Solutions Manual

Engineering Mechanics: Statics

2nd Edition

Michael E. Plesha
University of Wisconsin–Madison

Gary L. Gray
The Pennsylvania State University

Francesco Costanzo
The Pennsylvania State University

With the assistance of:
Chris Punshon
Andrew J. Miller
Justin High
Chris O'Brien
Chandan Kumar
Joseph Wyne
Jonathan Fleischmann

Version: May 11, 2012

The McGraw-Hill Companies, Inc.

Important Information about this Solutions Manual

We encourage you to occasionally visit

`http://www.mhhe.com/pgc2e`

to obtain the most up-to-date version of this solutions manual.

Contact the Authors

If you find any errors and/or have questions concerning a solution, please do not hesitate to contact the authors and editors via email at:

`plesha@engr.wisc.edu`, and
`stat_solns@email.esm.psu.edu`

We welcome your input.

Accuracy of Numbers in Calculations

Throughout this solutions manual, we will generally assume that the data given for problems is accurate to 3 significant digits. When calculations are performed, all intermediate numerical results are reported to 4 significant digits. Final answers are usually reported with 3 or 4 significant digits. If you verify the calculations in this solutions manual using the rounded intermediate numerical results that are reported, you should obtain the final answers that are reported to 3 significant digits.

Chapter 1 Solutions

Problem 1.1

(a) Consider a situation in which the force F applied to a particle of mass m is zero. Multiply the scalar form of Eq. (1.2) on page 6 (i.e., $a = dv/dt$) by dt, and integrate both sides to show that the velocity v (also a scalar) is constant. Then use the scalar form of Eq. (1.1) to show that the (scalar) position r is a linear function of time.

(b) Repeat Part (a) when the force applied to the particle is a nonzero constant, to show that the velocity and position are linear and quadratic functions of time, respectively.

Solution

Part (a) Consider the scalar form of Eq. (1.3) on page 7 for the case with $F = 0$,

$$F = ma \quad \Rightarrow \quad 0 = ma \quad \Rightarrow \quad a = 0. \tag{1}$$

Next, consider the scalar form of Eq. (1.2) on page 6,

$$\frac{dv}{dt} = a \quad \Rightarrow \quad dv = a dt \quad \Rightarrow \quad \int dv = v = \int a\, dt. \tag{2}$$

Substituting $a = 0$ into Eq. (2) and evaluating the integral provides

$$\boxed{v = \text{constant} = v_0,} \tag{3}$$

demonstrating that the velocity v is constant when the acceleration is zero. Next, consider the scalar form of Eq. (1.1),

$$\frac{dr}{dt} = v \quad \Rightarrow \quad dr = v dt \quad \Rightarrow \quad \int dr = r = \int v\, dt. \tag{4}$$

For the case with constant velocity given by Eq. (3), it follows that

$$\boxed{r = \int v_0\, dt = v_0 \int dt = v_0\, t + c_1,} \tag{5}$$

where c_1 is a constant of integration. Thus, the position r is a linear function of time when the acceleration is zero. Note that in the special case that $v_0 = 0$, then the position r does not change with time.

Part (b) When the force F is constant, then Newton's second law provides

$$F = \text{constant} = ma \quad \Rightarrow \quad a = F/m = \text{constant}. \tag{6}$$

Following the same procedure as used in Part (a), we find that

$$\boxed{v = \int a\, dt = \int \frac{F}{m}\, dt = \frac{F}{m} t + c_2,} \tag{7}$$

where c_2 is a constant of integration and v is shown to be a linear function of time. Likewise, recalling Eq. (4)

$$\boxed{u = \int v\,dt = \int \left(\frac{F}{m}t + c_2\right) dt = \frac{F}{2m}t^2 + c_2 t + c_3,} \tag{8}$$

which is a general quadratic function of time. To determine the constants of integration requires that initial conditions be specified. That is, at some instant of time (usually $t = 0$), we must specify the position and velocity of the particle.

Problem 1.2

Using the length and force conversion factors in Table 1.2 on p. 10, verify that 1 slug = 14.59 kg.

Solution

$$\boxed{1\,\text{slug} = 1\,\text{slug}\left(\frac{\text{lb}\cdot\text{s}^2/\text{ft}}{\text{slug}}\right)\left(\frac{4.448\,\text{N}}{\text{lb}}\right)\left(\frac{\text{ft}}{0.3048\,\text{m}}\right)\left(\frac{\text{kg}\cdot\text{m/s}^2}{\text{N}}\right) = 14.59\,\text{kg}.} \tag{1}$$

Problem 1.3

Convert the numbers given in U.S. Customary units to the corresponding SI units indicated.

(a) *Length*: Convert $l = 123$ in. to m.

(b) *Mass*: Convert $m = 2.87$ slug to kg.

(c) *Force* (weight): Convert $F = 18.9$ lb to N.

(d) *Moment* (torque): Convert $M = 433$ ft·lb to N·m.

Solution

Part (a)

$$\boxed{l = 123 \text{ in.} \left(\frac{0.0254 \text{ m}}{\text{in.}}\right) = 3.124 \text{ m.}} \tag{1}$$

Part (b)

$$\boxed{m = 2.87 \text{ slug} \left(\frac{14.59 \text{ kg}}{\text{slug}}\right) = 41.87 \text{ kg.}} \tag{2}$$

Part (c)

$$\boxed{F = 18.9 \text{ lb} \left(\frac{4.448 \text{ N}}{\text{lb}}\right) = 84.07 \text{ N.}} \tag{3}$$

Part (d)

$$\boxed{M = 433 \text{ ft} \cdot \text{lb} \left(\frac{0.3048 \text{ m}}{\text{ft}}\right) \left(\frac{4.448 \text{ N}}{\text{lb}}\right) = 587.0 \text{ N} \cdot \text{m.}} \tag{4}$$

Problem 1.4

Convert the numbers given in U.S. Customary units to the corresponding SI units indicated.

(a) *Length*: Convert $l = 45.6$ ft to m.

(b) *Mass*: Convert $m = 6.36 \times 10^4$ slug to kg.

(c) *Force* (weight): Convert $F = 22.1$ kip to kN.

(d) *Moment* (torque): Convert $M = 7660$ ft·lb to kN·m.

Solution

Part (a)

$$\boxed{l = 45.6\ \text{ft}\left(\frac{0.3048\ \text{m}}{\text{ft}}\right) = 13.90\ \text{m}.} \tag{1}$$

Part (b)

$$\boxed{m = 6.36 \times 10^4\ \text{slug}\left(\frac{14.59\ \text{kg}}{\text{slug}}\right) = 9.279 \times 10^5\ \text{kg}.} \tag{2}$$

Part (c)

$$\boxed{F = 22.1\ \text{kip}\left(\frac{4.448\ \text{kN}}{\text{kip}}\right) = 98.30\ \text{kN}.} \tag{3}$$

Part (d)

$$\boxed{M = 7660\ \text{ft}\cdot\text{lb}\left(\frac{0.3048\ \text{m}}{\text{ft}}\right)\left(\frac{4.448\ \text{N}}{\text{lb}}\right)\left(\frac{\text{kN}}{10^3\ \text{N}}\right) = 10.39\ \text{kN}\cdot\text{m}.} \tag{4}$$

Problem 1.5

Convert the numbers given in U.S. Customary units to the corresponding SI units indicated.

(a) *Length*: Convert $l = 2.35$ in. to m.

(b) *Mass*: Convert $m = 0.156$ slug to kg.

(c) *Force* (weight): Convert $F = 100$ lb to N.

(d) *Moment* (torque): Convert $M = 32.9$ ft·lb to N·m.

Solution

Part (a)

$$\boxed{l = 2.35\,\text{in.}\left(\frac{25.4\,\text{mm}}{\text{in.}}\right)\left(\frac{\text{m}}{10^3\,\text{mm}}\right) = 0.0597\,\text{m}.} \tag{1}$$

Part (b)

$$\boxed{m = 0.156\,\text{slug}\left(\frac{14.59\,\text{kg}}{\text{slug}}\right) = 2.28\,\text{kg}.} \tag{2}$$

Part (c)

$$\boxed{F = 100\,\text{lb}\left(\frac{4.448\,\text{N}}{\text{lb}}\right) = 445\,\text{N}.} \tag{3}$$

Part (d)

$$\boxed{M = 32.9\,\text{ft}\cdot\text{lb}\left(\frac{0.3048\,\text{m}}{\text{ft}}\right)\left(\frac{4.448\,\text{N}}{\text{lb}}\right) = 44.6\,\text{N}\cdot\text{m}.} \tag{4}$$

Problem 1.6

Convert the numbers given in U.S. Customary units to the corresponding SI units indicated.

(a) *Length*: Convert $l = 0.001$ in. to μm.

(b) *Mass*: Convert $m = 0.305$ lb·s^2/in. to kg.

(c) *Force* (weight): Convert $F = 2.56$ kip to kN. (Recall: 1 kip = 1000 lb.)

(d) *Mass moment of inertia*: Convert $I_{\text{mass}} = 23.0$ in.·lb·s^2 to N·m·s^2.

Solution

Part (a)

$$\boxed{l = 0.001\,\text{in.}\left(\frac{25.4\,\text{mm}}{\text{in.}}\right)\left(\frac{\text{m}}{10^3\,\text{mm}}\right)\left(\frac{10^6\ \mu\text{m}}{\text{m}}\right) = 25.4\,\mu\text{m}.} \tag{1}$$

Part (b)

$$\boxed{m = 0.305\,\text{lb}\cdot\text{s}^2/\text{in.}\left(\frac{4.448\,\text{N}}{\text{lb}}\right)\left(\frac{\text{kg}\cdot\text{m/s}^2}{\text{N}}\right)\left(\frac{\text{in.}}{0.0254\,\text{m}}\right) = 53.4\,\text{kg}.} \tag{2}$$

Part (c)

$$\boxed{F = 2.56\,\text{kips}\left(\frac{4.448\,\text{kN}}{\text{kip}}\right) = 11.4\,\text{kN}.} \tag{3}$$

Part (d)

$$\boxed{I_{\text{mass}} = 23.0\,\text{in.}\cdot\text{lb}\cdot\text{s}^2\left(\frac{0.0254\,\text{m}}{\text{in.}}\right)\left(\frac{4.448\,\text{N}}{\text{lb}}\right) = 2.60\,\text{N}\cdot\text{m}\cdot\text{s}^2.} \tag{4}$$

Problem 1.7

Convert the numbers given in U.S. Customary units to the corresponding SI units indicated.

(a) *Pressure*: Convert $p = 25\ \text{lb/ft}^2$ to N/m^2.

(b) *Elastic modulus*: Convert $E = 30 \times 10^6\ \text{lb/in.}^2$ to GN/m^2.

(c) *Area moment of inertia*: Convert $I_{\text{area}} = 63.2\ \text{in.}^4$ to mm^4.

(d) *Mass moment of inertia*: Convert $I_{\text{mass}} = 15.4\ \text{in.}\cdot\text{lb}\cdot\text{s}^2$ to $\text{kg}\cdot\text{m}^2$.

Solution

Part (a)

$$\boxed{p = 25\,\text{lb/ft}^2\left(\frac{4.448\,\text{N}}{\text{lb}}\right)\left(\frac{\text{ft}}{0.3048\,\text{m}}\right)^2 = 1.20\times10^3\,\text{N/m}^2.} \tag{1}$$

Part (b)

$$\boxed{E = 30\times10^6\,\text{lb/in}^2\left(\frac{\text{in.}}{0.0254\,\text{m}}\right)^2\left(\frac{4.448\,\text{N}}{\text{lb}}\right)\left(\frac{\text{GN}}{10^9\,\text{N}}\right) = 207\,\text{GN/m}^2.} \tag{2}$$

Part (c)

$$\boxed{I_{\text{area}} = 63.2\,\text{in.}^4\left(\frac{25.4\,\text{mm}}{\text{in.}}\right)^4 = 26.3\times10^6\,\text{mm}^4.} \tag{3}$$

Part (d)

$$\boxed{I_{\text{mass}} = 15.4\,\text{in.}\cdot\text{lb}\cdot\text{s}^2\left(\frac{0.0254\,\text{m}}{\text{in.}}\right)\left(\frac{4.448\,\text{N}}{\text{lb}}\right)\left(\frac{\text{kg}\cdot\text{m/s}^2}{\text{N}}\right) = 1.74\,\text{kg}\cdot\text{m}^2.} \tag{4}$$

Problem 1.8

Convert the numbers given in SI units to the corresponding U.S. Customary units indicated.

(a) *Length*: Convert $l = 18.4$ m to ft.

(b) *Mass*: Convert $m = 4.32$ kg to slug.

(c) *Force* (weight): Convert $F = 2120$ N to lb.

(d) *Moment* (torque): Convert $M = 865$ N·m to in.·lb.

Solution

Part (a)

$$\boxed{l = 18.4\ \text{m}\left(\frac{\text{ft}}{0.3048\ \text{m}}\right) = 60.37\ \text{ft.}} \tag{1}$$

Part (b)

$$\boxed{m = 4.32\ \text{kg}\left(\frac{\text{slug}}{14.59\ \text{kg}}\right) = 0.2961\ \text{slug.}} \tag{2}$$

Part (c)

$$\boxed{F = 2120\ \text{N}\left(\frac{\text{lb}}{4.448\ \text{N}}\right) = 476.6\ \text{lb.}} \tag{3}$$

Part (d)

$$\boxed{M = 865\ \text{N}\cdot\text{m}\left(\frac{\text{lb}}{4.448\ \text{N}}\right)\left(\frac{\text{in.}}{0.0254\ \text{m}}\right) = 7656\ \text{in.}\cdot\text{lb.}} \tag{4}$$

Problem 1.9

Convert the numbers given in SI units to the corresponding U.S. Customary units indicated.

(a) *Length*: Convert $l = 312$ mm to in.

(b) *Mass*: Convert $m = 2100$ kg to slug.

(c) *Force* (weight): Convert $F = 25$ kN to lb.

(d) *Moment* (torque): Convert $M = 1.86$ kN·m to ft·lb.

Solution

Part (a)

$$\boxed{l = 312\text{ mm}\left(\frac{\text{in.}}{25.4\text{ mm}}\right) = 12.28\text{ in.}} \tag{1}$$

Part (b)

$$\boxed{m = 2100\text{ kg}\left(\frac{\text{slug}}{14.59\text{ kg}}\right) = 143.9\text{ slug.}} \tag{2}$$

Part (c)

$$\boxed{F = 25\text{ kN}\left(\frac{\text{kip}}{4.448\text{ kN}}\right)\left(\frac{1000\text{ lb}}{\text{kip}}\right) = 5621\text{ lb.}} \tag{3}$$

Part (d)

$$\boxed{M = 1.86\text{ kN}\cdot\text{m}\left(\frac{\text{kip}}{4.448\text{ kN}}\right)\left(\frac{1000\text{ lb}}{\text{kip}}\right)\left(\frac{\text{ft}}{0.3048\text{ m}}\right) = 1372\text{ ft}\cdot\text{lb.}} \tag{4}$$

Problem 1.10

Convert the numbers given in SI units to the corresponding U.S. Customary units indicated.

(a) *Length*: Convert $l = 1.53$ m to in.

(b) *Mass*: Convert $m = 65$ kg to slug.

(c) *Force* (weight): Convert $F = 89.2$ N to lb.

(d) *Moment* (torque): Convert $M = 32.9$ N·m to in.·lb.

Solution

Part (a)

$$\boxed{l = 1.53\,\text{m}\left(\frac{\text{ft}}{0.3048\,\text{m}}\right)\left(\frac{12\,\text{in.}}{\text{ft}}\right) = 60.2\,\text{in.}} \tag{1}$$

Part (b)

$$\boxed{m = 65\,\text{kg}\left(\frac{\text{slug}}{14.59\,\text{kg}}\right) = 4.46\,\text{slug}.} \tag{2}$$

Part (c)

$$\boxed{F = 89.2\,\text{N}\left(\frac{\text{lb}}{4.448\,\text{N}}\right) = 20.1\,\text{lb}.} \tag{3}$$

Part (d)

$$\boxed{M = 32.9\,\text{N}\cdot\text{m}\left(\frac{\text{lb}}{4.448\,\text{N}}\right)\left(\frac{\text{in.}}{0.0254\,\text{m}}\right) = 291\,\text{in.}\cdot\text{lb}.} \tag{4}$$

Problem 1.11

Convert the numbers given in SI units to the corresponding U.S. Customary units indicated.

(a) *Length*: Convert $l = 122$ nm to in.

(b) *Mass*: Convert $m = 3.21$ kg to lb·s^2/in.

(c) *Force* (weight): Convert $F = 13.2$ kN to lb.

(d) *Mass moment of inertia*: Convert $I_{\text{mass}} = 93.2$ kg·m^2 to slug·in.^{2}.

Solution

Part (a)

$$\boxed{l = 122\,\text{nm}\left(\frac{10^{-9}\,\text{m}}{\text{nm}}\right)\left(\frac{\text{in.}}{0.0254\,\text{m}}\right) = 4.80\times 10^{-6}\,\text{in.}} \tag{1}$$

Part (b)

$$\boxed{m = 3.21\,\text{kg}\left(\frac{\text{slug}}{14.59\,\text{kg}}\right)\left(\frac{\text{lb}\cdot\text{s}^2/\text{ft}}{\text{slug}}\right)\left(\frac{\text{ft}}{12\,\text{in.}}\right) = 0.0183\,\text{lb}\cdot\text{s}^2/\text{in.}} \tag{2}$$

Part (c)

$$\boxed{F = 13.2\,\text{kN}\left(\frac{10^3\,\text{N}}{\text{kN}}\right)\left(\frac{\text{lb}}{4.448\,\text{N}}\right) = 2970\,\text{lb.}} \tag{3}$$

Part (d)

$$\boxed{I_{\mathbf{mass}} = 93.2\,\text{kg}\cdot\text{m}^2\left(\frac{\text{slug}}{14.59\,\text{kg}}\right)\left(\frac{\text{in.}}{0.0254\,\text{m}}\right)^2 = 9.90\times 10^3\,\text{slug}\cdot\text{in.}^2.} \tag{4}$$

Problem 1.12

Convert the numbers given in SI units to the corresponding U.S. Customary units indicated.

(a) *Pressure*: Convert $p = 25\ \mathrm{kN/m^2}$ to $\mathrm{lb/in.^2}$.

(b) *Elastic modulus*: Convert $E = 200\ \mathrm{GN/m^2}$ to $\mathrm{lb/in.^2}$.

(c) *Area moment of inertia*: Convert $I_{\mathrm{area}} = 23.5 \times 10^5\ \mathrm{mm^4}$ to $\mathrm{in.^4}$.

(d) *Mass moment of inertia*: Convert $I_{\mathrm{mass}} = 12.3\ \mathrm{kg{\cdot}m^2}$ to $\mathrm{in.{\cdot}lb{\cdot}s^2}$.

Solution

Part (a)

$$\boxed{p = 25\,\mathrm{kN/m^2}\left(\frac{10^3\,\mathrm{N}}{\mathrm{kN}}\right)\left(\frac{\mathrm{lb}}{4.448\,\mathrm{N}}\right)\left(\frac{0.0254\,\mathrm{m}}{\mathrm{in.}}\right)^2 = 3.63\,\mathrm{lb/in.^2}.} \tag{1}$$

Part (b)

$$\boxed{E = 200\,\mathrm{GN/m^2}\left(\frac{10^9\,\mathrm{N}}{\mathrm{GN}}\right)\left(\frac{\mathrm{lb}}{4.448\,\mathrm{N}}\right)\left(\frac{0.0254\,\mathrm{m}}{\mathrm{in.}}\right)^2 = 29.0\times10^6\,\mathrm{lb/in.^2}.} \tag{2}$$

Part (c)

$$\boxed{I_{\mathbf{area}} = 23.5\times10^5\,\mathrm{mm^4}\left(\frac{\mathrm{in.}}{25.4\,\mathrm{mm}}\right)^4 = 5.65\,\mathrm{in.^4}.} \tag{3}$$

Part (d)

$$\boxed{I_{\mathbf{mass}} = 12.3\,\mathrm{kg\cdot m^2}\left(\frac{\mathrm{lb{\cdot}s^2/ft}}{14.59\,\mathrm{kg}}\right)\left(\frac{\mathrm{ft}}{12\,\mathrm{in.}}\right)\left(\frac{\mathrm{in.}}{0.0254\,\mathrm{m}}\right)^2 = 109\,\mathrm{lb\cdot s^2\cdot in.}} \tag{4}$$

Problem 1.13

(a) Convert the kinetic energy $T = 0.379\ \mathrm{kg{\cdot}m^2/s^2}$ to $\mathrm{slug{\cdot}in.^2/s^2}$.

(b) Convert the kinetic energy $T = 10.1\ \mathrm{slug{\cdot}in.^2/s^2}$ to $\mathrm{kg{\cdot}m^2/s^2}$.

Solution

Part (a)

$$\boxed{T = 0.379\,\mathrm{kg\cdot m^2/s^2}\left(\frac{\mathrm{slug}}{14.59\,\mathrm{kg}}\right)\left(\frac{\mathrm{in.}}{0.0254\,\mathrm{m}}\right)^2 = 40.3\,\mathrm{slug\cdot in.^2/s^2}.} \tag{1}$$

Part (b)

$$\boxed{T = 10.1\,\mathrm{slug\cdot in.^2/s^2}\left(\frac{14.59\,\mathrm{kg}}{\mathrm{slug}}\right)\left(\frac{0.0254\,\mathrm{m}}{\mathrm{in.}}\right)^2 = 0.0951\,\mathrm{kg\cdot m^2/s^2}.} \tag{2}$$

Problem 1.14

If the weight of a certain object on the surface of the Earth is 0.254 lb, determine its mass in kilograms.

Solution

We first determine the mass of the object in slugs using

$$w = mg \quad \Rightarrow \quad m = w/g = (0.254\,\mathrm{lb})/(32.2\,\mathrm{ft/s^2}) = 7.888\times10^{-3}\,\mathrm{lb\cdot s^2/ft} = 7.888\times10^{-3}\,\mathrm{slug}. \tag{1}$$

Next, we convert the mass into SI units, such that

$$\boxed{m = 7.888\times10^{-3}\,\mathrm{slug}\left(\frac{14.59\,\mathrm{kg}}{\mathrm{slug}}\right) = 0.115\,\mathrm{kg}.} \tag{2}$$

Problem 1.15

If the mass of a certain object is 69.1 kg, determine its weight on the surface of the Earth in pounds.

Solution

We first determine the weight of the object in newtons using

$$w = mg = (69.1\,\mathrm{kg})\left(9.81\,\mathrm{m/s^2}\right) = 677.9\,\mathrm{kg\cdot m/s^2} = 677.9\,\mathrm{N}. \tag{1}$$

Next, we convert the weight into pounds using

$$\boxed{w = 677.9\,\mathrm{N}\left(\frac{\mathrm{lb}}{4.448\,\mathrm{N}}\right) = 152\,\mathrm{lb}.} \tag{2}$$

Problem 1.16

Use Eq. (1.11) on p. 16 to compute a theoretical value of acceleration due to gravity g, and compare this value with the actual acceleration due to gravity at the Earth's poles, which is about 0.3% higher than the value reported in Eq. (1.12). Comment on the agreement.

Solution

The values $G = 66.74\times10^{-12}\,\mathrm{m}^3/(\mathrm{kg}\cdot\mathrm{s}^2)$, $m_{\mathrm{Earth}} = 5.9736\times10^{24}\,\mathrm{kg}$, and $r_{\mathrm{Earth}} = 6.371\times10^6\,\mathrm{m}$ are given in the text in the discussion of Eqs. (1.10) and (1.11). Using these values, the theoretical value of acceleration due to gravity, Eq. (1.11), is

$$\boxed{g_{\mathrm{theory}} = G\frac{m_{\mathrm{Earth}}}{r^2_{\mathrm{Earth}}} = 66.74\times10^{-12}\,\mathrm{m}^3/(\mathrm{kg}\cdot\mathrm{s}^2)\frac{5.9736\times10^{24}\,\mathrm{kg}}{\left[6.371\times10^6\,\mathrm{m}\right]^2} = 9.822\,\mathrm{m/s}^2.} \tag{1}$$

The commonly accepted value for acceleration due to gravity, namely $g = 9.81\,\mathrm{m/s}^2$, is most accurate at $\pm45°$ latitude; it accounts for the Earth not being perfectly spherical and the Earth's rotation. As stated in the problem description, the acceleration due to gravity at the poles is about 0.3% higher than at $\pm45°$ latitude and therefore, at the poles, the acceleration due to gravity is approximately

$$g_{\mathrm{poles}} = 9.81\,\mathrm{m/s}^2(1 + 0.003) = 9.839\,\mathrm{m/s}^2. \tag{2}$$

Note that the poles are useful locations for comparing the theoretical value of acceleration due to gravity, Eq. (1), with the actual value, given approximately by Eq. (2), because the effects of the Earth's rotation (i.e., centripetal acceleration) are absent at the poles. While the agreement between Eqs. (1) and (2) is quite good, the differences between these two values is due to the Earth not being perfectly spherical.

Problem 1.17

Two identical asteroids travel side by side while touching one another. If the asteroids are composed of homogeneous pure iron and are spherical, what diameter in feet must they have for their mutual gravitational attraction to be 1 lb?

Solution

Begin by considering Eq. (1.10) on p. 16,

$$F = G\,\frac{m_1 m_2}{r^2}, \tag{1}$$

where $F = 1\,\text{lb}$ according to the problem statement. The mass of each asteroid is given by $m = \rho(4\pi r^3/3)$, where ρ is the density and r is the radius of the spherical asteroids. Thus, Eq. (1) becomes

$$F = G\,\frac{\left[\rho(4\pi r^3/3)\right]^2}{(2r)^2} = \frac{4\pi^2 G\rho^2 r^4}{9}. \tag{2}$$

Solving for r, we obtain

$$r = \left(\frac{9F}{4\pi^2 G\rho^2}\right)^{1/4} = \left[\frac{9(4.448\,\text{N})}{4\pi^2(66.74\times 10^{-12}\,\text{m}^3/(\text{kg}\cdot\text{s}^2))(7860\,\text{kg}/\text{m}^3)^2}\right]^{1/4} = 3.960\,\text{m}, \tag{3}$$

where $F = 1\,\text{lb} = 4.448\,\text{N}$. Since the diameter d of the asteroid is twice its radius r (i.e., $d = 2r$), it follows that

$$\boxed{d = 2(3.960\,\text{m})\left(\frac{\text{ft}}{0.3048\,\text{m}}\right) = 26.0\,\text{ft}.} \tag{4}$$

Problem 1.18

The mass of the Moon is approximately 7.35×10^{22} kg, and its mean distance from the Earth is about 3.80×10^8 km. Determine the force of mutual gravitational attraction in newtons between the Earth and Moon. In view of your answer, discuss why the Moon does not crash into the Earth.

Solution

Use Eq. (1.10) to solve for the force F:

$$F = G\,\frac{m_1 m_2}{r^2} = 66.74\times 10^{-12}\,\mathrm{m^3/(kg\cdot s^2)}\,\frac{(5.9736\times 10^{24}\,\mathrm{kg})(7.35\times 10^{22}\,\mathrm{kg})}{\left[3.80\times 10^{8}\,\mathrm{km}(10^{3}\,\mathrm{m/km})\right]^2} = 2.02\times 10^{14}\,\mathrm{kg\cdot m/s^2}, \tag{1}$$

so that the force may be written as

$$\boxed{F = 2.02\times 10^{14}\,\mathrm{N}.} \tag{2}$$

Although this force is large, the force due to centripetal acceleration equilibrates this, so that the Moon maintains its orbit.

Problem 1.19

> Consider a spacecraft that is positioned directly between the Earth and Moon. The mass of the Moon is approximately 7.35×10^{22} kg, and at the instant under consideration, the Moon is 3.80×10^{8} km from Earth. Determine the distances the spacecraft must be from the Earth and Moon for the gravitational force of the Earth on the spacecraft to be the same as the gravitational force of the Moon on the spacecraft.

Solution

The mass of the Moon is given in the problem statement as

$$m_{\mathrm{M}} = 7.35 \times 10^{22}\ \mathrm{kg}, \tag{1}$$

and the mass of the Earth is given in the text as

$$m_{\mathrm{E}} = 5.9736 \times 10^{24}\ \mathrm{kg}. \tag{2}$$

The gravitational force the Earth applies to the spacecraft is

$$F_{\mathrm{ES}} = G\frac{m_{\mathrm{E}}\, m_{\mathrm{S}}}{r_{\mathrm{ES}}^2} \tag{3}$$

where m_{S} is the mass of the spacecraft and r_{ES} is the distance between the Earth and the spacecraft. The gravitational force the Moon applies to the spacecraft is

$$F_{\mathrm{MS}} = G\frac{m_{\mathrm{M}}\, m_{\mathrm{S}}}{r_{\mathrm{MS}}^2} \tag{4}$$

where r_{MS} is the distance between the Moon and the spacecraft. According to the problem statement, $F_{\mathrm{ES}} = F_{\mathrm{MS}}$. Hence, we equate Eqs. (3) and (4) to obtain

$$G\,\frac{m_{\mathrm{E}}\, m_{\mathrm{S}}}{r_{\mathrm{ES}}^2} = G\,\frac{m_{\mathrm{M}}\, m_{\mathrm{S}}}{r_{\mathrm{MS}}^2}. \tag{5}$$

Canceling G and m_{S}, and rearranging slightly provides

$$m_{\mathrm{E}}\, r_{\mathrm{MS}}^2 = m_{\mathrm{M}}\, r_{\mathrm{ES}}^2. \tag{6}$$

The distance between the Earth and Moon is 3.80×10^{8} km, thus

$$r_{\mathrm{ES}} + r_{\mathrm{MS}} = 3.80 \times 10^{8}\ \mathrm{km}. \tag{7}$$

Combining Eqs. (6) and (7) provides

$$m_{\mathrm{E}}\, r_{\mathrm{MS}}^2 = m_{\mathrm{M}}\,(3.80 \times 10^{8}\ \mathrm{km} - r_{\mathrm{MS}})^2 \tag{8}$$

$$m_{\mathrm{E}}\, r_{\mathrm{MS}}^2 = m_{\mathrm{M}}\left[(3.80 \times 10^{8}\ \mathrm{km})^2 - 2(3.80 \times 10^{8}\ \mathrm{km})r_{\mathrm{MS}} + r_{\mathrm{MS}}^2\right] \tag{9}$$

$$\underbrace{\left(\frac{m_{\mathrm{E}}}{m_{\mathrm{M}}} - 1\right)}_{80.2735} r_{\mathrm{MS}}^2 + 2(3.80 \times 10^{8}\ \mathrm{km})\, r_{\mathrm{MS}} - (3.80 \times 10^{8}\ \mathrm{km})^2 = 0 \tag{10}$$

Using the quadratic formula, the solutions to Eq. (10) are

$$r_{\mathrm{MS}} = -4.741 \times 10^{7}\ \mathrm{km} \quad \text{and} \quad 3.794 \times 10^{7}\ \mathrm{km}, \tag{11}$$

and using Eq. (7), the corresponding solutions for r_{ES} are

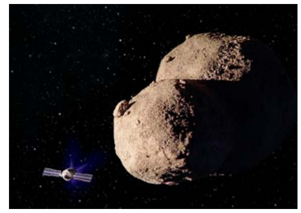
$$r_{\text{ES}} = 4.274 \times 10^8 \text{ km} \quad \text{and} \quad 3.421 \times 10^8 \text{ km}. \tag{12}$$

The first of the above solutions ($r_{\text{MS}} = -4.741 \times 10^7$ km and $r_{\text{ES}} = 4.274 \times 10^8$ km) is physically possible, but it corresponds to a spacecraft position where the moon is between the Earth and spacecraft; hence this is not the solution for the problem that is asked. Hence, the second solution is correct, and

$$\boxed{r_{\text{MS}} = 3.794 \times 10^7 \text{ km} \quad \text{and} \quad r_{\text{ES}} = 3.421 \times 10^8 \text{ km}.} \tag{13}$$

Problem 1.20

The *gravity tractor*, as shown in the artist's rendition, is a proposed spacecraft that will fly close to an asteroid whose trajectory threatens to impact the Earth. Due to the gravitational attraction between the two objects and a prolonged period of time over which it acts (several years), the asteroid's trajectory is changed slightly, thus hopefully diverting it from impacting the Earth. If the gravity tractor's weight on earth is 20,000 lb and it flies with its center of gravity 160 ft from the surface of the asteroid, and the asteroid is homogeneous pure iron with 1290 ft diameter spherical shape, determine the force of mutual attraction. Idealize the gravity tractor to be a particle.



Solution

The mass of the gravity tractor is

$$m_{\mathrm{GT}} = \frac{W}{g} = \frac{20{,}000\,\mathrm{lb}}{32.2\ \mathrm{ft/s^2}} = 621.1\ \mathrm{slug}. \tag{1}$$

The mass of the asteroid is

$$\begin{aligned} m_{\mathrm{A}} = \rho V = \frac{\gamma}{g}V &= \frac{491\ \mathrm{lb/ft^3}}{32.2\ \mathrm{ft/s^2}}\ \frac{4}{3}\pi\left(\frac{1290\ \mathrm{ft}}{2}\right)^3 \\ &= 1.714\times 10^{10}\ \mathrm{slug}. \end{aligned} \tag{2}$$

The distance between the gravity tractor (idealized as a particle) and the center of gravity of the asteroid is

$$r = \frac{1290\ \mathrm{ft}}{2} + 160\ \mathrm{ft} = 805\ \mathrm{ft}. \tag{3}$$

The gravitational constant G, expressed in U.S. Customary units (Example 1.2 carries out the conversion of G from SI units to U.S. Customary units) is

$$G = 34.39\times 10^{-9}\,\frac{\mathrm{ft^4}}{\mathrm{lb\cdot s^4}}\ \frac{\mathrm{lb\cdot s^2/ft}}{\mathrm{slug}} \tag{4}$$

$$= 34.39\times 10^{-9}\,\frac{\mathrm{ft^3}}{\mathrm{slug\cdot s^2}}. \tag{5}$$

Using Newton's law of gravitational attraction, the force of mutual attraction is

$$F = G\frac{m_{\mathrm{GT}}\,m_{\mathrm{A}}}{r^2} = 34.39\times 10^{-9}\ \frac{\mathrm{ft^3}}{\mathrm{slug\cdot s^2}}\,\frac{(621.1\ \mathrm{slug})(1.714\times 10^{10}\ \mathrm{slug})}{(805\ \mathrm{ft})^2} \tag{6}$$

$$= 0.5649\,\frac{\mathrm{ft\cdot slug}}{\mathrm{s^2}}\ \frac{\mathrm{lb\cdot s^2/ft}}{\mathrm{slug}} \tag{7}$$

$$= \boxed{0.5649\ \mathrm{lb.}} \tag{8}$$

Alternate solution To avoid the need for G in U.S. Customary units, we may carry out our calculations in SI units as follows (note: $\mathrm{N} = \mathrm{kg\cdot m/s^2}$):

$$m_{\mathrm{GT}} = \frac{20{,}000\,\mathrm{lb}}{32.2\ \mathrm{ft/s^2}}\ \frac{14.59\ \mathrm{kg}}{\mathrm{slug}} = 9062\ \mathrm{kg}, \tag{9}$$

$$m_{\mathrm{A}} = 1.714 \times 10^{10}\ \mathrm{slug}\frac{14.59\ \mathrm{kg}}{\mathrm{slug}} = 2.501 \times 10^{11}\ \mathrm{kg}, \tag{10}$$

$$r = 805\ \mathrm{ft}\ \frac{0.3048\ \mathrm{m}}{\mathrm{ft}} = 245.4\ \mathrm{m}, \tag{11}$$

$$F = G\frac{m_{\mathrm{GT}}\, m_{\mathrm{A}}}{r^2} = 66.74 \times 10^{-12}\ \frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}\ \frac{(9062\ \mathrm{kg})(2.501 \times 10^{11}\ \mathrm{kg})}{(245.4\ \mathrm{m})^2} \tag{12}$$

$$= 2.512\ \frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2} = 2.512\ \mathrm{N} \tag{13}$$

$$= 2.512\ \mathrm{N}\frac{\mathrm{lb}}{4.448\ \mathrm{N}} = \boxed{0.5649\ \mathrm{lb.}} \tag{14}$$

Problem 1.21

If a person standing at the first-floor entrance to the Sears Tower (recently renamed Willis Tower) in Chicago weighs exactly 150 lb, determine the weight while he or she is standing on top of the building, which is 1450 ft above the first-floor entrance. How high would the top of the building need to be for the person's weight to be 99% of its value at the first-floor entrance?

Solution

We will use Eq. (1.10) from p. 16,

$$F = G\,\frac{m_1 m_2}{r^2}, \tag{1}$$

where the person's weight is given by F in Eq. (1). Let the person's weight on the first floor be given by W_1, the weight on top of the building be given by W_{top}, and the height between the first floor and the top of the building be h. Using Eq. (1), we may write these two relations:

$$W_1 = G\,\frac{m_{\text{Earth}}\,m_{\text{person}}}{r_{\text{Earth}}^2}, \quad W_{\text{top}} = G\,\frac{m_{\text{Earth}} m_{\text{person}}}{(r_{\text{Earth}} + h)^2}. \tag{2}$$

Dividing W_{top} by W_1 and solving for W_{top} leads to

$$\frac{W_{\text{top}}}{W_1} = \frac{r_{\text{Earth}}^2}{(r_{\text{Earth}} + h)^2} \quad \Rightarrow \quad W_{\text{top}} = W_1\,\frac{r_{\text{Earth}}^2}{(r_{\text{Earth}} + h)^2}, \tag{3}$$

such that

$$\boxed{W_{\text{top}} = (150\,\text{lb})\,\frac{(6.371\times10^6\,\text{m})^2}{\left[6.371\times10^6\,\text{m} + 1450\,\text{ft}(0.3048\,\text{m/ft})\right]^2} = (150\,\text{lb})(0.99986) = 149.979\,\text{lb}.} \tag{4}$$

For the second part of the problem, we need to find the height h needed to make $W_{\text{top}} = 0.99\,W_1$, i.e.,

$$\frac{r_{\text{Earth}}^2}{(r_{\text{Earth}} + h)^2} = 0.99 \quad \Rightarrow \quad h = (0.005038)\,r_{\text{Earth}} = \frac{3.2095\times10^4\,\text{m}}{0.3048\,\text{m/ft}} = 1.053\times10^5\,\text{ft}. \tag{5}$$

Therefore, the top of the building must be h above the first floor, where h is given by

$$\boxed{h = 32.1\,\text{km} = 1.05\times10^5\,\text{ft} = 19.94\,\text{mi}.} \tag{6}$$

Problem 1.22

The specific weights of several materials are given in U.S. Customary units. Convert these to specific weights in SI units (kN/m^3), and also compute the densities of these materials in SI units (kg/m^3).

(a) Zinc die casting alloy, $\gamma = 0.242$ lb/in.^{3}.

(b) Oil shale (30 gal/ton rock), $\gamma = 133$ lb/ft^3.

(c) Styrofoam (medium density), $\gamma = 2.0$ lb/ft^3.

(d) Silica glass, $\gamma = 0.079$ lb/in.^{3}.

Solution

The equation to be used to convert the specific weight to density is

$$\gamma = \rho g \quad \Rightarrow \quad \rho = \gamma/g. \tag{1}$$

Part (a)

$$\gamma = 0.242\,\mathrm{lb/in^3}\left(\frac{4.448\,\mathrm{N}}{\mathrm{lb}}\right)\left(\frac{\mathrm{in.}}{0.0254\,\mathrm{m}}\right)^3 = 65.69\times 10^3\,\mathrm{N/m^3} = 65.7\,\mathrm{kN/m^3}, \tag{2}$$

$$\rho = \frac{65.69\times 10^3\,\mathrm{N/m^3}}{9.81\,\mathrm{m/s^2}} = \frac{65.69\times 10^3\,\mathrm{kg/(m^2\cdot s^2)}}{9.81\,\mathrm{m/s^2}} = 6.70\times 10^3\,\mathrm{kg/m^3}. \tag{3}$$

Part (b)

$$\gamma = 133\,\mathrm{lb/ft^3}\left(\frac{4.448\,\mathrm{N}}{\mathrm{lb}}\right)\left(\frac{\mathrm{ft}}{0.3048\,\mathrm{m}}\right)^3 = 20.89\times 10^3\,\mathrm{N/m^3} = 20.9\,\mathrm{kN/m^3}, \tag{4}$$

$$\rho = \frac{20.89\times 10^3\,\mathrm{N/m^3}}{9.81\,\mathrm{m/s^2}} = \frac{20.89\times 10^3\,\mathrm{kg/(m^2\cdot s^2)}}{9.81\,\mathrm{m/s^2}} = 2.13\times 10^3\,\mathrm{kg/m^3}. \tag{5}$$

Part (c)

$$\gamma = 2.0\,\mathrm{lb/ft^3}\left(\frac{4.448\,\mathrm{N}}{\mathrm{lb}}\right)\left(\frac{\mathrm{ft}}{0.3048\,\mathrm{m}}\right)^3 = 314.2\,\mathrm{N/m^3} = 0.314\,\mathrm{kN/m^3}, \tag{6}$$

$$\rho = \frac{314.2\,\mathrm{N/m^3}}{9.81\,\mathrm{m/s^2}} = \frac{314.2\,\mathrm{kg/(m^2\cdot s^2)}}{9.81\,\mathrm{m/s^2}} = 32.0\,\mathrm{kg/m^3}. \tag{7}$$

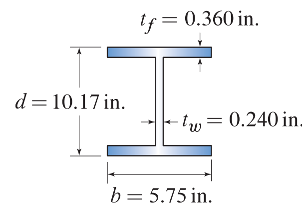
Part (d)

$$\gamma = 0.079\,\mathrm{lb/in^3}\left(\frac{4.448\,\mathrm{N}}{\mathrm{lb}}\right)\left(\frac{\mathrm{in.}}{0.0254\,\mathrm{m}}\right)^3 = 21.44\times 10^3\,\mathrm{N/m^3} = 21.4\,\mathrm{kN/m^3}, \tag{8}$$

$$\rho = \frac{21.44\times 10^3\,\mathrm{N/m^3}}{9.81\,\mathrm{m/s^2}} = \frac{21.44\times 10^3\,\mathrm{kg/(m^2\cdot s^2)}}{9.81\,\mathrm{m/s^2}} = 2.19\times 10^3\,\mathrm{kg/m^3}. \tag{9}$$

Problem 1.23

The cross-sectional dimensions for a steel W10 × 22 wide-flange I beam are shown (the top and bottom flanges have the same thickness). The beam is 18 ft long (into the plane of the figure), and the steel has 490 lb/ft^3 specific weight. Determine the cross-sectional dimensions in millimeters (show these on a sketch), the length in meters, and the mass of the beam in kilograms.



Solution

Flange thickness:

$$t_f = 0.360\text{ in.}\,\frac{25.4\text{ mm}}{\text{in.}} = \boxed{9.144\text{ mm.}} \tag{1}$$

Web thickness:

$$t_w = 0.240\text{ in.}\,\frac{25.4\text{ mm}}{\text{in.}} = \boxed{6.096\text{ mm.}} \tag{2}$$

Flange width:

$$b = 5.75\text{ in.}\,\frac{25.4\text{ mm}}{\text{in.}} = \boxed{146.1\text{ mm.}} \tag{3}$$

Cross section depth:

$$d = 10.17\text{ in.}\,\frac{25.4\text{ mm}}{\text{in.}} = \boxed{258.3\text{ mm.}} \tag{4}$$

Length:

$$l = 18\text{ ft}\,\frac{0.3048\text{ m}}{\text{ft}} = \boxed{5.486\text{ m.}} \tag{5}$$

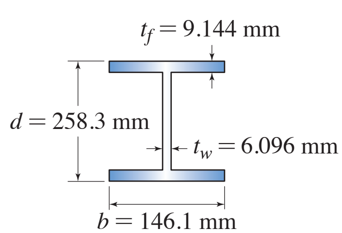
Several strategies may be used to determine the mass of the beam. We will determine its volume in in.^{3}, then its weight in lb, and then its mass in kg.

$$V = \Big\{2(0.360\text{ in.})(5.75\text{ in.}) + (0.240\text{ in.})\big[10.17\text{ in.} - 2(0.360\text{ in.})\big]\Big\}\,(18\text{ ft})\,\frac{12\text{ in.}}{\text{ft}} \tag{6}$$

$$= 1384\text{ in.}^3, \tag{7}$$

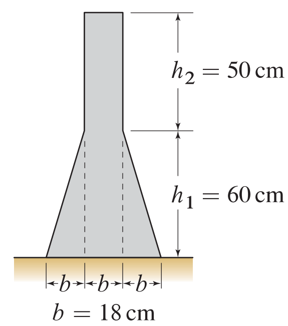
$$W = \gamma V = 490\,\frac{\text{lb}}{\text{ft}^3}\,(1384\text{ in.}^3)\left(\frac{\text{ft}}{12\text{ in.}}\right)^3 = 392.5\text{ lb}, \tag{8}$$

$$m = \frac{W}{g} = 392.5\text{ lb}\,\frac{4.448\text{ N}}{\text{lb}}\,\frac{\text{kg}\cdot\text{m/s}^2}{\text{N}}\,\frac{1}{9.81\text{ m/s}^2} = \boxed{178.0\text{ kg.}} \tag{9}$$



Problem 1.24

The cross-sectional dimensions of a concrete traffic barrier are shown. The barrier is 2 m long (into the plane of the figure), and the concrete has 2400 kg/m^3 density. Determine the cross-sectional dimensions in inches (show these on a sketch), the length in feet, and the weight of the barrier in pounds.



Solution

$$b = 18\text{ cm}\,\frac{\text{in.}}{2.54\text{ cm}} = \boxed{7.087\text{ in.}} \tag{1}$$

$$h_1 = 60\text{ cm}\,\frac{\text{in.}}{2.54\text{ cm}} = \boxed{23.62\text{ in.}} \tag{2}$$

$$h_2 = 50\text{ cm}\,\frac{\text{in.}}{2.54\text{ cm}} = \boxed{19.69\text{ in.}} \tag{3}$$

Length:

$$L = 2\text{ m}\,\frac{\text{ft}}{0.3048\text{ m}} = \boxed{6.562\text{ ft.}} \tag{4}$$

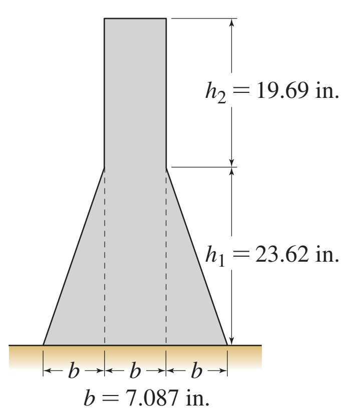
Several strategies may be used to determine the weight of the barrier. We will determine its volume in m^3, then its mass in kg, and then its weight in lb.

$$V = \left\{2\left[\frac{1}{2}(18\text{ cm})(60\text{ cm})\right] + (18\text{ cm})(110\text{ cm})\right\}(2\text{ m})\left(\frac{\text{m}}{10^2\text{ cm}}\right)^2 \tag{5}$$

$$= 0.6120\text{ m}^3, \tag{6}$$

$$m = \rho V = 2400\frac{\text{kg}}{\text{m}^3}(0.6120\text{ m}^3) = 1469\text{ kg}, \tag{7}$$

$$W = mg = 1469\text{ kg}\;9.81\frac{\text{m}}{\text{s}^2}\left(\frac{\text{N}}{\text{kg}\cdot\text{m/s}^2}\right)\frac{\text{lb}}{4.448\text{ N}} = \boxed{3239\text{ lb.}} \tag{8}$$



Problem 1.25

The densities of several materials are given in SI units. Convert these to densities in U.S. Customary units (slug/ft^3), and also compute the specific weights of these materials in U.S. Customary units (lb/ft^3).

(a) Lead (pure), $\rho = 11.34$ g/cm^3.

(b) Ceramic (alumina Al_2O_3), $\rho = 3.90$ Mg/m^3.

(c) Polyethylene (high density), $\rho = 960$ kg/m^3.

(d) Balsa wood, $\rho = 0.2$ Mg/m^3.

Solution

The equation to be used to determine the specific weight is

$$\gamma = \rho g. \tag{1}$$

Part (a)

$$\rho = 11.34\,\mathrm{g/cm^3}\left(\frac{\mathrm{kg}}{10^3\,\mathrm{g}}\right)\left(\frac{\mathrm{slug}}{14.59\,\mathrm{kg}}\right)\left(\frac{100\,\mathrm{cm}}{\mathrm{m}}\right)^3\left(\frac{0.3048\,\mathrm{m}}{\mathrm{ft}}\right)^3 = 22.0\,\mathrm{slug/ft^3}, \tag{2}$$

$$\gamma = 22.01\,\mathrm{slug/ft^3}(32.2\,\mathrm{ft/s^2})\left(\frac{\mathrm{lb{\cdot}s^2/ft}}{\mathrm{slug}}\right) = 709\,\mathrm{lb/ft^3}. \tag{3}$$

Part (b)

$$\rho = 3.90\,\mathrm{Mg/m^3}\left(\frac{10^3\,\mathrm{kg}}{\mathrm{Mg}}\right)\left(\frac{\mathrm{slug}}{14.59\,\mathrm{kg}}\right)\left(\frac{0.3048\,\mathrm{m}}{\mathrm{ft}}\right)^3 = 7.569\,\mathrm{slug/ft^3} = 7.57\,\mathrm{slug/ft^3}, \tag{4}$$

$$\gamma = 7.569\,\mathrm{slug/ft^3}(32.2\,\mathrm{ft/s^2})\left(\frac{\mathrm{lb{\cdot}s^2/ft}}{\mathrm{slug}}\right) = 244\,\mathrm{lb/ft^3}. \tag{5}$$

Part (c)

$$\rho = 960\,\mathrm{kg/m^3}\left(\frac{\mathrm{slug}}{14.59\,\mathrm{kg}}\right)\left(\frac{0.3048\,\mathrm{m}}{\mathrm{ft}}\right)^3 = 1.863\,\mathrm{slug/ft^3} = 1.86\,\mathrm{slug/ft^3}, \tag{6}$$

$$\gamma = 1.863\,\mathrm{slug/ft^3}(32.2\,\mathrm{ft/s^2})\left(\frac{\mathrm{lb{\cdot}s^2/ft}}{\mathrm{slug}}\right) = 60.0\,\mathrm{lb/ft^3}. \tag{7}$$

Part (d)

$$\rho = 0.2\,\mathrm{Mg/m^3}\left(\frac{10^3\,\mathrm{kg}}{\mathrm{Mg}}\right)\left(\frac{\mathrm{slug}}{14.59\,\mathrm{kg}}\right)\left(\frac{0.3048\,\mathrm{m}}{\mathrm{ft}}\right)^3 = 0.3882\,\mathrm{slug/ft^3} = 0.388\,\mathrm{slug/ft^3}, \tag{8}$$

$$\gamma = 0.3882\,\mathrm{slug/ft^3}(32.2\,\mathrm{ft/s^2})\left(\frac{\mathrm{lb{\cdot}s^2/ft}}{\mathrm{slug}}\right) = 12.5\,\mathrm{lb/ft^3}. \tag{9}$$

Problem 1.26

A Super Ball is a toy ball made of hard synthetic rubber called Zectron. This material has a high coefficient of restitution so that if it is dropped from a certain height onto a hard fixed surface, it rebounds to a substantial portion of its original height. If the Super Ball has 5 cm diameter and the density of Zectron is about 1.5 $\mathrm{Mg/m^3}$, determine the weight of the Super Ball on the surface of the Earth in U.S. Customary units.

Solution

Keeping in mind that the mass m is given by $m = \rho V$ and the volume V for a sphere is given by $V = 4\pi r^3/3$ (where r is the radius), it follows that the weight w is

$$w = mg = (\rho V)g = \frac{4\pi r^3 \rho g}{3} = \frac{\pi d^3 \rho g}{6}, \tag{1}$$

where d is the diameter. To obtain the weight in pounds,

$$\boxed{w = \frac{\pi(5\,\mathrm{cm})^3(1.5\,\mathrm{Mg/m^3})(9.81\,\mathrm{m/s^2})}{6}\left(\frac{\mathrm{m}}{10^2\,\mathrm{cm}}\right)^3\left(\frac{10^3\,\mathrm{kg}}{\mathrm{Mg}}\right)\left(\frac{\mathrm{lb}}{4.448\,\mathrm{kg\cdot m/s^2}}\right) = 0.217\,\mathrm{lb}.} \tag{2}$$

Problem 1.27

An ice hockey puck is a short circular cylinder, or disk, of vulcanized rubber with 3.00 in. diameter and 1.00 in. thickness, with weight between 5.5 and 6.0 oz (16 oz = 1 lb). Compute the range of densities for the rubber, in conventional SI units, that will provide for a puck that meets these specifications.

Solution

Based on the problem statement, the weight w of the hockey puck should be in the range

$$5.5\,\mathrm{oz}\left(\frac{\mathrm{lb}}{16\,\mathrm{oz}}\right) \le w \le 6.0\,\mathrm{oz}\left(\frac{\mathrm{lb}}{16\,\mathrm{oz}}\right) \quad \Rightarrow \quad 0.34375\,\mathrm{lb} \le w \le 0.375\,\mathrm{lb}. \tag{1}$$

The weight w, mass m, density ρ, and volume V of the hockey puck are related by

$$w = mg = (\rho V)g = (\rho\pi r^2 h)g \quad \Rightarrow \quad \rho = \frac{w}{\pi r^2 hg}, \tag{2}$$

where, in the above expressions, we have used the volume V of a cylinder as $V = \pi r^2 h$ (where r is the radius and h is the thickness). Multiplying all three terms of Eq. (1) by $1/\pi r^2 hg$ leads to (with $r = 1.5$ in and $g = 32.2\,\mathrm{ft/s^2}$)

$$\frac{0.34375\,\mathrm{lb}}{\pi(1.5\,\mathrm{in.}\frac{\mathrm{ft}}{12\,\mathrm{in.}})^2(1\,\mathrm{in.}\frac{\mathrm{ft}}{12\,\mathrm{in.}})(32.2\,\mathrm{ft/s^2})} \le \rho \le \frac{0.375\,\mathrm{lb}}{\pi(1.5\,\mathrm{in.}\frac{\mathrm{ft}}{12\,\mathrm{in.}})^2(1\,\mathrm{in.}\frac{\mathrm{ft}}{12\,\mathrm{in.}})(32.2\,\mathrm{ft/s^2})}, \tag{3}$$

which simplifies to obtain

$$2.610\,\mathrm{lb\cdot s^2/ft^4} \le \rho \le 2.847\,\mathrm{lb\cdot s^2/ft^4}, \tag{4}$$

Since 1 slug = $1\,\mathrm{lb\cdot s^2/ft}$, it follows that

$$2.610\,\mathrm{slug/ft^3} \le \rho \le 2.847\,\mathrm{slug/ft^3}. \tag{5}$$

Converting the above results to SI units provides

$$2.610\frac{\mathrm{slug}}{\mathrm{ft^3}}\left(\frac{14.59\,\mathrm{kg}}{\mathrm{slug}}\right)\left(\frac{\mathrm{ft}}{0.3048\,\mathrm{m}}\right)^3 \le \rho \le 2.847\frac{\mathrm{slug}}{\mathrm{ft^3}}\left(\frac{14.59\,\mathrm{kg}}{\mathrm{slug}}\right)\left(\frac{\mathrm{ft}}{0.3048\,\mathrm{m}}\right)^3, \tag{6}$$

which yields

$$\boxed{1340\,\mathrm{kg/m^3} \le \rho \le 1470\,\mathrm{kg/m^3}.} \tag{7}$$

Problem 1.28

Convert the angles given to the units indicated.

(a) Convert $\theta = 35.6^\circ$ to rad.

(b) Convert $\theta = (1.08 \times 10^{-3})^\circ$ to mrad.

(c) Convert $\theta = 4.65$ rad to degrees.

(d) Convert $\theta = 0.254$ mrad to degrees.

Solution

Part (a)

$$\boxed{\theta = 35.6^\circ \left(\frac{\pi \text{ rad}}{180^\circ}\right) = 0.621 \text{ rad.}} \tag{1}$$

Part (b)

$$\boxed{\theta = 1.08 \times 10^{-3\circ} \left(\frac{\pi \text{ rad}}{180^\circ}\right)\left(\frac{10^3 \text{ mrad}}{\text{rad}}\right) = 0.0188 \text{ mrad.}} \tag{2}$$

Part (c)

$$\boxed{\theta = 4.65 \text{ rad}\left(\frac{180^\circ}{\pi \text{ rad}}\right) = 266^\circ.} \tag{3}$$

Part (d)

$$\boxed{\theta = 0.254 \text{ mrad}\left(\frac{\text{rad}}{10^3 \text{ mrad}}\right)\left(\frac{180^\circ}{\pi \text{ rad}}\right) = 0.0146^\circ.} \tag{4}$$

Problem 1.29

Many of the examples of failure discussed in Section 1.7 have common causes, such as loads that were not anticipated, overestimation of the strength of materials, unanticipated use, etc. Using several paragraphs, identify those examples that have common causes of failure and discuss what these causes were.

Solution

Essay-type answer.