CHAPTER 1

The Celestial Sphere

1.1 From Fig. 1.7, Earth makes $S/P_\oplus$ orbits about the Sun during the time required for another planet to make S/P orbits. If that other planet is a superior planet then Earth must make one extra trip around the Sun to overtake it, hence

$$\frac{S}{P_\oplus} = \frac{S}{P} + 1.$$

Similarly, for an inferior planet, that planet must make the extra trip, or

$$\frac{S}{P} = \frac{S}{P_\oplus} + 1.$$

Rearrangement gives Eq. (1.1).

1.2 For an inferior planet at greatest elongation, the positions of Earth (E), the planet (P), and the Sun (S) form a right triangle ($\angle EPS = 90°$). Thus $\cos(\angle PES) = \overline{EP}/\overline{ES}$.

From Fig. S1.1, the time required for a superior planet to go from opposition (point P_1) to quadrature (P_2) can be combined with its sidereal period (from Eq. 1.1) to find the angle $\angle P_1SP_2$. In the same time interval Earth will have moved through the angle $\angle E_1SE_2$. Since P_1, E_1, and S form a straight line, the angle $\angle P_2SE_2 = \angle E_1SE_2 - \angle P_1SP_2$. Now, using the right triangle at quadrature, $\overline{P_2S}/\overline{E_2S} = 1/\cos(\angle P_2SE_2)$.

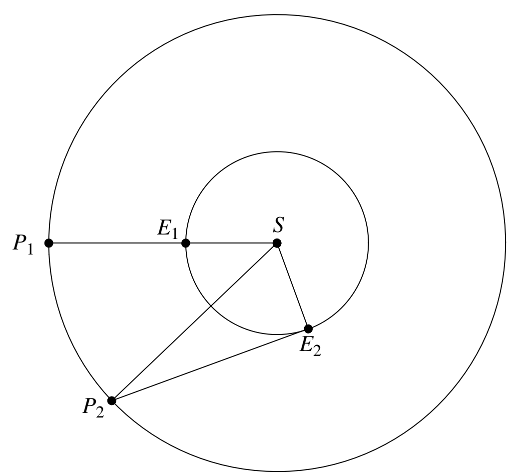

Figure S1.1: The relationship between synodic and sidereal periods for superior planets, as discussed in Problem 1.2.

1.3 (a) $P_{\text{Venus}} = 224.7$ d, $P_{\text{Mars}} = 687.0$ d

(b) Pluto. It travels the smallest fraction of its orbit before being "lapped" by Earth.

1.4 Vernal equinox: $\alpha = 0^{\text{h}}$, $\delta = 0°$
Summer solstice: $\alpha = 6^{\text{h}}$, $\delta = 23.5°$
Autumnal equinox: $\alpha = 12^{\text{h}}$, $\delta = 0°$
Winter solstice: $\alpha = 18^{\text{h}}$, $\delta = -23.5°$

1.5 (a) $(90° - 42°) + 23.5° = 71.5°$

(b) $(90° - 42°) - 23.5° = 24.5°$

1.6 (a) $90° - L < \delta < 90°$

(b) $L > 66.5°$

(c) Strictly speaking, only at $L = \pm 90°$. The Sun will move along the horizon at these latitudes.

1.7 (a) Both the year 2000 and the year 2004 were leap years, so each had 366 days. Therefore, the number of days between January 1, 2000 and January 1, 2006 is 2192 days. From January 1, 2006 to July 14, 2006 there are 194 days. Finally, from noon on July 14, 2006 to 16:15 UT is 4.25 hours, or 0.177 days. Thus, July 14, 2006 at 16:15 UT is JD 2453931.177.

(b) MJD 53930.677.

1.8 (a) $\Delta\alpha = 9^{\mathrm{m}}53.55^{\mathrm{s}} = 2.4731°$, $\Delta\delta = 2°9'16.2'' = 2.1545°$. From Eq. (1.8), $\Delta\theta = 2.435°$.

(b) $d = r\,\Delta\theta = 1.7 \times 10^{15}$ m $= 11{,}400$ AU.

1.9 (a) From Eqs. (1.2) and (1.3), $\Delta\alpha = 0.193628° = 0.774512^{\mathrm{m}}$ and $\Delta\delta = -0.044211° = -2.65266'$. This gives the 2010.0 precessed coordinates as $\alpha = 14^{\mathrm{h}}30^{\mathrm{m}}29.4^{\mathrm{s}}$, $\delta = -62°43'25.26''$.

(b) From Eqs. (1.6) and (1.7), $\Delta\alpha = -5.46^{\mathrm{s}}$ and $\Delta\delta = 7.984''$.

(c) Precession makes the largest contribution.

1.10 In January the Sun is at a right ascension of approximately 19^{h}. This implies that a right ascension of roughly 7^{h} is crossing the meridian at midnight. With about 14 hours of darkness this would imply observations of objects between right ascensions of 0 h and 14 h would be crossing the meridian during the course of the night (sunset to sunrise).

1.11 Using the identities, $\cos(90° - t) = \sin t$ and $\sin(90° - t) = \cos t$, together with the small-angle approximations $\cos\Delta\theta \approx 1$ and $\sin\Delta\theta \approx 1$, the expression immediately reduces to

$$\sin(\delta + \Delta\delta) = \sin\delta + \Delta\theta\cos\delta\cos\theta.$$

Using the identity $\sin(a + b) = \sin a\cos b + \cos a\sin b$, the expression now becomes

$$\sin\delta\cos\Delta\delta + \cos\delta\sin\Delta\delta = \sin\delta + \Delta\theta\cos\delta\cos\theta.$$

Assuming that $\cos\Delta\delta \approx 1$ and $\sin\Delta\delta \approx \Delta\delta$, Eq. (1.7) is obtained.

CHAPTER 2

Celestial Mechanics

2.1 From Fig. 2.4, note that

$$r^2 = (x - ae)^2 + y^2 \qquad \text{and} \qquad r'^2 = (x + ae)^2 + y^2.$$

Substituting Eq. (2.1) into the second expression gives

$$r = 2a - \sqrt{(x + ae)^2 + y^2}$$

which is now substituted into the first expression. After some rearrangement,

$$\frac{x^2}{a^2} + \frac{y^2}{a^2(1 - e^2)} = 1.$$

Finally, from Eq. (2.2),

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

2.2 The area integral in Cartesian coordinates is given by

$$A = \int_{-a}^{a} \int_{-b\sqrt{1-x^2/a^2}}^{b\sqrt{1-x^2/a^2}} dy\, dx = \frac{2b}{a} \int_{-a}^{a} \sqrt{a^2 - x^2}\, dx = \pi ab.$$

2.3 (a) From Eq. (2.3) the radial velocity is given by

$$v_r = \frac{dr}{dt} = \frac{a(1 - e^2)}{(1 + e\cos\theta)^2}\, e \sin\theta\, \frac{d\theta}{dt}. \qquad \text{(S2.1)}$$

Using Eqs. (2.31) and (2.32)

$$\frac{d\theta}{dt} = \frac{2}{r^2}\frac{dA}{dt} = \frac{L}{\mu r^2}.$$

The angular momentum can be written in terms of the orbital period by integrating Kepler's second law. If we further substitute $A = \pi ab$ and $b = a(1 - e^2)^{1/2}$ then

$$L = 2\mu\pi a^2(1 - e^2)^{1/2}/P.$$

Substituting L and r into the expression for $d\theta/dt$ gives

$$\frac{d\theta}{dt} = \frac{2\pi(1 + e\cos\theta)^2}{P(1 - e^2)^{3/2}}.$$

This can now be used in Eq. (S2.1), which simplifies to

$$v_r = \frac{2\pi ae \sin\theta}{P(1 - e^2)^{1/2}}.$$

Similarly, for the transverse velocity

$$v_\theta = r\frac{d\theta}{dt} = \frac{2\pi a(1 + e\cos\theta)}{(1 - e^2)^{1/2}P}.$$

(b) Equation (2.36) follows directly from $v^2 = v_r^2 + v_\theta^2$, Eq. (2.37) (Kepler's third law), and Eq. (2.3).

2.4 The total energy of the orbiting bodies is given by

$$E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - G\frac{m_1 m_2}{r}$$

where $r = |\mathbf{r}_2 - \mathbf{r}_1|$. Now,

$$v_1 = \dot{r}_1 = -\frac{m_2}{m_1 + m_2}\dot{r} \qquad \text{and} \qquad v_2 = \dot{r}_2 = \frac{m_1}{m_1 + m_2}\dot{r}.$$

Finally, using $M = m_1 + m_2$, $\mu = m_1 m_2 / (m_1 + m_2)$, and $m_1 m_2 = \mu M$, we obtain Eq. (2.25).

2.5 Following a procedure similar to Problem 2.4,

$$\begin{aligned}
\mathbf{L} &= m_1 \mathbf{r}_1 \times \mathbf{v}_1 + m_2 \mathbf{r}_2 \times \mathbf{v}_2 \\
&= m_1 \left[-\frac{m_2}{m_1 + m_2}\right] \mathbf{r} \times \left[-\frac{m_2}{m_1 + m_2}\right] \mathbf{v} \\
&\quad + m_2 \left[\frac{m_1}{m_1 + m_2}\right] \mathbf{r} \times \left[\frac{m_1}{m_1 + m_2}\right] \mathbf{v} \\
&= \mu \mathbf{r} \times \mathbf{v} = \mathbf{r} \times \mathbf{p}
\end{aligned}$$

2.6 (a) The total orbital angular momentum of the Sun–Jupiter system is given by Eq. (2.30). Referring to the data in Appendicies A and C, $M_\odot = 1.989 \times 10^{30}$ kg, $M_J = 1.899 \times 10^{27}$ kg, $M = M_J + M_\odot = 1.991 \times 10^{30}$ kg, and $\mu = M_J M_\odot / (M_J + M_\odot) = 1.897 \times 10^{27}$ kg. Furthermore, $e = 0.0489$, $a = 5.2044$ AU $= 7.786 \times 10^{11}$ m. Substituting,

$$L_{\text{total orbit}} = \mu\sqrt{GMa\left(1 - e^2\right)} = 1.927 \times 10^{43} \text{ kg m}^2 \text{ s}^{-1}.$$

(b) The distance of the Sun from the center of mass is $a_\odot = \mu a / M_\odot = 7.426 \times 10^8$ m. The Sun's orbital speed is $v_\odot = 2\pi a_\odot / P_J = 12.46$ m s^{-1}, where $P_J = 3.743 \times 10^8$ s is the system's orbital period. Thus, for an assumed circular orbit,

$$L_{\text{Sun orbit}} = M_\odot a_\odot v_\odot = 1.840 \times 10^{40} \text{ kg m}^2 \text{ s}^{-1}.$$

(c) The distance of Jupiter from the center of mass is $a_J = \mu a / M_J = 7.778 \times 10^{11}$ m, and its orbital speed is $v_J = 2\pi a_J / P_J = 1.306 \times 10^4$ m s^{-1}. Again assuming a circular orbit,

$$L_{\text{Jupiter orbit}} = M_J a_J v_J = 1.929 \times 10^{43} \text{ kg m}^2 \text{ s}^{-1}.$$

This is in good agreement with

$$L_{\text{total orbit}} - L_{\text{Sun orbit}} = 1.925 \times 10^{43} \text{ kg m}^2 \text{ s}^{-1}.$$

(d) The moment of inertia of the Sun is approximately

$$I_\odot \sim \frac{2}{5} M_\odot R_\odot^2 \sim 3.85 \times 10^{47} \text{ kg m}^2$$

and the moment of inertia of Jupiter is approximately

$$I_J \sim \frac{2}{5} M_J R_J^2 \sim 3.62 \times 10^{42} \text{ kg m}^2.$$

(*Note:* Since the Sun and Jupiter are centrally condensed, these values are overestimates; see Section 23.2.) Using $\omega = 2\pi/P$,

$$L_{\text{Sun rotate}} = 1.078 \times 10^{42} \text{ kg m}^2 \text{ s}^{-1}$$

$$L_{\text{Jupiter rotate}} = 6.312 \times 10^{38} \text{ kg m}^2 \text{ s}^{-1}.$$

(e) Jupiter's orbital angular momentum.

2.7 (a) $v_{\text{esc}} = \sqrt{2GM_J/R_J} = 60.6 \text{ km s}^{-1}$

(b) $v_{\text{esc}} = \sqrt{2GM_\odot/1 \text{ AU}} = 42.1 \text{ km s}^{-1}$.

2.8 (a) From Kepler's third law (Eq. 2.37) with $a = R_\oplus + h = 6.99 \times 10^6$ m, $P = 5820$ s $= 96.9$ min.

(b) The orbital period of a geosynchronous satellite is the same as Earth's sidereal rotation period, or $P = 8.614 \times 10^4$ s. From Eq. (2.37), $a = 4.22 \times 10^7$ m, implying an altitude of $h = a - R_\oplus = 3.58 \times 10^7$ m $= 5.6$ $R_\oplus$.

(c) A geosynchronous satellite must be "parked" over the equator and orbiting in the direction of Earth's rotation. This is because the center of the satellite's orbit is the center of mass of the Earth–satellite system (essentially Earth's center).

2.9 The integral average of the potential energy is given by

$$\langle U \rangle = \frac{1}{P} \int_0^P U(t)\, dt = -\frac{1}{P} \int_0^P \frac{GM\mu}{r(t)}\, dt.$$

Using Eqs. (2.31) and (2.32) to solve for dt in terms of $d\theta$, and making the appropriate changes in the limits of integration,

$$\langle U \rangle = -\frac{1}{P} \int_0^{2\pi} \frac{GM\mu^2 r}{L}\, d\theta.$$

Writing r in terms of θ via Eq. (2.3) leads to

$$\begin{aligned} \langle U \rangle &= -\frac{GM\mu^2 a\left(1 - e^2\right)}{PL} \int_0^{2\pi} \frac{d\theta}{1 + e\cos\theta} \\ &= -\frac{2\pi GM\mu^2 a\left(1 - e^2\right)^{1/2}}{PL}. \end{aligned}$$

Using Eq. (2.30) to eliminate the total orbital angular momentum L, and Kepler's third law (Eq. 2.37) to replace the orbital period P, we arrive at

$$\langle U \rangle = -G\frac{M\mu}{a}.$$

2.10 Using the integral average from Problem 2.9

$$\langle r \rangle = \frac{1}{P} \int_0^P r(t)\, dt.$$

Using substitutions similar to the solution of Problem 2.9 we eventually arrive at

$$\langle r \rangle = \frac{a}{2\pi}\left(1 - e^2\right)^{5/2} \int_0^{2\pi} \frac{d\theta}{\left(1 + e\cos\theta\right)^3}. \tag{S2.2}$$

It is evident that for $e = 0$, $\langle r \rangle = a$, as expected for perfectly circular motion. However, $\langle r \rangle$ deviates from a for other values of e. This function is most easily evaluted numerically. Employing a simple trapezoid method with 10^6 intervals, gives the results shown in Table S2.1.