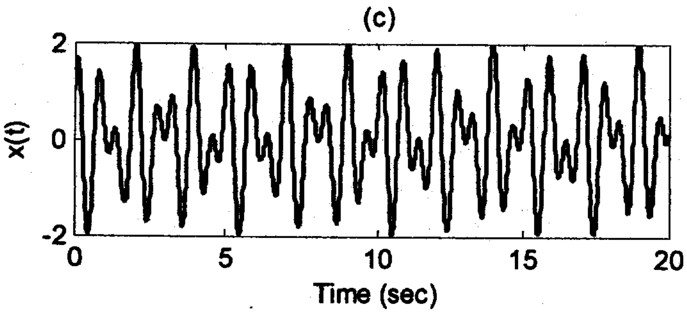
(b) $T_1 = \frac{2\pi}{2\pi} = 1,\ T_2 = \frac{2\pi}{5\pi} = \frac{2}{5},\ \frac{T_1}{T_2} = \frac{1}{2/5} = \frac{5}{2}$, so the signal is periodic with fundamental period equal to $5T_1 = 2$

```
t=0:.01:10;
x=cos(2*pi*(t-4))+sin(5*pi*t);
plot(t,x,'LineWidth',1.5)
xlabel('Time (sec)')
ylabel('x(t)')
title('(b)')
```



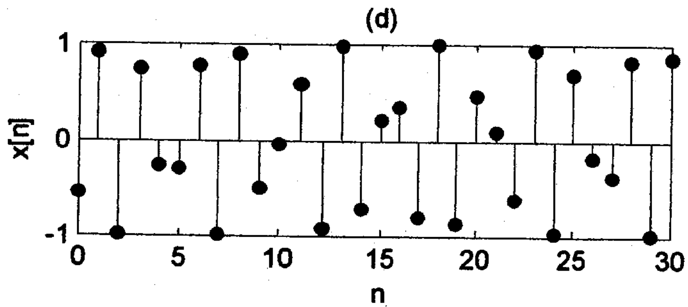
(c) $T_1 = \frac{2\pi}{2\pi} = 1,\ T_2 = \frac{2\pi}{10} = \frac{\pi}{5},\ \frac{T_1}{T_2} = \frac{1}{\pi/5} = \frac{5}{\pi} \neq \frac{q}{r}$ for any positive integers q and r, so the signal is not periodic.

```
t=0:.01:20;
x=cos(2*pi*t)+sin(10*t);
plot(t,x,'LineWidth',1.5)
xlabel('Time (sec)')
ylabel('x(t)')
title('(c)')
```



(d) $10 \neq \frac{2\pi q}{r}$ for any positive integers q and r, so the signal is not periodic.

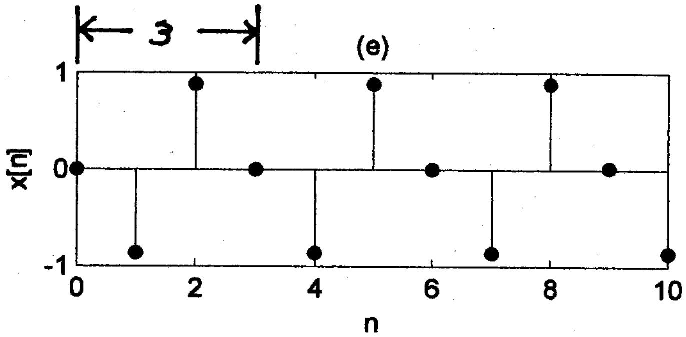
```
n=1:31;
x=sin(10*n);
n=0:30;
stem(n,x,'filled')
xlabel('n')
ylabel('x[n]')
title('(d)')
```



1.9 continued

(e) $\frac{10\pi}{3} = \frac{2\pi q}{r}$ with $q = 5$ and $r = 3$, so the signal is periodic with fundamental period equal to $r = 3$.

```
n=1:11;
x=sin(10*pi*(n-1)/3);
n=0:10;
stem(n,x,'filled')
xlabel('n')
ylabel('x[n]')
title('(e)')
```

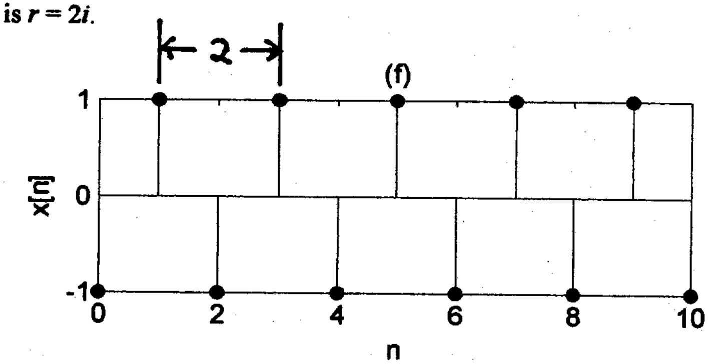


(f) Is there a positive integer r such that $x[n + r] = x[n]$? Since

$$x[n+r] = \cos\left(\pi(n+r)^2\right) = \cos\left(\pi n^2 + \pi(2nr + r^2)\right)$$

the signal is periodic if $\pi(2nr + r^2) = 2\pi m$ where m is an integer, or $(2nr + r^2) = 2m$. If $r = 2$, $m = 2(n + 1)$, which is an integer for any integer value of n, so $r = 2$ is the fundamental period. To generalize, consider the periodicity of $\cos\left(\frac{\pi n^2}{i}\right)$ for some integer i. This is a periodic signal if $\frac{2nr + r^2}{2i} = m$. The signal is periodic if i is even, in which case the fundamental period is $r = i$, and the signal is periodic if if i is odd, in which case the fundamental period is $r = 2i$.

```
n=1:11;
x=cos(pi*n.^2);
n=0:10;
stem(n,x,'filled')
xlabel('n')
ylabel('x[n]')
title('(f)')
```



1.10 (a) When $x[n] = \delta[n]$,

$$y[n] = \frac{1}{N}\left(\delta[n] + \delta[n-1] + \cdots + \delta[n - N + 1]\right)$$

$$y[n] = \begin{cases} \frac{1}{N}, & n = 0, 1, 2, \ldots, N-1 \\ 0, & \text{otherwise} \end{cases}$$

1.10 (b) When $x[n] = u[n]$, $y[n] = \frac{1}{N}\left(u[n]+u[n-1]+\cdots+u[n-N+1]\right)$.

For $n < 0$, $y[n] = 0$.

For $0 \le n \le N\text{-}1$, $y[n] = \frac{n+1}{N}$.

For $n \ge N$, $y[n] = 1$.

(c) When $x[n] = r[n]$, $y[n] = \frac{1}{N}\left(r[n]+r[n-1]+\cdots+r[n-N+1]\right)$

For $n < 0$, $y[n] = 0$.

For $0 \le n \le N\text{-}1$,

$$y[n] = \frac{1}{N}\left(n+n-1+n-2+\cdots+2+1+0\right)$$

Performing the addition

$$\begin{array}{l} n + n\text{-}1 + n\text{-}2 + \cdots + 2 + 1 + 0 \\ 0 + 1 + 2 + \cdots + n\text{-}2 + n\text{-}1 + n \\ \hline n + n + n + \cdots + n + n + n = n(n+1) \end{array}$$

shows that $2(n+n-1+n-2+\cdots+2+1+0) = n(n+1)$ and thus

$$y[n] = \frac{1}{N}\left(\frac{n(n+1)}{2}\right) = \frac{n(n+1)}{2N} \text{ for } 0 \le n \le N\text{-}1.$$

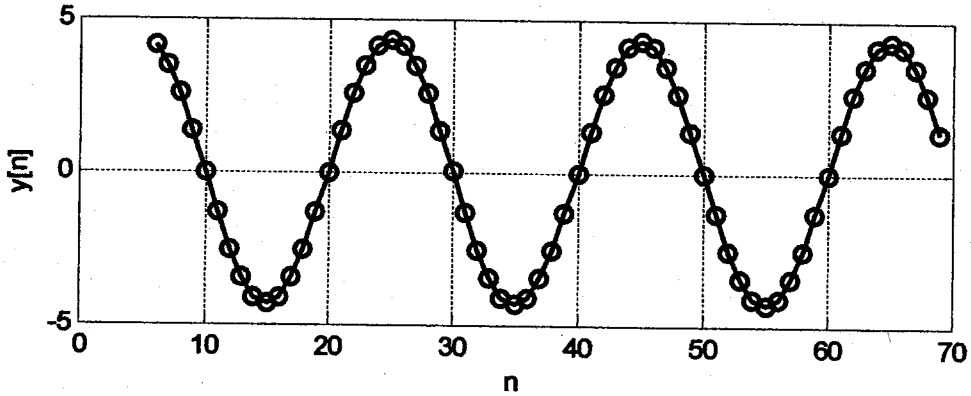
Finally, for $n \ge N$, $y[n] = \frac{1}{N}\left(n+n-1+n-2+\cdots+n-N+2+n-N+1\right)$. Via an addition process similar to the one given above, we have that $y[n] = \frac{1}{N}\left(\frac{N(2n-N+1)}{2}\right) = \frac{2n-N+1}{2}$ for $n \ge N$.

1.11 By the result in Problem 1.10, $y[n] = \frac{2n-N+1}{2}$ for $n \ge N$ and thus $y[n] = n - \frac{N-1}{2} = r[n-(N-1)/2]$ for $n \ge N$, which shows that the N-point MA filter delays the ramp $r[n]$ by $(N\text{-}1)/2$ time units.

1.12

```
n=1:69;
x=5*sin(pi*n/10+pi/4);
for n=6:69;
y(n)=(1/6)*sum(x(n-5:n));
end;
n=6:69;
plot(n,y(n),n,y(n),'o','LineWidth',1.5)
grid
xlabel('n')
ylabel('y[n]')
```

1.12 continued



1.13 The first zero crossing of $x(t) = 5\sin(\pi t/10 + \pi/4)$ occurs when $\pi t/10 + \pi/4 = \pi$, which implies that $t = 7.5$. From the plot in the solution to Problem 1.12, the first zero crossing occurs when $n = 10$. Hence, the time delay is $10 - 7.5 = 2.5$. Note that this results corresponds to the statement in Section 1.4 that the time delay through the N-point MA filter is $(N-1)/2 = 5/2 = 2.5$ when $N = 6$.

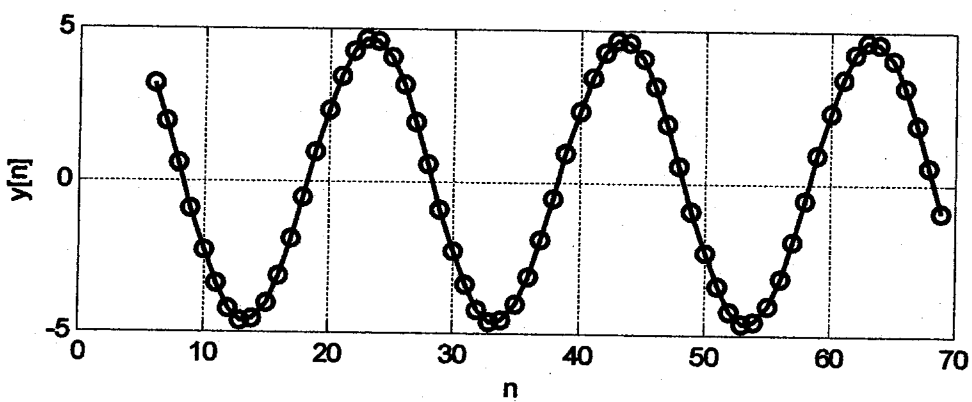
1.14 **(a)** When $x[n] = c$ for $n \geq 0$, for $n \geq 5$,

$$y[n] = \frac{32}{63}\left(c + \frac{c}{2} + \frac{c}{4} + \frac{c}{8} + \frac{c}{16} + \frac{c}{32}\right)$$

$$y[n] = \frac{32}{63}\left(\frac{32+16+8+4+2+1}{32}\right)c = \frac{32}{63}\left(\frac{63}{32}\right)c = c$$

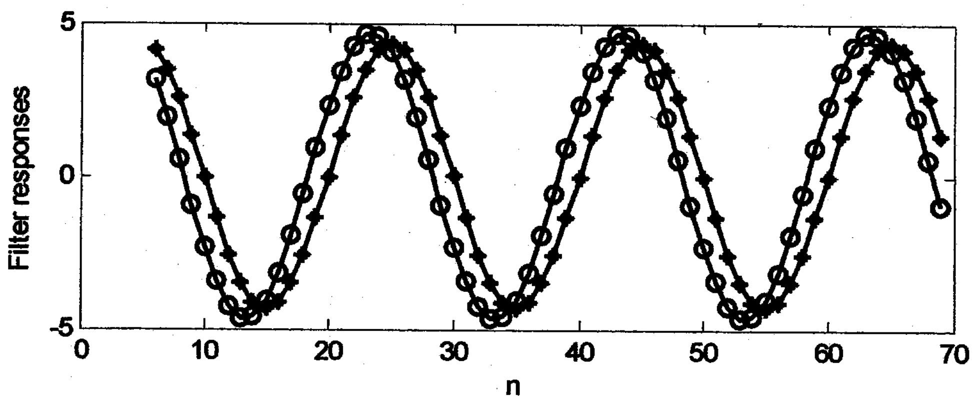
(b)

```
n=1:69;
x=5*sin(pi*n/10+pi/4);
w=[1/63 2/63 4/63 8/63 16/63 32/63];
for n=6:69;
 y(n)=w*(x(n-5:n)');
end;
n=6:69;
plot(n,y(n),n,y(n),'o','LineWidth',1.5)
grid
xlabel('n')
ylabel('y[n]')
```



1.14 continued

(c) The 6-point MA filter response is plotted below using *'s and the response of the system (which is also a filter) in Problem 1.14 is plotted below using o's.



From this plot it is seen that the filter responses are similar, except that the response to the filter in Problem 1.14 is faster (by about 2 time units) than the 6-point MA filter. The filter in Problem 1.14 is an example of a 6-point EWMA filter, which is studied in the next chapter and in later chapters of the book.

1.15 (a) When $x(t) = \delta(t)$,

$$y(t) = \frac{1}{I}\int_{t-I}^{t} \delta(\lambda)d\lambda = \begin{cases} 1/I \text{ for } 0 < t < I \\ 0, \text{ for all other } t \end{cases}$$

(b) When $x(t) = u(t)$, $y(t) = \frac{1}{I}\int_{t-I}^{t} u(\lambda)d\lambda$

For $t < 0$, $y(t) = 0$.

For $0 < t < I$, $y(t) = \frac{1}{I}\int_{0}^{t} 1d\lambda = \frac{1}{I}[\lambda]_{\lambda=0}^{\lambda=t} = \frac{t}{I}$

For $t > I$, $y(t) = \frac{1}{I}\int_{t-I}^{t} 1d\lambda = \frac{1}{I}[\lambda]_{\lambda=t-I}^{\lambda=t} = \frac{1}{I}[t-(t-I)] = 1$

(c) When $x(t) = r(t)$, $y(t) = \frac{1}{I}\int_{t-I}^{t} r(\lambda)d\lambda$

For $t < 0$, $y(t) = 0$.

For $0 < t < I$, $y(t) = \frac{1}{I}\int_{0}^{t} \lambda d\lambda = \frac{1}{I}\left[\frac{\lambda^2}{2}\right]_{\lambda=0}^{\lambda=t} = \frac{t^2}{2I}$

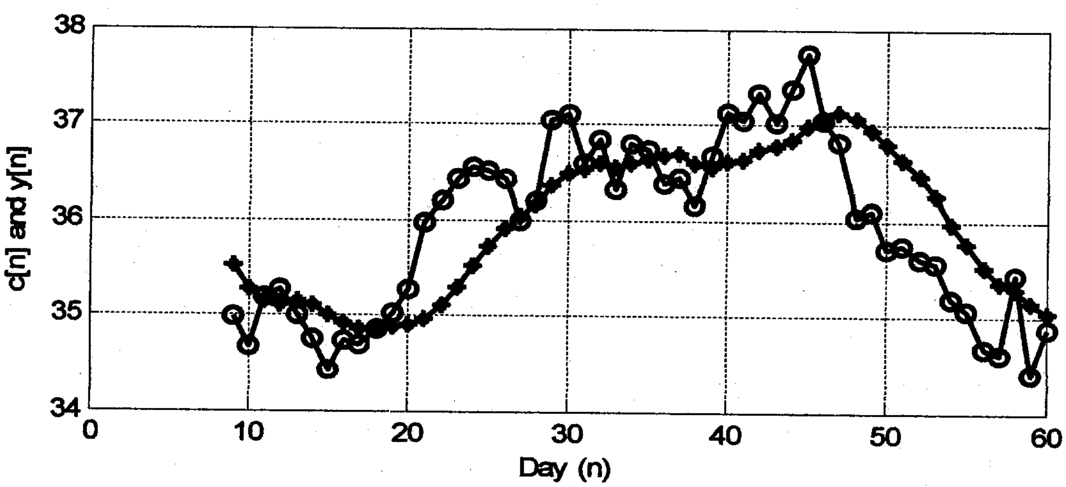
For $t > I$, $y(t) = \frac{1}{I}\int_{t-I}^{t} \lambda d\lambda = \frac{1}{I}\left[\frac{\lambda^2}{2}\right]_{\lambda=t-I}^{\lambda=t} = \frac{1}{I}\left[\frac{t^2}{2} - \frac{(t-I)^2}{2}\right] = \frac{1}{I}\left[It - \frac{I^2}{2}\right] = t - \frac{I}{2}$

1.15 continued

(d) By the result in Part (c), for $t > I$, $y(t) = t - I/2 = r(t-I/2)$, and thus the I-interval MA filter delays the ramp $r(t)$ by $I/2$ sec.

1.16

```
% c[n] is plotted using o's
% y[n] is plotted using *'s
c=csvread('QQQQdata5.csv',1,4,[1 4 60 4]);
for n=9:60;
y(n)=(1/9)*sum(c(n-8:n));
end;
n=9:60;
plot(n,c(n),n,c(n),'o','LineWidth',1.5)
hold on
plot(n,y(n),n,y(n),'*','LineWidth',1.5)
grid
xlabel('Day (n)')
ylabel('c[n] and y[n]')
hold off
```



1.17 (a) Causal and memoryless since the output $y(t)$ at time t depends only on the input $x(t)$ at time t.

(b) Same answer as that in Part (a).

(c) Same answer as that in Part (a).

(d) Same answer as that in Part (a).

(e) Causal and has memory since the output $y(t)$ at time t depends on the input $x(\lambda)$ for $\lambda = 0$ to $\lambda = t$.

(f) Same answer as that in Part (e).

1.18 (i) Let $y_1(t)$ be the response to $x_1(t)$ and $y_2(t)$ be the response to $x_2(t)$. Then

$$y_1(t) = \int_0^t x_1(\lambda)d\lambda \text{ and } y_2(t) = \int_0^t x_2(\lambda)d\lambda$$

1.18 (i) continued

Let $\tilde{y}(t)$ denote the response to $ax_1(t) + bx_2(t)$ where a and b are scalars. Then

$$\tilde{y}(t) = \int_0^t [ax_1(\lambda) + bx_2(\lambda)] d\lambda$$

By linearity of the integration operation,

$$\tilde{y}(t) = \int_0^t [ax_1(\lambda) + bx_2(\lambda)] d\lambda = a\int_0^t x_1(\lambda)d\lambda + b\int_0^t x_2(\lambda)d\lambda$$

Hence $\tilde{y}(t) = ay_1(t) + by_2(t)$, which shows that the system is linear.

(ii) With the same notation as in Part (i),

$$\tilde{y}(t) = \frac{d}{dt}[ax_1(t) + bx_2(t)]$$

By linearity of the differentiation operation,

$$\tilde{y}(t) = \frac{d}{dt}[ax_1(t) + bx_2(t)] = a\frac{d}{dt}x_1(t) + b\frac{d}{dt}x_2(t)$$

Hence $\tilde{y}(t) = ay_1(t) + by_2(t)$, which shows that the system is linear.

1.19 (a) Let $y(t)$ be the response to $x(t)$ so that $y(t) = |x(t)|$. Then the response to $-x(t)$ is $y(t) = |-x(t)| = |x(t)|$, which is not equal to $-y(t)$, and thus the system is not homogeneous, which shows that the system is not linear.

(b) Let $y(t)$ be the response to $x(t)$ so that $y(t) = e^{x(t)}$. Then the response to $-x(t)$ is $y(t) = e^{-x(t)} = 1/e^{x(t)}$, which is not equal to $-y(t)$, and thus the system is not homogeneous, which shows that the system is not linear.

(c) Let $y_1(t)$ be the response to $x_1(t)$ and $y_2(t)$ be the response to $x_2(t)$. Then

$$y_1(t) = (\sin t)x_1(t) \text{ and } y_2(t) = (\sin t)x_2(t)$$

Let $\tilde{y}(t)$ denote the response to $ax_1(t) + bx_2(t)$ where a and b are scalars. Then

$$\tilde{y}(t) = (\sin t)[ax_1(t) + bx_2(t)] = a(\sin t)x_1(t) + b(\sin t)x_2(t)$$

Hence $\tilde{y}(t) = ay_1(t) + by_2(t)$, which shows that the system is linear.

(d) Let $y_1(t)$ be the response to $x_1(t) = 6u(t)$ and $y_2(t)$ be the response to $x_2(t) = 6u(t)$, where $u(t)$ is the unit-step function. Then $y_1(t) = 6u(t)$ and $y_2(t) = 6u(t)$, and thus, $y_1(t) + y_2(t) = 12u(t)$. But the response to $x_1(t) + x_2(t) = 12u(t)$ is equal to 10, which is not equal to $y_1(t) + y_2(t)$. Hence, the system is not additive, and thus is not linear.

(e) Let $y_1(t)$ be the response to $x_1(t)$ and $y_2(t)$ be the response to $x_2(t)$. Then

$$y_1(t) = \int_0^t (t-\lambda)x_1(\lambda)d\lambda \quad \text{and} \quad y_2(t) = \int_0^t (t-\lambda)x_2(\lambda)d\lambda$$

1.19 (e) continued

Let $\tilde{y}(t)$ denote the response to $ax_1(t) + bx_2(t)$ where a and b are scalars. Then

$$\tilde{y}(t) = \int_0^t [(t-\lambda)(ax_1(\lambda) + bx_2(\lambda))]d\lambda$$

By linearity of the integration operation,

$$\tilde{y}(t) = \int_0^t [(t-\lambda)(ax_1(\lambda) + bx_2(\lambda))]d\lambda = a\int_0^t (t-\lambda)x_1(\lambda)d\lambda + b\int_0^t (t-\lambda)x_2(\lambda)d\lambda$$

Hence $\tilde{y}(t) = ay_1(t) + by_2(t)$, which shows that the system is linear.

(f) The system is linear. The proof is similar to that given in Part (e).

1.20 **(a)** Let $y(t)$ be the response to $x(t)$ so that $y(t) = |x(t)|$. For any t_1, the response to $x(t - t_1)$ is equal to $|x(t - t_1)|$, which is equal to $y(t - t_1)$, so the system is time invariant.

(b) For any t_1, the response to $x(t - t_1)$ is equal to $e^{x(t-t_1)}$, which is equal to $y(t - t_1)$, and thus the system is time invariant.

(c) For any t_1, the response to $x(t - t_1)$ is equal to $(\sin t)x(t - t_1)$, which is not equal to $y(t - t_1)$, since $y(t - t_1)$ is equal to $[\sin (t-t_1)]x(t - t_1)$. Therefore, the system is time varying.

(d) For any t_1, the response to $x(t - t_1)$ is equal to $\begin{cases} x(t-t_1) \text{ when } |x(t-t_1)| \le 10 \\ 0 \text{ when } |x(t-t_1)| > 10 \end{cases}$

This is equal to $y(t - t_1)$, so the system is time invariant.

(e) Let $\tilde{y}(t)$ denote the response to $x_1(t-t_1)$. Then $\tilde{y}(t) = \int_0^t (t-\lambda)x(\lambda - t_1)d\lambda$.

But $y(t-t_1) = \int_0^{t-t_1} (t - t_1 - \lambda)x(\lambda)d\lambda$. Let $\bar{\lambda} = t_1 + \lambda$, so $\bar{\lambda} = t_1$ when $\lambda = 0$, and $\bar{\lambda} = t$ when $\lambda = t - t_1$.

Also, $\lambda = \bar{\lambda} - t_1$, and inserting this into the integral expression for $y(t - t_1)$ gives

$$y(t-t_1) = \int_{t_1}^t (t - \bar{\lambda})x(\bar{\lambda} - t_1)d\bar{\lambda} = \int_{t_1}^t (t-\lambda)x(\lambda - t_1)d\lambda \neq \tilde{y}(t)$$

Thus the system is time varying.

(f) Same type of proof as given in part (e) shows that the system is time varying.

1.21 Let $x(t) = 0.8A$, then $y(t) = 0$. The response to $2x(t) = 1.6A$ is generally not zero, and thus is not equal to 2 times the response to $x(t)$. Therefore, the system is not homogeneous, and is not linear.

1.22 The input/output equation for the circuit is $y(t) = \begin{cases} x(t) \text{ if } x(t) > 0 \\ 0 \text{ if } x(t) < 0 \end{cases}$

The system is causal and memoryless since the output $y(t)$ at time t depends only on the input $x(t)$ at time t. Let $x(t)$ = the unit-step function $u(t)$. Then $y(t) = u(t)$. The response to $-u(t)$ is 0 for all t, which is not equal to -1 times the response $y(t)$ to $u(t)$. Hence, the system is not linear.

For any t_1, the response to $x(t - t_1)$ is equal to $\begin{cases} x(t-t_1) \text{ if } x(t-t_1) > 0 \\ 0 \text{ if } x(t-t_1) < 0 \end{cases}$

This is equal to $y(t - t_1)$, so the system is time invariant.

1.23 (a) Let $y(t)$ be the response to $x(t)$ and let T be a small positive number. Then

$$\int_0^t x(\lambda)d\lambda = \int_{t-T}^t x(\lambda)d\lambda + \int_{t-2T}^{t-T} x(\lambda)d\lambda + \int_{t-3T}^{t-2T} x(\lambda)d\lambda + \cdots$$

If T is sufficiently small, $\int_{t-nT-T}^{t-nT} x(\lambda)d\lambda \approx Tx(t-nT)$, $n = 0,1,2,\ldots$, where "≈" means approximately equal to. Thus $\int_0^t x(\lambda)d\lambda \approx Tx(t) + Tx(t-T) + Tx(t-2T) + \cdots$. If $x(t)$ is a continuous function of t, this expression for $\int_0^t x(\lambda)d\lambda$ becomes exact in the limit as $T \to 0$. By linearity and time invariance, the response to $Tx(t) + Tx(t-T) + Tx(t-2T) + \cdots$ is equal to $Ty(t) + Ty(t-T) + Ty(t-2T) + \cdots$, which is equal to $\int_0^t y(\lambda)d\lambda$ in the limit as $T \to 0$. Hence the property is proved.

(b) If $x(t)$ is differentiable, $\frac{dx(t)}{dt} = \lim_{h \to 0} \frac{x(t+h) - x(t)}{h}$. By linearity and time invariance, the response to $\frac{x(t+h) - x(t)}{h}$ is equal to $\frac{y(t+h) - y(t)}{h}$. Thus $\frac{dy(t)}{dt}$ is the response to $\frac{dx(t)}{dt}$, and the property is proved.

1.24 (a) $y(t) = 4(1 - e^{-t})u(t) - 4(1 - e^{-(t-1)})u(t-1)$

(b) $y(t) = (4)(1.414)\cos\left(2(t-2) - \pi/4\right) = 2.828\cos\left(2t - 4 - \pi/4\right)$

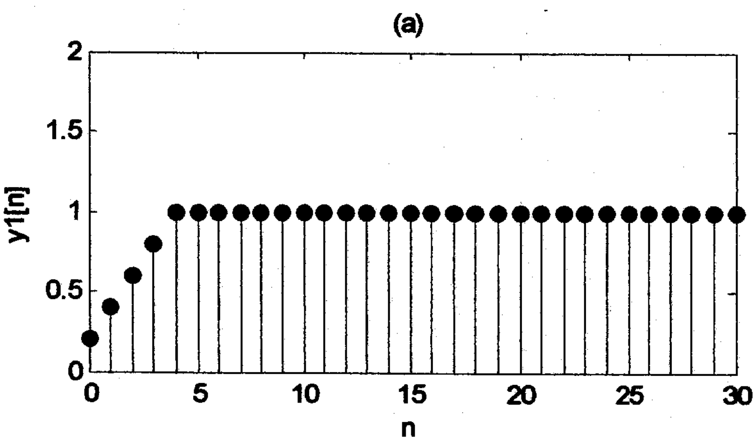
(c) $y(t) = 10(1 - e^{-t})u(t) + 14.14\cos(2t - \pi/4)$

(d) The ramp $tu(t)$ is equal to the integral of the step function $u(t)$. Hence, using the result in Part (a) of Problem 1.23 gives

$$y(t) = \int_0^t 2(1 - e^{-\lambda})d\lambda = 2\left[\lambda + e^{-\lambda}\right]_{\lambda=0}^{\lambda=t} = 2(t + e^{-t}) - 2, \; t \geq 0$$

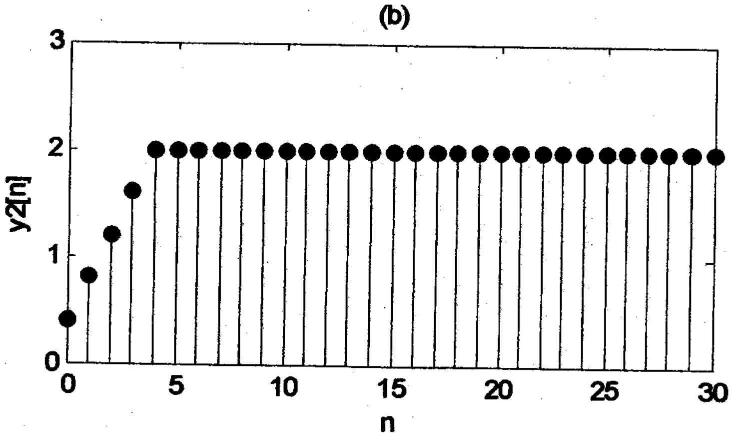
1.25 (a) Using the result in Problem 1.10 Part (b), $y[0] = 1/5$, $y[1] = 2/5$, $y[2] = 3/5$, $y[3] = 4/5$, $y[4] = 1$, $y[n] = 1$ for $n \geq 5$.

```
for n=1:5;
    y1(n)=n/5;
end;
for n=6:31;
    y1(n)=1;
end;
n=0:30;
stem(n,y1,'filled')
axis([0 30 0 2]);
xlabel('n')
ylabel('y1[n]')
title('(a)')
```



1.25 (b)

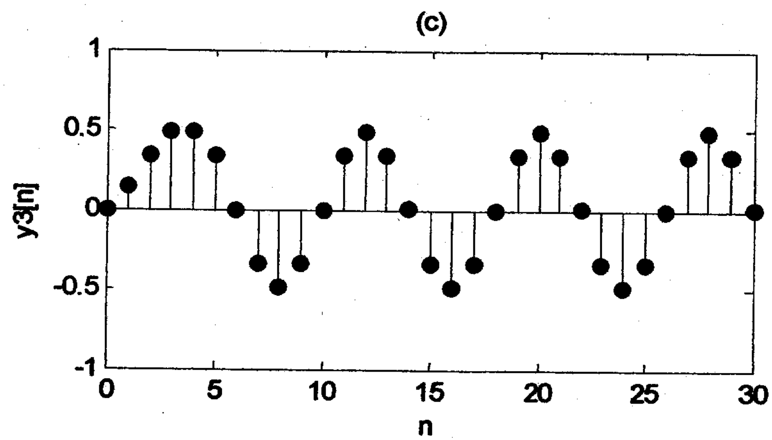
```
x=2*ones(1,31);
for n=1:5;
    y2(n)=(1/5)*sum(x(1:n));
end;
for n=6:31;
    y2(n)=(1/5)*sum(x(n-4:n));
end;
n=0:30;
stem(n,y2,'filled')
axis([0 30 0 3]);
xlabel('n')
ylabel('y2[n]')
title('(b)')
```



It is clear from the plots that $y_2[n]$ is equal to $2y_1[n]$.

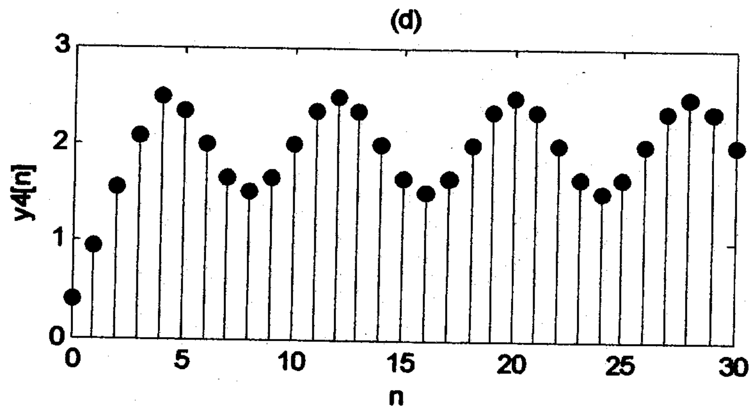
(c)

```
n=0:30;
x=sin(pi*n/4);
for n=1:5;
y3(n)=(1/5)*sum(x(1:n));
end;
for n=6:31;
y3(n)=(1/5)*sum(x(n-4:n));
end;
n=0:30;
stem(n,y3,'filled')
axis([0 30 -1 1]);
xlabel('n')
ylabel('y3[n]')
title('(c)')
```



(d)

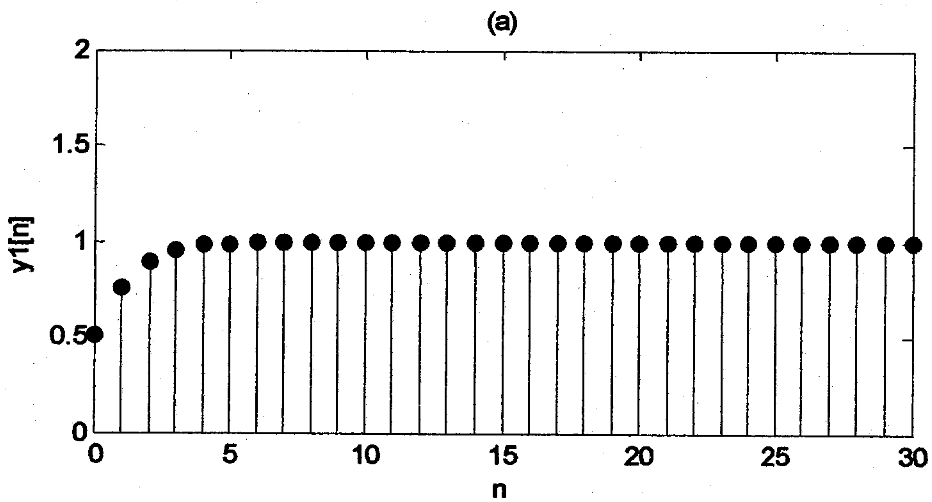
```
n=0:30;
x=2*ones(1,31)+sin(pi*n/4);
for n=1:5;
y4(n)=(1/5)*sum(x(1:n));
end;
for n=6:31;
y4(n)=(1/5)*sum(x(n-4:n));
end;
n=0:30;
stem(n,y4,'filled')
axis([0 30 0 3]);
xlabel('n')
ylabel('y4[n]')
title('(d)')
```



It can be seen from the plots that that $y_4[n]$ is equal to $2y_1[n] + y_3[n]$.

1.26 (a)

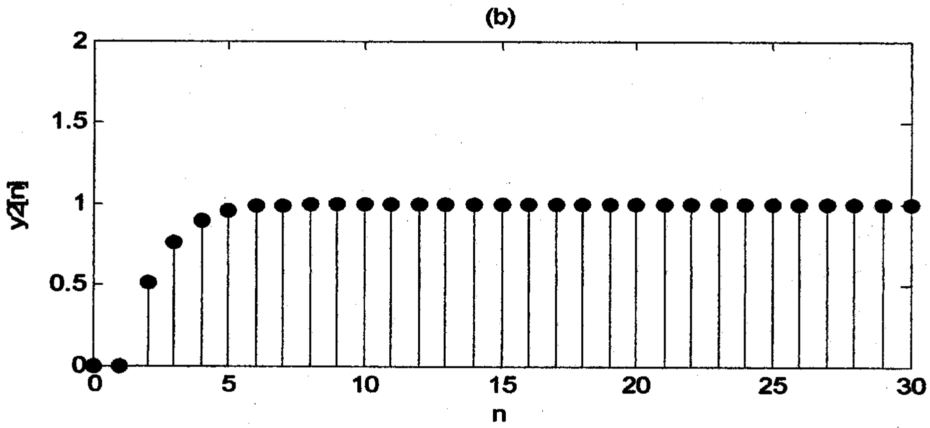
```
y1(1)=(32/63)*1;
y1(2)=(32/63)*(1+(1/2)*1);
y1(3)=(32/63)*(1+(1/2)*1+(1/4)*1);
y1(4)=(32/63)*(1+(1/2)*1+(1/4)*1+(1/8)*1);
y1(5)=(32/63)*(1+(1/2)*1+(1/4)*1+(1/8)*1+(1/16)*1);
y1(6)=(32/63)*(1+(1/2)*1+(1/4)*1+(1/8)*1+(1/16)*1);
for n=7:31;
y1(n)=1;
end
n=0:30;
stem(n,y1,'filled')
axis([0 30 0 2])
xlabel('n')
ylabel('y1[n]')
title('(a)')
```



(b)

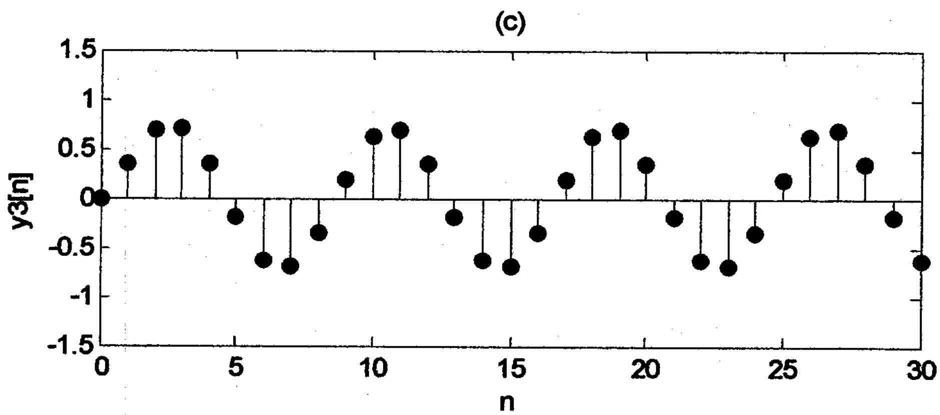
```
y2(1)=0;y2(2)=0;
y2(3)=(32/63)*1;
y2(4)=(32/63)*(1+(1/2)*1);
y2(5)=(32/63)*(1+(1/2)*1+(1/4)*1);
y2(6)=(32/63)*(1+(1/2)*1+(1/4)*1+(1/8)*1);
y2(7)=(32/63)*(1+(1/2)*1+(1/4)*1+(1/8)*1+(1/16)*1);
y2(8)=(32/63)*(1+(1/2)*1+(1/4)*1+(1/8)*1+(1/16)*1);
for n=9:31;
y2(n)=1;
end
n=0:30;
stem(n,y2,'filled')
axis([0 30 0 2])
xlabel('n')
ylabel('y2[n]')
title('(b)')
```

1.26 **(b) continued**



From the plots it is clear that $y_2[n] = y_1[n-2]$.

(c)
```
n=0:30;
x=sin(pi*n/4);
y3(1)=(32/63)*x(1);
i=1:2;w2=(32/63)*(.5.^(2-i));y3(2)=w2*(x(1:2)');
i=1:3;w3=(32/63)*(.5.^(3-i));y3(3)=w3*(x(1:3)');
i=1:4;w4=(32/63)*(.5.^(4-i));y3(4)=w4*(x(1:4)');
i=1:5;w5=(32/63)*(.5.^(5-i));y3(5)=w5*(x(1:5)');
i=1:6;w6=(32/63)*(.5.^(6-i));
for n=6:31;
y3(n)=w6*(x(n-5:n)');
end
n=0:30;
stem(n,y3,'filled')
axis([0 30 -1.5 1.5])
xlabel('n')
ylabel('y3[n]')
title('(c)')
```



1.26 **(d)** Replacing n by $n-4$ in the expression for $y[n]$ given in Problem 1.14 reveals that $y_4[n] = y_3[n-4]$. Hence, the plot of $y_4[n]$ is equal to a right shift of 4 time units of the plot in Part (c).

1.27 Let $y_1[n]$ be the response to $x_1[n]$ and $y_2[n]$ be the response to $x_2[n]$. Then for any scalars a and b, the response to $ax_1[n] + bx_2[n]$ is

$$\sum_{i=0}^{n} a_i\left(ax_1[n]+bx_2[n]\right)=a\sum_{i=0}^{n} a_i x_1[n]+b\sum_{i=0}^{n} a_i x_2[n]$$
$$=y_1[n]+y_2[n]$$

and therefore the system is linear.

1.28 The system is causal and has memory since the output $y[n]$ at time n depends only on the input $x[i]$ for $i = n\text{-}5, n\text{-}4, n\text{-}3, n\text{-}2, n\text{-}1\ n$. Replacing $x[n]$ by $ax_1[n] + bx_2[n]$ in the expression for $y[n]$ given in Problem 1.14 reveals that the response to $ax_1[n] + bx_2[n]$ is equal to a times the response to $x_1[n]$ plus b times the response to $x_2[n]$, and so the system is linear. Time invariance of the system follows by replacing n by $n-q$ for any integer q in the expression for $y[n]$ given in Problem 1.14.

1.29 **(a)** The system is causal and has memory since the output $y[n]$ at time n depends only on the input $x[i]$ for $i = n\text{-}2$ and $i = n$. The system is clearly linear and time invariant.

(b) The system is not causal since the output $y[n]$ at time n depends on the input $x[i]$ at future time $i = n+1$. The system is clearly linear and time invariant.

(c) The system is causal and memoryless since the output $y[n]$ at time n depends only on the input $x[n]$ at time n. The system is obviously linear, but it is time varying since the response to $x[n-1]$ is equal to $nx[n-1]$, which is not equal to $y[n-1] = (n-1)x[n-1]$.

(d) The system is causal and memoryless since the output $y[n]$ at time n depends only on the input $x[n]$ at time n. The system is linear, but is time varying since the response to $x[n-1]$ is equal to $u[n]x[n-1]$, which is not equal to $y[n-1] = u[n-1]x[n-1]$.

(e) The system is causal, memoryless, nonlinear and time invariant. The justifications are similar to those given in Problems 1.17 (a), 1.19 (a), and 1.20 (a).

(f) The system is causal, memoryless, nonlinear and time invariant. The justifications are similar to those given in Problems 1.17 (b), 1.19 (b), and 1.20 (b).

(g) The system is causal, has memory, is linear and time varying. The justifications are similar to those given in Problems 1.17 (f), 1.19 (f), and 1.20 (f).