CHAPTER 2

Light and Shading

PROBLEMS

2.1. We see a diffuse sphere centered at the origin, with radius one and albedo ρ, in an orthographic camera, looking down the z-axis. This sphere is illuminated by a distant point light source whose source direction is $(0, 0, 1)$. There is no other illumination. Show that the shading field in the camera is

$$\rho\sqrt{1 - x^2 - y^2}$$

Solution:

The surface is $(x, y, \sqrt{1 - x^2 - y^2})$. We get two tangent vectors by partial differentiation; they are $(1, 0, p)$ and $(0, 1, q)$ where $p = -x/\sqrt{1 - x^2 - y^2}$ and $q = -y/\sqrt{1 - x^2 - y^2}$. You can verify that the unit normal is

$$\vec{N} = \frac{(-p, -q, 1)}{\sqrt{1 + p^2 + q^2}}$$

and the shading must be $\rho\left((0, 0, 1) \cdot \vec{N}\right)$, which yields

$$\frac{1}{\sqrt{1 + p^2 + q^2}} = \sqrt{1 - x^2 - y^2}$$

2.2. What shapes can the shadow of a sphere take if it is cast on a plane and the source is a point source?

Solution:

These are conic sections with one important exception - you only get one half of the hyperbola.

2.3. We have a square area source and a square occluder, both parallel to a plane. The source is the same size as the occluder, and they are vertically above one another with their centers aligned.

(a) What is the shape of the umbra?

Solution:

Square. This is rather a special case. You can construct the umbra by constructing all points on the plane that can see no part of the source. This is a square directly below the occluder.

(b) What is the shape of the outside boundary of the penumbra?

Solution:

This is quite a nasty question. Easiest way to construct the answer is to think about an arbitrary point on the occluder; construct a cone over the source, whose vertex is this point. Now intersect that cone with the plane — the resulting square region on the plane consists of all points such that this point on the occluder blocks some point on the source. Finally, take the union of all such regions (i.e. over all points on the occluder); that's the penumbra. The envelope of the boundaries of these regions is the boundary of the penumbra. It's a square.

2.4. We have a square area source and a square occluder, both parallel to a plane. The edge length of the source is now twice that of the occluder, and they are vertically above one another with their centers aligned.

(a) What is the shape of the umbra?

Solution:

Depending on the distances between area source, occluder and plane, either there isn't one, or it's square.

(b) What is the shape of the outside boundary of the penumbra?

Solution:

same as in previous exercise; This is quite a nasty question. Easiest way to construct the answer is to think about an arbitrary point on the occluder; construct a cone over the source, whose vertex is this point. Now intersect that cone with the plane — the resulting square region on the plane consists of all points such that this point on the occluder blocks some point on the source. Finally, take the union of all such regions (i.e. over all points on the occluder); that's the penumbra. The envelope of the boundaries of these regions is the boundary of the penumbra. It's a square.

2.5. We have a square area source and a square occluder, both parallel to a plane. The edge length of the source is now half that of the occluder, and they are vertically above one another with their centers aligned.

(a) What is the shape of the umbra?

Solution:

Square

(b) What is the shape of the outside boundary of the penumbra?

Solution:

This is quite a nasty question. Easiest way to construct the answer is to think about an arbitrary point on the occluder; construct a cone over the source, whose vertex is this point. Now intersect that cone with the plane — the resulting square region on the plane consists of all points such that this point on the occluder blocks some point on the source. Finally, take the union of all such regions (i.e. over all points on the occluder); that's the penumbra. The envelope of the boundaries of these regions is the boundary of the penumbra. It's a square.

2.6. A small sphere casts a shadow on a larger sphere. Describe the possible shadow boundaries that occur.