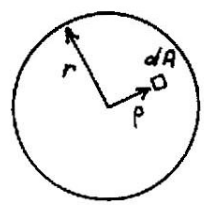
1.22	ALLOY STEEL FIG. 1.3 & 1.4	STRUCTURAL STEEL FIG. 1.5
YIELD POINT	N/A	265 MPa
YIELD STRENGTH	450 MPa	N/A
UPPER YIELD POINT	N/A	280 MPa
LOWER YIELD POINT	N/A	265 MPa
MODULUS OF RESILIENCE	N/A	0.1855 MPa
ULTIMATE TENSILE STRENGTH	715 MPa	470 MPa
STRAIN AT FRACTURE	0.23	0.26
PER CENT ELONGATION	23%	26%

1.23 ASSUME:
1. PLANE SECTIONS NORMAL TO THE AXIS OF THE ROD REMAIN PLANE UNDER APPLICATION OF THE LOAD.
2. SHEAR STRAINS VARY LINEARLY FROM THE LONGITUDINAL AXIS.
3. HOOKE'S LAW APPLIES



EQUILIBRIUM: $\Sigma M_x = 0$

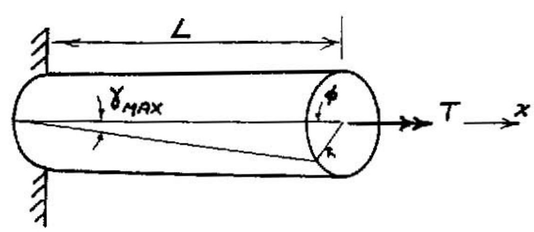
$$T = \int_A \rho \tau \, dA \quad (a)$$

COMPATIBILITY:

$$\gamma = \frac{\gamma_{max}}{r}\rho \quad (b)$$

HOOKE'S LAW:

$$\tau = G\gamma \quad (c)$$



SUB (c) INTO (b) & THEN (b) INTO (a)

$$T = \frac{\tau_{max}}{r}\int_A \rho^2 dA$$

$$T = \frac{\tau_{max}}{r} J \;;\; J = \int_A \rho^2 dA$$

$$\tau_{max} = \frac{Tr}{J} \quad \text{ON SURFACE}$$

$$\tau = \frac{T\rho}{J} \quad \text{AT ANY } \rho \quad (d)$$

FROM GEOMETRY OF DEFORMATION:

$$\gamma_{max} = \frac{r\phi}{L} \;;\; \gamma = \frac{\rho\phi}{L} \quad (e)$$

SUB (d) & (e) INTO (c)

$$\frac{T\rho}{J} = G\left(\frac{\rho\phi}{L}\right)$$

$$\phi = \frac{TL}{GJ}$$

1.24 ASSUME: 1. PLANE SECTIONS NORMAL TO THE AXIS OF THE BAR REMAIN PLANE UNDER APPLICATION OF THE LOAD.
2. HOOKE'S LAW APPLIES.

EQUILIBRIUM:
$$P = \sigma A \qquad (a)$$

HOOKE'S LAW:
$$\sigma = E\varepsilon \qquad (c)$$

CONTINUITY:
$$\Delta L = \int_0^L \varepsilon \, dx \qquad (b)$$

GEOMETRY
$$A(x) = b\left(d_o - \tfrac{x}{L}(d_o - d_1)\right) \qquad (d)$$

SUB (d) INTO (a)

$$\underline{\underline{\sigma = P\Big/\left[b\left(d_o - \tfrac{x}{L}(d_o - d_1)\right)\right]}} \qquad (e)$$

SUB. (c) & (e) INTO (b):

$$\Delta L = \int_0^L \frac{P}{Eb}\left[d_o - \tfrac{x}{L}(d_o - d_1)\right]^{-1} dx$$

$$\underline{\underline{\Delta L = \frac{PL}{Eb}\int_0^L \frac{dx}{d_o L - x(d_o - d_1)} = \frac{PL}{Eb}\left(\frac{1}{d_o - d_1}\right)\ln\frac{d_o}{d_1}}}$$

1.25

RODS: $A_R = 4\left(\frac{\pi}{4}15^2\right) = 706.9 \text{ mm}^2$

PIPE: $A_P = \frac{\pi}{4}(100^2 - 90^2) = 1492 \text{ mm}^2$

AFTER ASSEMBLY: $T_R = C_P = 4(65) = 260 \text{ kN}$

$$\sigma_R = \frac{4(65000)}{706.9} = 367.8 \text{ MPa}$$

$$\sigma_P = \frac{-4(65000)}{1492} = -174.3 \text{ MPa}$$

AFTER PRESSURE IS APPLIED:

EQUILIBRIUM:
$$p\left(\frac{\pi}{4}90^2\right) = \Delta T_R + \Delta C_P \qquad (a)$$

COMPATIBILITY:
$$\Delta L_R = \Delta L_P: \quad \frac{\Delta T_R L}{A_R E} = \frac{\Delta C_P L}{A_P E} \qquad (b)$$

LEAKAGE REQUIREMENT:
$$\Delta C_P = 260 \text{ kN} \qquad (c)$$

SUB (c) INTO (b):
$$\Delta T_R = 123.2 \text{ kN} \qquad (d)$$

SUB (c) & (d) INTO (a)

$$\underline{\underline{p = 60.23 \text{ MPa}}}$$

$$\underline{\underline{\Delta\sigma_R = \frac{123200}{706.9} = 174.3 \text{ MPa}}}$$

$$\underline{\underline{\sigma_{R(FINAL)} = 542.1 \text{ MPa}}}$$

1.26

(a) Figure a shows the bars subjected to force P. Figure b shows the free-body diagrams of the steel bar, the aluminum bar, and point A. By the free-body diagram of point A,

$$P = P_s + P_a \qquad \text{(a)}$$

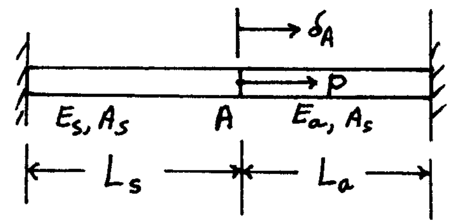


Figure a

By Eq. (1.2) and Figs. a and b,

$$\delta_a = \frac{P_s L_s}{E_s A_s} = \frac{P_a L_a}{E_a A_a} \qquad \text{(b)}$$

By Eqs. (a) and (b),

$$P_s = \frac{P_a E_s A_s L_a}{E_a A_a L_s} = \frac{(P - P_s)(E_s A_s L_a)}{E_a A_a L_s}$$

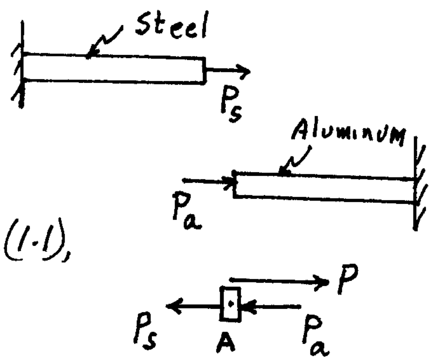


Figure b

Solving this equation for P_s, we find

$$P_s = \frac{P E_s A_s L_a}{E_s A_s L_a + E_a A_a L_s}.$$ Hence, by Eq. (1.1), the stress in the steel bar is

$$\sigma_s = \frac{P_s}{A_s} = \frac{P}{A_s}\left(\frac{E_s A_s L_a}{E_s A_s L_a + E_a A_a L_s}\right)$$

Similarly, by Eqs. (a) and (b),

$$P_a = \frac{P_s E_a A_a L_s}{E_s A_s L_a} = \frac{(P - P_a)(E_a A_a L_s)}{E_s A_s L_a}$$

or

$$P_a = \frac{P E_a A_a L_s}{E_s A_s L_a + E_a A_a L_s}$$

Hence, the stress in the aluminum bar is

$$\sigma_a = \frac{P}{A_a}\left(\frac{E_a A_a L_s}{E_s A_s L_a + E_a A_a L_s}\right)$$

(cont.)

1.26 cont.

(b) When the left wall is displaced to the right by an amount δ, the point A is displaced to the right by an amount δ_A (Fig. c).

By Eq. (1.2) and Fig. c,

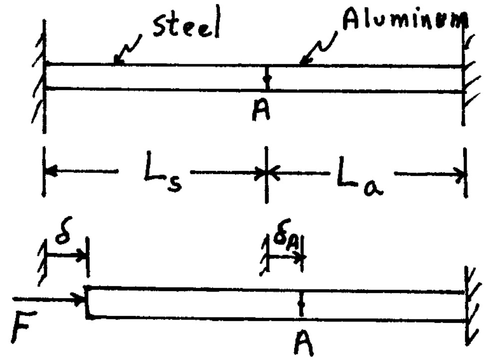


Figure C

$$\delta_A = \frac{F L_a}{E_a A_a} \quad (a)$$

and

$$\delta - \delta_A = \frac{F L_s}{E_s A_s} \quad (b)$$

By Eqs. (a) and (b),

$$\delta = \delta_A + \frac{F L_s}{E_s A_s} = F\left(\frac{L_a}{E_a A_a} + \frac{L_s}{E_s A_s}\right)$$

or

$$F = \frac{\delta E_a A_a E_s A_s}{E_s A_s L_a + E_a A_a L_s}$$

Hence, by Eq. (1.1), the stress in the steel bar is

$$\sigma_s = \frac{F}{A_s} = \frac{\delta E_a A_a E_s}{E_s A_s L_a + E_a A_a L_s}$$

and the stress in the aluminum bar is

$$\sigma_a = \frac{F}{A_a} = \frac{\delta E_a E_s A_s}{E_s A_s L_a + E_a A_a L_s}$$

1.27

(a) Consider the free-body diagram of a cable (Fig. a). By equilibrium,

$$\sum F_y = T - W = 0$$

$$W = AL\rho g, \quad g = \text{acceleration of gravity}$$

$$A = \pi D^2/4$$

Therefore,

$$T = AL\rho g \qquad (a)$$

so the maximum stress in the cable is

$$\sigma_{max} = \frac{T}{A} = L\rho g \qquad (b)$$

For the steel cable, by Eq. (b),

$$\sigma_{max} = \frac{1}{10}\sigma_u = \frac{1030}{10} \text{ MPa} = L(7920)(9.81)$$

or

$$L = 1325.7 \text{ m} \qquad (c)$$

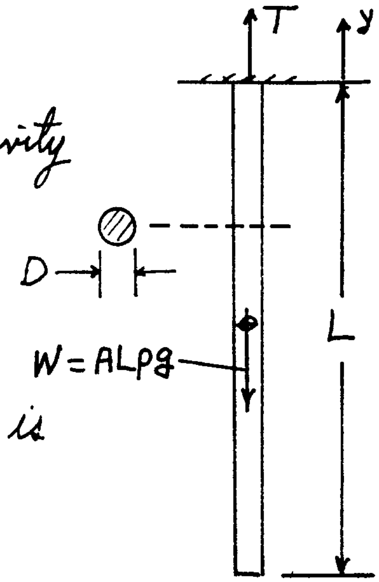


Figure a

Similarly for the aluminum cable

$$\sigma_{max} = \frac{1}{10}\sigma_u = \frac{570}{10} \text{ MPa} = L(2770)(9.81)$$

or

$$L = 2097.6 \text{ m} \qquad (d)$$

Consequently, the aluminum cable can be longer before exceeding a stress greater than $\frac{1}{10}\sigma_u$

(b) For a cable of length L subjected to a load P at its end, the elongation δ is [see Eq. (1.3)]

$$\delta = \frac{\sigma L}{E} \qquad (e)$$

(cont.)