CHAPTER 2
How to Calculate Present Values

Answers to Problem Sets

1. If the discount factor is .507, then $.507*1.12^6 = \$1$

2. 125/139 = .899

3. PV = $374/(1.09)^9 = 172.20$

4. PV = $432/1.15 + 137/(1.15^2) + 797/(1.15^3) = 376 + 104 + 524 = \$1,003$

5. FV = $100*1.15^8 = \$305.90$

6. NPV = -1,548 + 138/.09 = -14.67 (cost today plus the present value of the perpetuity)

7. PV = 4/(.14-.04) = \$40

8. a. PV = 1/.10 = \$10

   b. Since the perpetuity will be worth \$10 in year 7, and since that is roughly double the present value, the approximate PV equals \$5.
   PV = $(1 / .10)/(1.10)^7 = 10/2= \$5$ (approximately)

   c. A perpetuity paying \$1 starting now would be worth \$10, whereas a perpetuity starting in year 8 would be worth roughly \$5. The difference between these cash flows is therefore approximately \$5. PV = 10 – 5= \$5 (approximately)

   d. PV = C/(r-g) = 10,000/(.10-.05) = \$200,000.

9. a. PV = $10,000/(1.05^5) = \$7,835.26$ (assuming the cost of the car does not appreciate over those five years).

   b. You need to set aside (12,000 × 6-year annuity factor) = 12,000 × 4.623 =

$55,476.

c. At the end of 6 years you would have $1.08^6 \times (60{,}476 - 55{,}476) = \$7{,}934$.

10. a. $FV = 1{,}000e^{.12x5} = 1{,}000e^{.6} = \$1{,}822.12$.

b. $PV = 5e^{-.12 \times 8} = 5e^{-.96} = \$1.914$ million

c. $PV = C\,(1/r - 1/re^{rt}) = 2{,}000(1/.12 - 1/.12e^{.12 \times 15}) = \$13{,}912$

11.

a. $FV = 10{,}000{,}000x(1.06)^4 = 12{,}624{,}770$

b. $FV = 10{,}000{,}000x(1 + .06/12)^{(4x12)} = 12{,}704{,}892$

c. $FV = 10{,}000{,}000xe^{(4x.06)} = 12{,}712{,}492$

12.

a. $PV = \$100/1.01^{10} = \$90.53$

b. $PV = \$100/1.13^{10} = \$29.46$

c. $PV = \$100/1.25^{15} = \$\ 3.52$

d. $PV = \$100/1.12 + \$100/1.12^2 + \$100/1.12^3 = \$240.18$

13. a. $DF_1 = \frac{1}{1+r_1} = 0.905 \Rightarrow r_1 = 0.1050 = 10.50\%$

b. $DF_2 = \frac{1}{(1+r_2)^2} = \frac{1}{(1.105)^2} = 0.819$

c. $AF_2 = DF_1 + DF_2 = 0.905 + 0.819 = 1.724$

d. PV of an annuity = C $\times$ [Annuity factor at r% for t years]

Here:

$\$24.65 = \$10 \times [AF_3]$

$AF_3 = 2.465$

e. $AF_3 = DF_1 + DF_2 + DF_3 = AF_2 + DF_3$

$2.465 = 1.724 + DF_3$

$DF_3 = 0.741$

14. The present value of the 10-year stream of cash inflows is:

$$PV = \$170,000 \times \left[\frac{1}{0.14} - \frac{1}{0.14 \times (1.14)^{10}} \right] = \$886,739.66$$

Thus:

NPV = –$800,000 + $886,739.66 = +$86,739.66

At the end of five years, the factory's value will be the present value of the five remaining $170,000 cash flows:

$$PV = \$170,000 \times \left[\frac{1}{0.14} - \frac{1}{0.14 \times (1.14)^{5}} \right] = \$583,623.76$$

15.

$$NPV = \sum_{t=0}^{10} \frac{C_t}{(1.12)^t} = -\$380,000 + \frac{\$50,000}{1.12} + \frac{\$57,000}{1.12^2} + \frac{\$75,000}{1.12^3} + \frac{\$80,000}{1.12^4} + \frac{\$85,000}{1.12^5}$$

$$+ \frac{\$92,000}{1.12^6} + \frac{\$92,000}{1.12^7} + \frac{\$80,000}{1.12^8} + \frac{\$68,000}{1.12^9} + \frac{\$50,000}{1.12^{10}} = \$23,696.15$$

16. a. Let S_t = salary in year t

$$PV = \sum_{t=1}^{30} \frac{40,000\,(1.05)^{t-1}}{(1.08)^t}$$

$$= 40,000 \times \left[\frac{1}{(.08 - .05)} - \frac{(1.05)^{30}}{(.08 - .05) \times (1.08)^{30}} \right] = \$760,662.53$$

b. PV(salary) x 0.05 = $38,033.13

Future value = $38,018.96 x $(1.08)^{30}$ = $382,714.30

c.

$$PV = C \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^t}\right]$$

$$\$382{,}714.30 = C \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{20}}\right]$$

$$C = \$382{,}714.30 \Big/ \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{20}}\right] = \$38{,}980.30$$

17.

Period		Present Value
0		–400,000.00
1	+100,000/1.12 =	+ 89,285.71
2	$+200{,}000/1.12^2$ =	+159,438.78
3	$+300{,}000/1.12^3$ =	+213,534.07
	Total = NPV =	$62,258.56

18. We can break this down into several different cash flows, such that the sum of these separate cash flows is the total cash flow. Then, the sum of the present values of the separate cash flows is the present value of the entire project. (All dollar figures are in millions.)

- Cost of the ship is $8 million
  PV = –$8 million
- Revenue is $5 million per year, operating expenses are $4 million. Thus, operating cash flow is $1 million per year for 15 years.

$$PV = \$1\text{ million} \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{15}}\right] = \$8.559\text{ million}$$

- Major refits cost $2 million each, and will occur at times t = 5 and t = 10.
  PV = (–$2 million)/$1.08^5$ + (–$2 million)/$1.08^{10}$ = –$2.288 million
- Sale for scrap brings in revenue of $1.5 million at t = 15.
  PV = $1.5 million/$1.08^{15}$ = $0.473 million

Adding these present values gives the present value of the entire project:

NPV = –$8 million + $8.559 million – $2.288 million + $0.473 million
NPV = –$1.256 million

19. a. PV = $100,000

b. PV = $180,000/1.12^5 = $102,136.83

c. PV = $11,400/0.12 = $95,000

d. $$PV = \$19{,}000 \times \left[\frac{1}{0.12} - \frac{1}{0.12 \times (1.12)^{10}} \right] = \$107{,}354.24$$

e. PV = $6,500/(0.12 – 0.05) = $92,857.14

Prize (d) is the most valuable because it has the highest present value.

20. Mr. Basset is buying a security worth $20,000 now. That is its present value. The unknown is the annual payment. Using the present value of an annuity formula, we have:

$$PV = C \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^t} \right]$$

$$\$20{,}000 = C \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{12}} \right]$$

$$C = \$20{,}000 \Big/ \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{12}} \right] = \$2{,}653.90$$

21. Assume the Zhangs will put aside the same amount each year. One approach to solving this problem is to find the present value of the cost of the boat and then equate that to the present value of the money saved. From this equation, we can solve for the amount to be put aside each year.

PV(boat) = $20,000/(1.10)^5 = $12,418

$$PV(savings) = \text{Annual savings} \times \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^5} \right]$$

Because PV(savings) must equal PV(boat):

$$\text{Annual savings} \times \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^5} \right] = \$12{,}418$$

$$\text{Annual savings} = \$12{,}418 \Bigg/ \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^5}\right] = \$3{,}276$$

Another approach is to use the future value of an annuity formula:

$$\text{Annual savings} \times \left[\frac{(1+.10)^5 - 1}{.10}\right] = \$20{,}000$$

Annual savings = $ 3,276

22. The fact that Kangaroo Autos is offering "free credit" tells us what the cash payments are; it does not change the fact that money has time value. A 10% annual rate of interest is equivalent to a monthly rate of 0.83%:

$$r_{monthly} = r_{annual} /12 = 0.10/12 = 0.0083 = 0.83\%$$

The present value of the payments to Kangaroo Autos is:

$$\$1{,}000 + \$300 \times \left[\frac{1}{0.0083} - \frac{1}{0.0083 \times (1.0083)^{30}}\right] = \$8{,}938$$

A car from Turtle Motors costs $9,000 cash. Therefore, Kangaroo Autos offers the better deal, i.e., the lower present value of cost.

23. The NPVs are:

$$\text{at } 5\% \quad \Rightarrow \text{NPV} = -\$170{,}000 - \frac{\$100{,}000}{1.05} + \frac{\$320{,}000}{(1.05)^2} = \$25{,}011$$

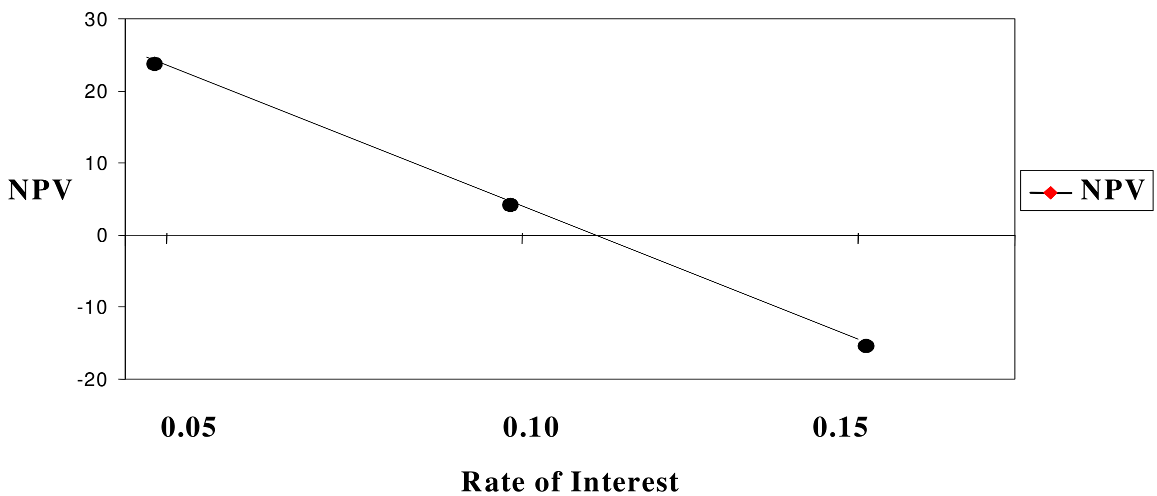
$$\text{at } 10\% \quad \Rightarrow \text{NPV} = -\$170{,}000 - \frac{\$100{,}000}{1.10} + \frac{320{,}000}{(1.10)^2} = \$3{,}554$$

$$\text{at } 15\% \quad \Rightarrow \text{NPV} = -\$170{,}000 - \frac{\$100{,}000}{1.15} + \frac{320{,}000}{(1.15)^2} = -\$14{,}991$$

The figure below shows that the project has zero NPV at about 11%.

As a check, NPV at 11% is:

$$NPV = -\$170,000 - \frac{\$100,000}{1.11} + \frac{320,000}{(1.11)^2} = -\$371$$



24. a. This is the usual perpetuity, and hence:

$$PV = \frac{C}{r} = \frac{\$100}{0.07} = \$1,428.57$$

b. This is worth the PV of stream (a) *plus* the immediate payment of $100:

PV = $100 + $1,428.57 = $1,528.57

c. The continuously compounded equivalent to a 7% annually compounded rate is approximately 6.77%, because:

$e^{0.0677} = 1.0700$

Thus:

$$PV = \frac{C}{r} = \frac{\$100}{0.0677} = \$1,477.10$$

Note that the pattern of payments in part (b) is more valuable than the pattern of payments in part (c). It is preferable to receive cash flows at the start of every year than to spread the receipt of cash evenly over the year; with the former pattern of payment, you receive the cash more quickly.

25. a. PV = \$1 billion/0.08 = \$12.5 billion

b. PV = \$1 billion/(0.08 – 0.04) = \$25.0 billion

c. $$PV = \$1\ \text{billion} \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{20}}\right] = \$9.818\ \text{billion}$$

d. The continuously compounded equivalent to an 8% annually compounded rate is approximately 7.7% , because:

$e^{0.0770} = 1.0800$

Thus:

$$PV = \$1\ \text{billion} \times \left[\frac{1}{0.077} - \frac{1}{0.077 \times e^{(0.077)(20)}}\right] = \$10.203\ \text{billion}$$

This result is greater than the answer in Part (c) because the endowment is now earning interest during the entire year.

26. With annual compounding: FV = \$100 $\times$ $(1.15)^{20}$ = \$1,636.65

With continuous compounding: FV = \$100 $\times$ $e^{(0.15 \times 20)}$ = \$2,008.55

27. One way to approach this problem is to solve for the present value of:

(1) \$100 per year for 10 years, and
(2) \$100 per year in perpetuity, with the first cash flow at year 11.

If this is a fair deal, these present values must be equal, and thus we can solve for the interest rate (r).

The present value of \$100 per year for 10 years is:

$$PV = \$100 \times \left[\frac{1}{r} - \frac{1}{(r) \times (1+r)^{10}}\right]$$

The present value, as of year 10, of \$100 per year forever, with the first payment in year 11, is: PV_{10} = \$100/r

At t = 0, the present value of PV_{10} is:

$$PV = \left[\frac{1}{(1+r)^{10}}\right] \times \left[\frac{\$100}{r}\right]$$

Equating these two expressions for present value, we have:

$$\$100 \times \left[\frac{1}{r} - \frac{1}{(r) \times (1+r)^{10}}\right] = \left[\frac{1}{(1+r)^{10}}\right] \times \left[\frac{\$100}{r}\right]$$

Using trial and error or algebraic solution, we find that r = 7.18%.

28. Assume the amount invested is one dollar.
Let A represent the investment at 12%, compounded annually.
Let B represent the investment at 11.7%, compounded semiannually.
Let C represent the investment at 11.5%, compounded continuously.

After one year:

$FV_A = \$1 \times (1 + 0.12)^1 = \$1.1200$

$FV_B = \$1 \times (1 + 0.0585)^2 = \$1.1204$

$FV_C = \$1 \times e^{(0.115 \times 1)} = \$1.1219$

After five years:

$FV_A = \$1 \times (1 + 0.12)^5 = \$1.7623$

$FV_B = \$1 \times (1 + 0.0585)^{10} = \$1.7657$

$FV_C = \$1 \times e^{(0.115 \times 5)} = \$1.7771$

After twenty years:

$FV_A = \$1 \times (1 + 0.12)^{20} = \$9.6463$

$FV_B = \$1 \times (1 + 0.0585)^{40} = \$9.7193$

$FV_C = \$1 \times e^{(0.115 \times 20)} = \$9.9742$

The preferred investment is C.

29. Because the cash flows occur every six months, we first need to calculate the equivalent semi-annual rate. Thus, $1.08 = (1 + r/2)^2$ => r = 7.85 semi-annually compounded APR. Therefore the rate for six months is 7.85/2 or 3.925%:

$$PV = \$100{,}000 + \$100{,}000 \times \left[\frac{1}{0.03925} - \frac{1}{0.03925 \times (1.03925)^9}\right] = \$846{,}081$$

30. a. Each installment is: $9,420,713/19 = $495,827

$$PV = \$495{,}827 \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{19}}\right] = \$4{,}761{,}724$$

b. If ERC is willing to pay $4.2 million, then:

$$\$4{,}200{,}000 = \$495{,}827 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{19}}\right]$$

Using Excel or a financial calculator, we find that r = 9.81%.

31. a. $$PV = \$70{,}000 \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{8}}\right] = \$402{,}264.73$$

b.

Year	Beginning-of-Year Balance	Year-end Interest on Balance	Total Year-end Payment	Amortization of Loan	End-of-Year Balance
1	402,264.73	32,181.18	70,000.00	37,818.82	364,445.91
2	364,445.91	29,155.67	70,000.00	40,844.33	323,601.58
3	323,601.58	25,888.13	70,000.00	44,111.87	279,489.71
4	279,489.71	22,359.18	70,000.00	47,640.82	231,848.88
5	231,848.88	18,547.91	70,000.00	51,452.09	180,396.79
6	180,396.79	14,431.74	70,000.00	55,568.26	124,828.54
7	124,828.54	9,986.28	70,000.00	60,013.72	64,814.82
8	64,814.82	5,185.19	70,000.00	64,814.81	0.01

32. This is an annuity problem with the present value of the annuity equal to $2 million (as of your retirement date), and the interest rate equal to 8% with 15 time periods. Thus, your annual level of expenditure (C) is determined as follows:

$$PV = C \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{t}}\right]$$

$$\$2{,}000{,}000 = C \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{15}}\right]$$

$$C = \$2{,}000{,}000 \Big/ \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{15}}\right] = \$233{,}659$$

With an inflation rate of 4% per year, we will still accumulate $2 million as of our retirement date. However, because we want to spend a constant amount per year in real terms (R, constant for all t), the nominal amount (C_t) must increase each year. For each year t: $R = C_t / (1 + \text{inflation rate})^t$

Therefore:

PV [all C_t] = PV [all $R \times (1 + \text{inflation rate})^t$] = $2,000,000

$$R \times \left[\frac{(1+0.04)^1}{(1+0.08)^1} + \frac{(1+0.04)^2}{(1+0.08)^2} + \ldots + \frac{(1+0.04)^{15}}{(1+0.08)^{15}} \right] = \$2{,}000{,}000$$

$R \times [0.9630 + 0.9273 + \ . \ . \ . \ + 0.5677] = \$2{,}000{,}000$

$R \times 11.2390 = \$2{,}000{,}000$

$R = \$177{,}952$

Alternatively, consider that the real rate is $\frac{(1+0.08)}{(1+0.04)} - 1 = .03846$. Then, redoing the steps above using the real rate gives a real cash flow equal to:

$$C = \$2{,}000{,}000 \Big/ \left[\frac{1}{0.03846} - \frac{1}{0.03846 \times (1.03846)^{15}} \right] = \$177{,}952$$

Thus $C_1 = (\$177{,}952 \times 1.04) = \$185{,}070$, $C_2 = \$192{,}473$, etc.

33. a. $$PV = \$50{,}000 \times \left[\frac{1}{0.055} - \frac{1}{0.055 \times (1.055)^{12}} \right] = \$430{,}925.89$$

b. The annually compounded rate is 5.5%, so the semiannual rate is:

$(1.055)^{(1/2)} - 1 = 0.0271 = 2.71\%$

Since the payments now arrive six months earlier than previously:

$PV = \$430{,}925.89 \times 1.0271 = \$442{,}603.98$

34. In three years, the balance in the mutual fund will be:

$FV = \$1{,}000{,}000 \times (1.035)^3 = \$1{,}108{,}718$

The monthly shortfall will be: \$15,000 – (\$7,500 + \$1,500) = \$6,000

Annual withdrawals from the mutual fund will be: \$6,000 × 12 = \$72,000

Assume the first annual withdrawal occurs three years from today, when the balance in the mutual fund will be \$1,108,718. Treating the withdrawals as an annuity due, we solve for t as follows:

$$PV = C \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^t} \right] \times (1+r)$$

$$\$1{,}108{,}718 = \$72{,}000 \times \left[\frac{1}{0.035} - \frac{1}{0.035 \times (1.035)^t} \right] \times 1.035$$

Using Excel or a financial calculator, we find that t = 22.5 years.

35. a. PV = 2/.12 = $16.667 million

b. $PV = \$2\times\left[\frac{1}{0.12}-\frac{1}{0.12\times(1.12)^{20}}\right]=\$14.939$ million

c. PV = 2/(.12-.03) = $22.222 million

d. $PV = \$2\times\left[\frac{1}{(0.12-.03)}-\frac{1.03^{20}}{(0.12-.03)\times(1.12)^{20}}\right]=\$18.061$ million

36. a. Using the Rule of 72, the time for money to double at 12% is 72/12, or 6 years. More precisely, if x is the number of years for money to double, then:

$(1.12)^x = 2$

Using logarithms, we find:

x (ln 1.12) = ln 2

x = 6.12 years

b. With continuous compounding for interest rate r and time period x:

$e^{rx} = 2$

Taking the natural logarithm of each side:

r x = ln(2) = 0.693

Thus, if r is expressed as a percent, then x (the time for money to double) is: x = 69.3/(interest rate, in percent).

37. Spreadsheet exercise.

38. a. This calls for the growing perpetuity formula with a negative growth rate (g = –0.04):

$$PV = \frac{\$2\text{ million}}{0.10-(-0.04)} = \frac{\$2\text{ million}}{0.14} = \$14.29\text{ million}$$

b. The pipeline's value at year 20 (i.e., at t = 20), assuming its cash flows last forever, is:

$$PV_{20} = \frac{C_{21}}{r-g} = \frac{C_1(1+g)^{20}}{r-g}$$

With C_1 = \$2 million, g = –0.04, and r = 0.10:

$$PV_{20} = \frac{(\$2\ \text{million}) \times (1-0.04)^{20}}{0.14} = \frac{\$0.884\ \text{million}}{0.14} = \$6.314\ \text{million}$$

Next, we convert this amount to PV today, and subtract it from the answer to Part (a):

$$PV = \$14.29\ \text{million} - \frac{\$6.314\ \text{million}}{(1.10)^{20}} = \$13.35\ \text{million}$$